

Mathematics
Class XII
Sample Paper – 8 Solution

SECTION – A

1. The element '6' lies on 3rd row and 3rd column

So,

$$a_{33} = 6$$

2. $2x + 3y = \cos x$

Differentiating w.r.t. x, we get,

$$\frac{d}{dx}(2x + 3y) = \frac{d}{dx}\cos x$$

$$2 + 3\frac{dy}{dx} = -\sin x$$

$$\frac{dy}{dx} = \frac{-\sin x - 2}{3}$$

3. DE:

$$s^2 \frac{d^2y}{dx^2} + sy \frac{dy}{dx} = s$$

It is nonlinear, since we have product of dependent variable and differential coefficient $y \frac{dy}{dx}$

4. $\frac{x-5}{3} = \frac{y-(-4)}{7} = \frac{z-6}{2}$

Clearly, it passes through (5, -4, 6) and has a direction ratios proportional to 3, 7, 2.

So its vector equation is

$$\vec{r} = (5\hat{i} - 4\hat{j} + 6\hat{k}) + \lambda(3\hat{i} + 7\hat{j} + 2\hat{k})$$

OR

Let θ be the angles between, the given two lines

So, the angle between them given their direction cosines is given by

$$\cos \theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

substituting we get

$$\theta = \cos^{-1} \left(\frac{8}{5\sqrt{3}} \right)$$

SECTION – B

5. The binary operation * on the set {1, 2, 3, 4, 5} is defined by $a * b = \min \{a, b\}$
 The operation table for the given operation * on the given set is as follows

*	1	2	3	4	5
1	1	1	1	1	1
2	1	2	2	2	2
3	1	2	3	3	3
4	1	2	3	4	4
5	1	2	3	4	5

6. We have,

$$\begin{bmatrix} x^2 \\ y^2 \end{bmatrix} - 3 \begin{bmatrix} x \\ 2y \end{bmatrix} = \begin{bmatrix} -2 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} x^2 - 3x \\ y^2 - 6y \end{bmatrix} = \begin{bmatrix} -2 \\ 9 \end{bmatrix}$$

$$x^2 - 3x = -2$$

$$y^2 - 6y = 9$$

$$x^2 - 3x + 2 = 0$$

$$(x-2)(x-1) = 0$$

$$x = 2 \text{ or } x = 1$$

$$y^2 - 6y - 9 = 0$$

$$y = \frac{6 \pm \sqrt{36+36}}{2} = 3 \pm 3\sqrt{2}$$

$$\begin{aligned}
 7. \quad & \int \frac{5x - 2}{1 + 2x + 3x^2} dx \\
 &= 5 \int \frac{x - \frac{2}{5}}{1 + 2x + 3x^2} dx \\
 &= \frac{5}{6} \int \frac{6x - \frac{12}{5}}{1 + 2x + 3x^2} dx \\
 &= \frac{5}{6} \int \frac{6x + 2 - \frac{12}{5} - 2}{1 + 2x + 3x^2} dx \\
 &= \frac{5}{6} \int \frac{6x + 2 - \frac{22}{5}}{1 + 2x + 3x^2} dx \\
 &= \frac{5}{6} \int \frac{6x + 2}{1 + 2x + 3x^2} dx - \frac{5}{6} \times \frac{22}{5} \int \frac{1}{3 \left[\left(x + \frac{1}{3} \right)^2 + \frac{2}{9} \right]} dx \\
 &= \frac{5}{6} \log|1 + 2x + 3x^2| - \frac{11}{9} \int \frac{1}{\left(x + \frac{1}{3} \right)^2 + \frac{2}{9}} dx \\
 &= \frac{5}{6} \log|1 + 2x + 3x^2| - \frac{11}{9} \times \frac{3}{\sqrt{2}} \tan^{-1} \left[\frac{\left(x + \frac{1}{3} \right)}{\frac{\sqrt{2}}{3}} \right] + C \\
 &= \frac{5}{6} \log|1 + 2x + 3x^2| - \frac{11}{3\sqrt{2}} \times \tan^{-1} \left(\frac{3x + 1}{\sqrt{2}} \right) + C
 \end{aligned}$$

8. Let $x^2 = y$

$$\frac{x^2}{x^2 + 4} \cdot \frac{x^2 + 9}{x^2 + 9} = \frac{y}{y+4} \cdot \frac{y+9}{y+9} = \frac{A}{y+4} + \frac{B}{y+9}$$

$$y = A(y+9) + B(y+4)$$

Comparing both sides,

$$A+B=1 \text{ and } 9A+4B=0$$

Solving, we get $A = -\frac{4}{5}$ and $B = \frac{9}{5}$

$$\begin{aligned} \therefore I &= \int \left[\frac{-4}{5x^2 + 4} + \frac{9}{5x^2 + 9} \right] dx \\ &= -\frac{4}{5} \times \frac{1}{2} \tan^{-1}\left(\frac{x}{2}\right) + \frac{9}{5} \times \frac{1}{3} \tan^{-1}\left(\frac{x}{3}\right) + C \\ &= -\frac{2}{5} \tan^{-1}\frac{x}{2} + \frac{3}{5} \tan^{-1}\frac{x}{3} + C \end{aligned}$$

OR

$$\begin{aligned} &\int \frac{(x+3)e^x}{(x+5)^3} dx \\ &= \int \frac{(x+5-2)e^x}{(x+5)^3} dx \\ &= \int \left[\frac{(x+5)}{(x+5)^3} - \frac{2}{(x+5)^3} \right] e^x dx \\ &= \int \left[\frac{1}{(x+5)^2} - \frac{2}{(x+5)^3} \right] e^x dx \end{aligned}$$

This is of the form

$$\begin{aligned} &\int e^x [f(x) + f'(x)] dx = e^x f(x) + C \\ &\Rightarrow \int \left[\frac{1}{(x+5)^2} - \frac{2}{(x+5)^3} \right] e^x dx \\ &= \frac{e^x}{(x+5)^2} + C \end{aligned}$$

9. We have to differentiate it w.r.t. x two times differentiating

$$\frac{dy}{dx} = ab \cos(bx + c) \dots\dots(1)$$

differentiating again

$$\frac{d^2y}{dx^2} = -ab^2 \sin(bx + c) \dots\dots(2)$$

$$\frac{d^2y}{dx^2} = -b^2y \dots\dots\dots\dots \left\{ \because y = a \sin(bx + c) \right\}$$

which is the required differential equation

10. ABCD is a parallelogram with,

$$\vec{AB} = 2\hat{i} - 4\hat{j} + 5\hat{k}; \vec{AD} = \hat{i} - 2\hat{j} - 3\hat{k}$$

Using the parallelogram law of vector addition, diagonal is given by

$$\vec{AC} = \vec{AB} + \vec{AD} = 2\hat{i} - 4\hat{j} + 5\hat{k} + \hat{i} - 2\hat{j} - 3\hat{k} = 3\hat{i} - 6\hat{j} + 2\hat{k}$$

Unit vector parallel to diagonal \vec{AC}

$$\begin{aligned} &= \frac{\vec{AC}}{|\vec{AC}|} = \frac{3\hat{i} - 6\hat{j} + 2\hat{k}}{|3\hat{i} - 6\hat{j} + 2\hat{k}|} = \frac{3\hat{i} - 6\hat{j} + 2\hat{k}}{\sqrt{3^2 + (-6)^2 + (2)^2}} \\ &= \frac{3\hat{i} - 6\hat{j} + 2\hat{k}}{\sqrt{49}} = \frac{1}{7} 3\hat{i} - 6\hat{j} + 2\hat{k} \end{aligned}$$

Area of the parallelogram ABCD = $|\vec{AB} \times \vec{AD}|$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -4 & 5 \\ 1 & -2 & -3 \end{vmatrix}$$

$$= \left| \hat{i}(12 + 10) - \hat{j}(-6 - 5) + \hat{k}(-4 + 4) \right| = \left| \hat{i}(22) - \hat{j}(-11) + \hat{k}(0) \right| = \left| \hat{i}(22) + \hat{j}(11) \right|$$

$$= \sqrt{(22)^2 + (11)^2 + 0^2} = 11\sqrt{5} \text{ sq units}$$

OR

We have $\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$.

$$\text{Now, } \vec{b} + \vec{c} = \hat{i} + 2\hat{j} - 2\hat{k} + 2\hat{i} - \hat{j} + 4\hat{k}$$

$$= 3\hat{i} + \hat{j} + 2\hat{k}$$

$$|\vec{a}| = |2\hat{i} - 2\hat{j} + \hat{k}| = \sqrt{4 + 4 + 1} = 3$$

$$\text{Also, } \vec{a} \cdot (\vec{b} + \vec{c}) = 2\hat{i} - 2\hat{j} + \hat{k} \cdot 3\hat{i} + \hat{j} + 2\hat{k}$$

$$= 6 - 2 + 2 = 6$$

$$\text{So projection of } (\vec{b} + \vec{c}) \text{ on vector } \vec{a} = \frac{\vec{a} \cdot (\vec{b} + \vec{c})}{|\vec{a}|} = \frac{6}{3} = 2 \text{ units}$$

11. This is a case of Bernoulli trials.

$$p = P(\text{Success}) = P(\text{getting a spade in a single draw}) = \frac{13}{52} = \frac{1}{4}$$

$$q = P(\text{Failure}) = 1 - p = 1 - \frac{1}{4} = \frac{3}{4}$$

$$(i) \text{All the four cards are spades} = P(X = 4) = {}^4C_4 p^4 q^0 = \left(\frac{1}{4}\right)^4 = \frac{1}{256}$$

$$(ii) \text{Only 3 cards are spades} = P(X = 3) = {}^4C_3 p^3 q^1 = \frac{12}{256} = \frac{3}{64}$$

$$(iii) \text{None is a spade} = P(X = 0) = {}^4C_0 p^0 q^4 = \left(\frac{3}{4}\right)^4 = \frac{81}{256}$$

$$\begin{aligned}
 12. (i) \sum_{i=0}^7 P(X_i) &= 1 \\
 \Rightarrow [0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k] &= 1 \\
 \Rightarrow 10k^2 + 9k - 1 &= 0 \Rightarrow 10k^2 + 10k - k - 1 = 0 \\
 \Rightarrow 10k(k+1) - (k+1) &= 0 \\
 \Rightarrow (k+1)(10k-1) &= 0 \\
 \Rightarrow k = -1, k = \frac{1}{10} &
 \end{aligned}$$

k , also being a probability cannot be negative

$$\Rightarrow k = \frac{1}{10}$$

$$(ii) P(X < 3) = P(0) + P(1) + P(2) = 0 + k + 2k = 3k = \frac{3}{10}$$

$$(iii) P(X > 6) = P(7) = 7k^2 + k = 7\left(\frac{1}{10}\right)^2 + \left(\frac{1}{10}\right) = \frac{17}{100}$$

$$(iv) P(1 \leq X < 3) = P(1) + P(2) = k + 2k = 3k = \frac{3}{10}$$

OR

Let S denote the success (getting a '6') and F denote the failure (not getting a '6').

$$\text{Thus, } P(S) = \frac{1}{6}; P(F) = \frac{5}{6}$$

$$P(A \text{ wins in 1 throw}) = P(S) = \frac{1}{6}$$

$$P(A \text{ wins in 3 throw}) = P(FFS) = \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}$$

$$P(A \text{ wins in 5 throw}) = P(FFFFS) = \left(\frac{5}{6}\right)^4 \times \frac{1}{6}$$

$$P(A \text{ wins}) = P(S) + P(FFS) + P(FFFFS) = \frac{1}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + \left(\frac{5}{6}\right)^4 \times \frac{1}{6} + \dots$$

$$\begin{aligned}
 &= \frac{\frac{1}{6}}{1 - \frac{25}{36}} = \frac{6}{11}
 \end{aligned}$$

$$P(B \text{ wins}) = 1 - \frac{6}{11} = \frac{5}{11}$$

SECTION – C

13. Given that $A = Q \times Q$ and $(a, b) * (c, d) = (ac, b + ad)$

(i) We know that $a * e = a$ for identity element.

Let, $a = (a_1, a_2)$ and $e = (e_1, e_2)$

$$\Rightarrow a * e = (a_1 e_1, a_1 e_2 + a_2)$$

It should be equals (a_1, a_2)

$$(a_1 e_1, a_1 e_2 + a_2) = (a_1, a_2) \text{ at } e = (1, 0)$$

Hence, $e = (1, 0)$ satisfies condition.

(ii) Condition for invertible element $a * b = b * a = e$

Let, $a(a_1, a_2)$ and $b(x_1, x_2)$

$$a * b = (a_1 x_1, a_1 x_2 + a_2) = (1, 0)$$

This will satisfy when $x_1 = \frac{1}{a_1}$ and $x_2 = \frac{-a_2}{a_1}$

$$\text{Hence, invertible element} = \left(\frac{1}{a_1}, \frac{-a_2}{a_1} \right)$$

OR

$$(a, b) * (c, d) = (ac, ad + b)$$

$$(c, d) * (a, b) = (ca, cb + d)$$

$$(ac, ad + b) \neq (ca, cb + d)$$

So, '*' is not commutative

Let $(a, b), (c, d), (e, f) \in A$, Then

$$((a, b) * (c, d)) * (e, f) = (ac, ad + b) * (e, f) = ((ac)e, (ac)f + (ad + b))$$

$$= (ace, acf + ad + b)$$

$$(a, b) * ((c, d) * (e, f)) = (a, b) * (ce, cf + d) =$$

$$(a(ce), a(cf + d) + b) = (ace, acf + ad + b)$$

$$((a, b) * (c, d)) * (e, f) = (a, b) * ((c, d) * (e, f))$$

Hence, '*' is associative.

Let $(x, y) \in A$. Then (x, y) is an identity element, if and only if

$$(x, y) * (a, b) = (a, b) = (a, b) * (x, y), \text{ for every } (a, b) \in A$$

$$\text{Consider } (x, y) * (a, b) = (xa, xb + y)$$

$$(a, b) * (x, y) = (ax, ay + b)$$

$$(xa, xb + y) = (a, b) = (ax, ay + b)$$

$$ax = x a = a \Rightarrow x = 1$$

$$xb + y = b = ay + b \Rightarrow b + y = b = ay + b \Rightarrow y = 0 = ay \Rightarrow y = 0$$

Therefore, $(1, 0)$ is the identity element

14.

$$\text{Let } y = \cot^{-1} \left(\sqrt{1+x^2} - x \right)$$

$$\text{Let } x = \tan \theta \Rightarrow \theta = \tan^{-1} x$$

$$y = \cot^{-1} \left(\sqrt{1+\tan^2 \theta} - \tan \theta \right)$$

$$y = \cot^{-1} (\sec \theta - \tan \theta)$$

$$y = \cot^{-1} \left(\frac{1}{\cos \theta} - \frac{\sin \theta}{\cos \theta} \right)$$

$$y = \cot^{-1} \left(\frac{1 - \sin \theta}{\cos \theta} \right)$$

$$y = \cot^{-1} \left[\frac{1 - \cos \left(\frac{\pi}{2} - \theta \right)}{\sin \left(\frac{\pi}{2} - \theta \right)} \right]$$

$$y = \cot^{-1} \left[\frac{2 \sin^2 \left(\frac{\pi}{4} - \frac{\theta}{2} \right)}{2 \sin \left(\frac{\pi}{4} - \frac{\theta}{2} \right) \cos \left(\frac{\pi}{4} - \frac{\theta}{2} \right)} \right]$$

$$y = \cot^{-1} \left[\tan \left(\frac{\pi}{4} - \frac{\theta}{2} \right) \right]$$

$$y = \cot^{-1} \left[\cot \left(\frac{\pi}{2} - \frac{\pi}{4} + \frac{\theta}{2} \right) \right]$$

$$y = \frac{\pi}{4} + \frac{\theta}{2}$$

$$\therefore y = \frac{\pi}{4} + \frac{1}{2} \tan^{-1} x.$$

15. Let a, b, c be positive numbers not all are zero.

$$\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

Applying $C_1 \rightarrow C_1 + C_2 + C_3$

$$\Delta = \begin{vmatrix} a+b+c & b & c \\ b+c+a & c & a \\ c+a+b & a & b \end{vmatrix} = \begin{vmatrix} a+b+c & b & c \\ a+b+c & c & a \\ a+b+c & a & b \end{vmatrix}$$

$$\Delta = (a+b+c) \begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix}$$

Applying $R_2 \rightarrow R_2 - R_1; R_3 \rightarrow R_3 - R_1$

$$\begin{aligned} &= (a+b+c) \begin{vmatrix} 1 & b & c \\ 0 & c-b & a-c \\ 0 & a-b & b-c \end{vmatrix} \\ &= -(a+b+c) [a^2 + b^2 + c^2 - ab - bc - ca] \\ &= -\frac{1}{2}(a+b+c) [(a-b)^2 + (b-c)^2 + (c-a)^2] \end{aligned}$$

$$= -\frac{1}{2}(A \text{ positive real number}) [At \text{ least one non zero positive real number}]$$

$$= -\frac{1}{2} \times \text{positive real number}$$

$$= A \text{ negative real number}$$

16. Differentiating both sides of the given relation with respect to x, we get

$$\frac{d}{dx}(\sin x) = \frac{d}{dx}\{x \sin(a+y)\}$$

$$\cos y \frac{dy}{dx} = 1 \times \sin(a+y) + x \cos(a+y) \frac{d}{dx}(a+y)$$

$$\cos y \frac{dy}{dx} = \sin(a+y) + x \cos(a+y) \frac{dy}{dx}$$

$$\{\cos y - x \cos(a+y)\} \frac{dy}{dx} = \sin(a+y)$$

$$\frac{dy}{dx} = \frac{\sin(a+y)}{\cos y - x \cos(a+y)}$$

$$\text{put } x = \frac{\sin y}{\sin(a+y)}$$

$$\frac{dy}{dx} = \frac{\sin(a+y)}{\cos y - \frac{\sin y}{\sin(a+y)} \times \cos(a+y)}$$

$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin(a+y) \cos y - \sin y \times \cos(a+y)}$$

$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin(a+y-y)}$$

$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin(a)}$$

OR

we have

$$y = b \tan^{-1} \left(\frac{x}{a} + \tan^{-1} \frac{y}{x} \right)$$

$$\frac{y}{b} = \tan^{-1} \left(\frac{x}{a} + \tan^{-1} \frac{y}{x} \right)$$

$$\tan \frac{y}{b} = \frac{x}{a} + \tan^{-1} \frac{y}{x}$$

differntiating w.r.t. x

$$\frac{1}{b} \sec^2 \left(\frac{y}{b} \right) \frac{dy}{dx} = \frac{1}{a} + \frac{1}{1 + \left(\frac{y}{x} \right)^2} \times \frac{x \frac{dy}{dx} - y}{x^2}$$

$$\frac{1}{b} \sec^2 \left(\frac{y}{b} \right) \frac{dy}{dx} = \frac{1}{a} + \frac{x \frac{dy}{dx} - y}{x^2 + y^2}$$

$$\frac{d}{dx} \left\{ \frac{1}{b} \sec^2 \left(\frac{y}{b} \right) - \frac{x}{x^2 + y^2} \right\} = \frac{1}{a} - \frac{y}{x^2 + y^2}$$

$$\frac{dy}{dx} = \frac{\frac{1}{a} - \frac{y}{x^2 + y^2}}{\frac{1}{b} \sec^2 \left(\frac{y}{b} \right) - \frac{x}{x^2 + y^2}}$$

17. We have

$$y = \cos^{-1} \sqrt{\frac{\cos 3x}{\cos^3 x}}$$

$$\cos y = \sqrt{\frac{\cos 3x}{\cos^3 x}}$$

$$\cos y = \sqrt{\frac{4\cos^3 x - 3\cos x}{\cos^3 x}}$$

$$\cos y = \sqrt{4 - 3\sec^2 x}$$

$$\cos^2 y = 4 - 3\sec^2 x$$

$$\cos^2 y = 4 - 3(1 + \tan^2 x)$$

$$\cos^2 y = 4 - 3 - 3\tan^2 x$$

$$1 - \cos^2 y = 3\tan^2 x$$

$$\sin^2 y = 3\tan^2 x$$

$$\sin y = \sqrt{3} \tan x$$

differentiating w.r.t. x

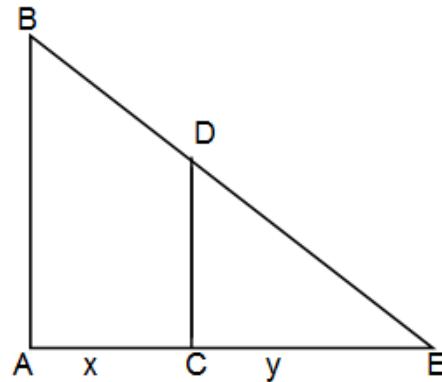
$$\cos y \frac{dy}{dx} = \sqrt{3} \sec^2 x$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sqrt{3}}{\cos y \cos^2 x}$$

$$\text{sub, } \cos y = \sqrt{\frac{\cos 3x}{\cos^3 x}}$$

$$\Rightarrow \frac{dy}{dx} = \sqrt{\frac{3}{\cos x \cos 3x}}$$

18.



Let AB – lamp post

CD – man

$$AC = x$$

$$CE = y$$

$$\text{Given } \frac{dx}{dt} = 5 \text{ km/h}$$

$$\text{To find } \frac{dy}{dt}$$

We have $\Delta ABE \sim \Delta CDE$ (by AA)

$$\Rightarrow \frac{AB}{CD} = \frac{AE}{CE}$$

$$\Rightarrow \frac{6}{2} = \frac{x+y}{y}$$

$$\Rightarrow x = 2y$$

$$\text{Differentiating } \frac{dx}{dt} = 2 \frac{dy}{dt}$$

$$\frac{dy}{dt} = \frac{1}{2} \frac{dx}{dt} = \frac{5}{2} \text{ km/h}$$

19. $\int \frac{dx}{\sin^4 x + \sin^2 x \cos^2 x + \cos^4 x}$

Let $I = \int \frac{dx}{\sin^4 x + \sin^2 x \cos^2 x + \cos^4 x}$

Multiply the numerator and the denominator by $\sec^4 x$, we have

$$I = \int \frac{\sec^4 x dx}{\tan^4 x + \tan^2 x + 1}$$

$$I = \int \frac{\sec^2 x \times \sec^2 x dx}{\tan^4 x + \tan^2 x + 1}$$

We know that $\sec^2 x = 1 + \tan^2 x$

Thus,

$$I = \int \frac{(1 + \tan^2 x) \sec^2 x dx}{\tan^4 x + \tan^2 x + 1}$$

Now substitute $t = \tan x$; $dt = \sec^2 x dx$

Therefore,

$$I = \int \frac{(1+t^2) dt}{1+t^2+t^4}$$

Let us rewrite the integrand as

$$\frac{(1+t^2)}{1+t^2+t^4} = \frac{(1+t^2)}{(t^2-t+1)(t^2+t+1)}$$

Using partial fractions, we have

$$\begin{aligned} \frac{(1+t^2)}{1+t^2+t^4} &= \frac{At+B}{t^2-t+1} + \frac{Ct+D}{t^2+t+1} \\ \Rightarrow \frac{(1+t^2)}{1+t^2+t^4} &= \frac{(At+B)(t^2+t+1) + (Ct+D)(t^2-t+1)}{(t^2-t+1)(t^2+t+1)} \\ \Rightarrow \frac{(1+t^2)}{1+t^2+t^4} &= \frac{At^3 + At^2 + At + Bt^2 + Bt + B + Ct^3 - Ct^2 + Ct + Dt^2 - Dt + D}{(t^2-t+1)(t^2+t+1)} \\ &= \frac{At^3 + At^2 + At + Bt^2 + Bt + B + Ct^3 - Ct^2 + Ct + Dt^2 - Dt + D}{(t^2-t+1)(t^2+t+1)} \end{aligned}$$

$$\Rightarrow \frac{(1+t^2)}{1+t^2+t^4} = \frac{t^3(A+C)+t^2(A+B-C+D)+t(A+B+C-D)+(B+D)}{(t^2-t+1)(t^2+t+1)}$$

So we have,

$$A+C=0; A+B-C+D=1; A+B+C-D=0; B+D=1$$

Solving the above equations, we have

$$A=C=0 \text{ and } B=D=\frac{1}{2}$$

$$\begin{aligned} I &= \int \frac{(1+t^2)dt}{1+t^2+t^4} \\ &= \int \left[\frac{1}{2(t^2-t+1)} + \frac{1}{2(t^2+t+1)} \right] dt \\ &= \int \frac{dt}{2(t^2-t+1)} + \int \frac{dt}{2(t^2+t+1)} \\ &= \frac{1}{2} \int \frac{dt}{t^2-t+1} + \frac{1}{2} \int \frac{dt}{t^2+t+1} \\ &= I_1 + I_2 \end{aligned}$$

$$\text{where, } I_1 = \frac{1}{2} \int \frac{dt}{t^2-t+1} \text{ and } I_2 = \frac{1}{2} \int \frac{dt}{t^2+t+1}$$

Consider I_1 :

$$\begin{aligned} I_1 &= \frac{1}{2} \int \frac{dt}{t^2-t+1} \\ &= \frac{1}{2} \int \frac{dt}{t^2-t+\frac{1}{4}+1-\frac{1}{4}} \\ &= \frac{1}{2} \int \frac{dt}{\left(t-\frac{1}{2}\right)^2 + \frac{3}{4}} \\ &= \frac{1}{2} \times \frac{1}{\sqrt{\frac{3}{4}}} \tan^{-1} \left(\frac{t-\frac{1}{2}}{\sqrt{\frac{3}{4}}} \right) \\ &= \frac{1}{\sqrt{3}} \tan^{-1} \frac{2t-1}{\sqrt{3}} \\ &= \frac{1}{\sqrt{3}} \tan^{-1} \frac{2\tan x - 1}{\sqrt{3}} \end{aligned}$$

Similarly,

Consider I_2 :

$$\begin{aligned}
 I_2 &= \frac{1}{2} \int \frac{dt}{t^2 + t + 1} \\
 &= \frac{1}{2} \int \frac{dt}{t^2 + t + \frac{1}{4} + 1 - \frac{1}{4}} \\
 &= \frac{1}{2} \int \frac{dt}{\left(t + \frac{1}{2}\right)^2 + \frac{3}{4}} \\
 &= \frac{1}{2} \times \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{t + \frac{1}{2}}{\sqrt{\frac{3}{4}}} \right) \\
 &= \frac{1}{\sqrt{3}} \tan^{-1} \frac{2t + 1}{\sqrt{3}} \\
 &= \frac{1}{\sqrt{3}} \tan^{-1} \frac{2\tan x + 1}{\sqrt{3}}
 \end{aligned}$$

Thus, $I = I_1 + I_2$

$$\begin{aligned}
 \Rightarrow I &= \frac{1}{\sqrt{3}} \tan^{-1} \frac{2\tan x - 1}{\sqrt{3}} + \frac{1}{\sqrt{3}} \tan^{-1} \frac{2\tan x + 1}{\sqrt{3}} \\
 I &= \frac{1}{\sqrt{3}} \left[\tan^{-1} \frac{2\tan x - 1}{\sqrt{3}} + \tan^{-1} \frac{2\tan x + 1}{\sqrt{3}} \right] + C
 \end{aligned}$$

20. $\int_{-1}^2 (7x - 5) dx;$

$$a = -1, b = 2; h = \frac{2+1}{n} \Rightarrow nh = 3, f(x) = 7x - 5$$

$$\lim_{h \rightarrow 0} h \left[f(-1) + f(-1+h) + f(-1+2h) + \dots + f(-1+n-1)h \right] \quad (i)$$

$$= f(-1) = -7 - 5 = -12; f(-1+h) = 7(-1+h) - 5 = 7h - 12$$

$$= f(-1+n-1)h = 7\{-1+n-1\}h - 5 = 7(n-1)h - 12$$

Substituting in (i)

$$\int_{-1}^2 (7x - 5) dx = \lim_{h \rightarrow 0} h \left[(-12) + (7h - 12) + (14h - 12) + \dots + \{7(n-1)h - 12\} \right]$$

$$= \lim_{h \rightarrow 0} h \left[7h(1+2+\dots+n-1) - 12n \right] = \lim_{h \rightarrow 0} h \left[7h \frac{(n-1)n}{2} - 12n \right]$$

$$= \lim_{h \rightarrow 0} \left[\frac{7}{2}(nh)(nh-h) - 12nh \right] = \lim_{h \rightarrow 0} \left[\frac{7}{2}(3)(3-h) - 36 \right]$$

$$= \frac{7}{2} \times 9 - 36 = \frac{63}{2} - 36 = -\frac{9}{2}$$

21. The given D.E. can be written as

$$\frac{dy}{dx} = \frac{1}{\cos(x+y)}$$

let $x+y=v$. then,

$$1 + \frac{dy}{dx} = \frac{dv}{dx}$$

D.E. becomes

$$\Rightarrow \frac{dv}{dx} - 1 = \frac{1}{\cos(v)}$$

$$\Rightarrow \frac{dv}{dx} = \frac{1 + \cos v}{\cos v}$$

$$\Rightarrow \int \frac{\cos v}{1 + \cos v} dv = \int dx$$

$$\Rightarrow \int \frac{\cos v(1 - \cos v)}{1 - \cos^2 v} dv = \int dx$$

$$\Rightarrow \int \frac{\cos v - \cos^2 v}{\sin^2 v} dv = \int dx$$

$$\Rightarrow \int \cot v \operatorname{cosec} v dv - \int \cot^2 v dv = x + c$$

$$\Rightarrow \int \cot v \operatorname{cosec} v dv - \int \operatorname{cosec}^2 v dv + \int dv = x + c$$

$$\Rightarrow -\operatorname{cosec} v + \cot v + v = x + c$$

$$\Rightarrow -\operatorname{cosec}(x+y) + \cot(x+y) + (x+y) = x + c$$

$$\Rightarrow -\frac{1 - \cos(x+y)}{\sin(x+y)} + y = c$$

$$\Rightarrow -\frac{2 \sin^2 \frac{x+y}{2}}{2 \sin \frac{x+y}{2} \cos \frac{x+y}{2}} + y = c$$

$$\Rightarrow -\tan\left(\frac{x+y}{2}\right) + y = c$$

$$\Rightarrow -\operatorname{cosec}(x+y) + \cot(x+y) + (x+y) = x + c$$

$$\Rightarrow -\frac{1 - \cos(x+y)}{\sin(x+y)} + y = c$$

$$\Rightarrow -\tan\left(\frac{x+y}{2}\right) + y = c$$

given $y = 0$ when $x = 0$.

$$\Rightarrow 0 = c$$

so,

$$y = \tan\left(\frac{x+y}{2}\right)$$

as required.

OR

$$(x+y)^2 \frac{dy}{dx} = a^2$$

let $x+y=v$

$$1 + \frac{dy}{dx} = \frac{dv}{dx}$$

so,

$$v^2 \left(\frac{dv}{dx} - 1 \right) = a^2$$

$$\Rightarrow v^2 \frac{dv}{dx} - v^2 = a^2$$

$$\Rightarrow v^2 \frac{dv}{dx} = v^2 + a^2$$

$$\Rightarrow \frac{v^2}{v^2 + a^2} dv = dx$$

$$\Rightarrow \int \frac{v^2}{v^2 + a^2} dv = \int dx$$

$$\Rightarrow \int \left(1 - \frac{a^2}{v^2 + a^2} \right) dv = \int dx$$

$$\Rightarrow v - a \tan^{-1} \frac{v}{a} = x + c$$

$$\Rightarrow (x+y) - a \tan^{-1} \left(\frac{x+y}{a} \right) = x + c$$

as required

22.

a) Given $\vec{\alpha} = 3\hat{i} - \hat{j}$, $\vec{\beta} = 2\hat{i} + \hat{j} - 3\hat{k}$

Since $\vec{\beta}_1 \parallel \vec{\alpha}$: let $\vec{\beta}_1 = \lambda \vec{\alpha} = 3\lambda \hat{i} - \lambda \hat{j}$

Now

$$\vec{\beta}_2 = \vec{\beta} - \vec{\beta}_1 = (2\hat{i} + \hat{j} - 3\hat{k}) - ((3\lambda)\hat{i} - \lambda\hat{j}) = (2 - 3\lambda)\hat{i} + (1 + \lambda)\hat{j} - 3\hat{k}$$

since $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$

we get $\vec{\alpha} \cdot \vec{\beta}_2 = 0$

$$\Rightarrow 3(2 - 3\lambda) - (1 + \lambda) = 0$$

$$\Rightarrow \lambda = \frac{1}{2}$$

Hence $\vec{\beta}_1 = \frac{3}{2}\hat{i} - \frac{1}{2}\hat{j}$ and $\vec{\beta}_2 = \frac{1}{2}\hat{i} + \frac{3}{2}\hat{j} - 3\hat{k}$

b) Since each of the vectors is perpendicular to the sum of other two

$$\Rightarrow \vec{a} \cdot (\vec{b} + \vec{c}) = 0, \quad \vec{b} \cdot (\vec{c} + \vec{a}) = 0, \quad \vec{c} \cdot (\vec{a} + \vec{b}) = 0$$

$$\text{Now } |\vec{a} + \vec{b} + \vec{c}|^2 = (\vec{a} + \vec{b} + \vec{c}) \cdot (\vec{a} + \vec{b} + \vec{c})$$

$$= \vec{a} \cdot \vec{a} + \vec{a} \cdot (\vec{b} + \vec{c}) + \vec{b} \cdot \vec{b} + \vec{b} \cdot (\vec{c} + \vec{a}) + \vec{c} \cdot \vec{c} + \vec{c} \cdot (\vec{a} + \vec{b})$$

$$= |\vec{a}|^2 + 0 + |\vec{b}|^2 + 0 + |\vec{c}|^2 + 0$$

$$= 3^2 + 4^2 + 5^2 = 50 \quad \text{Hence } |\vec{a} + \vec{b} + \vec{c}| = 5\sqrt{2}$$

23.

The equation of the given line is $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{2}$ (1)

Any point on the given line is $(3\lambda + 2, 4\lambda - 1, 2\lambda + 2)$.

If this point lies on the given plane $x - y + z - 5 = 0$, then

$$3\lambda + 2 - (4\lambda - 1) + 2\lambda + 2 - 5 = 0$$

$$\Rightarrow \lambda = 0$$

Putting $\lambda = 0$ in $(3\lambda + 2, 4\lambda - 1, 2\lambda + 2)$, we get the point of intersection of the given line and the plane is $(2, -1, 2)$.

Let θ be the angle between the given line and the plane.

$$\therefore \sin \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} = \frac{(3\vec{i} + 4\vec{j} + 2\vec{k}) \cdot (\vec{i} - \vec{j} + \vec{k})}{\sqrt{3^2 + 4^2 + 2^2} \sqrt{1^2 + 1^2 + 1^2}} = \frac{3 - 4 + 2}{\sqrt{29} \sqrt{3}} = \frac{1}{\sqrt{87}}$$

$$\Rightarrow \theta = \sin^{-1} \left(\frac{1}{\sqrt{87}} \right)$$

Thus, the angle between the given line and the given plane is $\sin^{-1} \left(\frac{1}{\sqrt{87}} \right)$

SECTION - D
24.

We have $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 2 \\ 2 & 0 & 3 \end{bmatrix}$ and $f(x) = x^3 - 6x^2 + 7x + 2$

$$\therefore f(A) = A^3 - 6A^2 + 7A + 2I$$

Now,

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 2 \\ 2 & 0 & 3 \end{bmatrix}$$

$$A \cdot A = A^2 = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 2 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 2 \\ 2 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 8 \\ 4 & 4 & 10 \\ 8 & 0 & 13 \end{bmatrix}$$

$$A^2 \cdot A = A^3 = \begin{bmatrix} 5 & 0 & 8 \\ 4 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 2 \\ 2 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 21 & 0 & 34 \\ 24 & 8 & 46 \\ 34 & 0 & 55 \end{bmatrix}$$

$$f(A) = A^3 - 6A^2 + 7A + 2I$$

$$= \begin{bmatrix} 21 & 0 & 34 \\ 24 & 8 & 46 \\ 34 & 0 & 55 \end{bmatrix} - 6 \begin{bmatrix} 5 & 0 & 8 \\ 4 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 2 \\ 2 & 0 & 3 \end{bmatrix} + 2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

Hence, A is the root of the polynomials $f(x) = x^3 - 6x^2 + 7x + 2$.

OR

$$A = \begin{bmatrix} -1 & -1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$AB = \begin{bmatrix} -1 & -1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$AB = \begin{bmatrix} -1 & -3 & -3 \\ 1 & 0 & 0 \\ 6 & 11 & 6 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & -1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$$

$$BA = \begin{bmatrix} 5 & 6 & 14 \\ 0 & -1 & 1 \\ -1 & -2 & 1 \end{bmatrix}$$

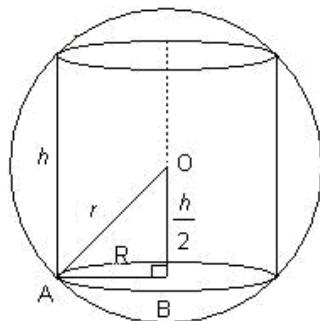
thus on comparing we get $AB \neq BA$

- 25.** The given sphere is of radius r . Let h be the height and R be the radius of the cylinder inscribed in the sphere.

Volume of cylinder

$$V = \pi R^2 h \quad \dots(1)$$

In right ΔOBA



$$AB^2 + OB^2 = OA^2$$

$$R^2 + \frac{h^2}{4} = r^2$$

$$\text{So, } R^2 = r^2 - \frac{h^2}{4}$$

Putting the value of R^2 in equation (1), we get

$$V = \pi \left(r^2 - \frac{h^2}{4} \right) \cdot h$$

$$V = \pi \left(r^2 h - \frac{h^3}{4} \right) \quad \dots(3)$$

$$\therefore \frac{dV}{dh} = \pi \left(r^2 - \frac{3h^2}{4} \right) \quad \dots(4)$$

For stationary point, $\frac{dV}{dh} = 0$

$$\pi \left(r^2 - \frac{3h^2}{4} \right) = 0$$

$$r^2 = \frac{3h^2}{4} \Rightarrow h^2 = \frac{4r^2}{3} \quad \Rightarrow h = \frac{2r}{\sqrt{3}}$$

Now $\frac{d^2V}{dh^2} = \pi \left(-\frac{6}{4} h \right)$

$$\therefore \left[\frac{d^2V}{dh^2} \right]_{at \ h=\frac{2r}{\sqrt{3}}} = \pi \left(-\frac{3}{2} \cdot \frac{2r}{\sqrt{3}} \right) < 0$$

\therefore Volume is maximum at $h = \frac{2r}{\sqrt{3}}$

Maximum volume is

$$\begin{aligned}
 &= \pi \left(r^2 \cdot \frac{2r}{\sqrt{3}} - \frac{1}{4} \cdot \frac{8r^3}{3\sqrt{3}} \right) \\
 &= \pi \left(\frac{2r^3}{\sqrt{3}} - \frac{2r^3}{3\sqrt{3}} \right) \\
 &= \pi \left(\frac{6r^3 - 2r^3}{3\sqrt{3}} \right) \\
 &= \frac{4\pi r^3}{3\sqrt{3}} \text{ cu. unit}
 \end{aligned}$$

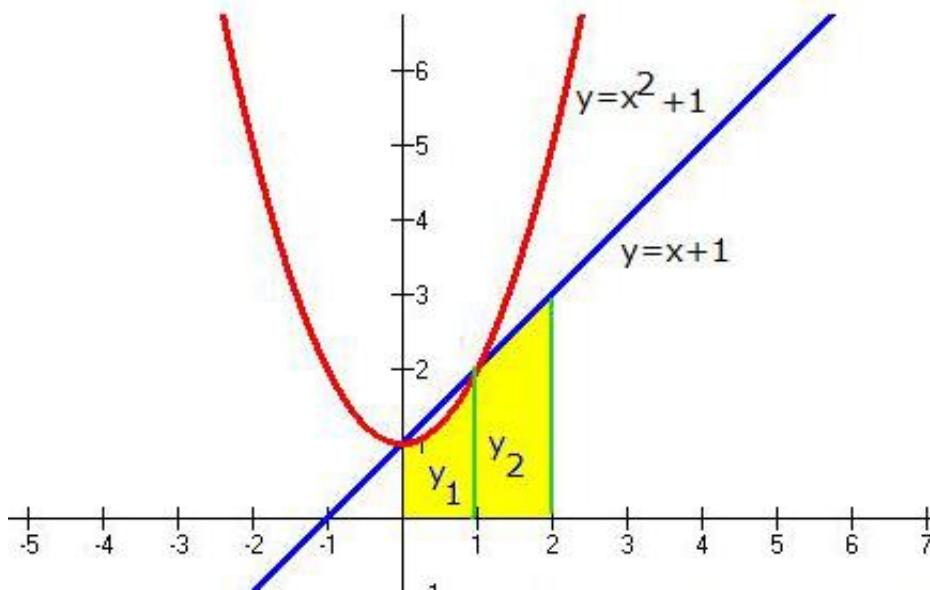
26. Points of intersection of $y = x^2 + 1$, $y = x + 1$

$$x^2 + 1 = x + 1$$

$$\Rightarrow x(x - 1) = 0$$

$$\Rightarrow x = 0, 1$$

So points of intersection are P(0, 1) and Q(1, 2). The graph is represented as



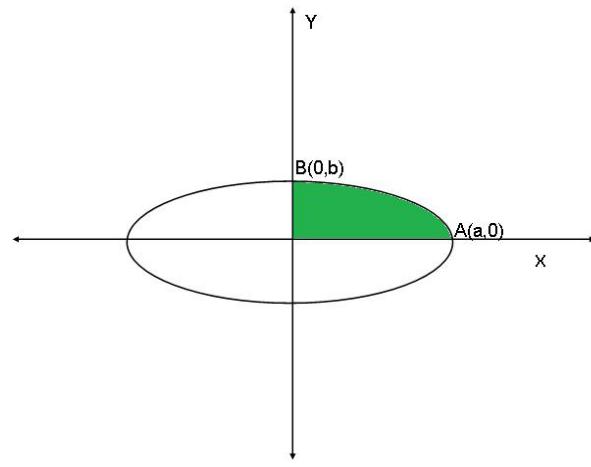
Required area is given by

$$A = \int_0^1 y_1 dx + \int_1^2 y_2 dx,$$

Where y_1 and y_2 represent the y co-ordinate of the parabola and straight line respectively.

$$\begin{aligned} \therefore A &= \int_0^1 (x^2 + 1) dx + \int_1^2 (x + 1) dx \\ &= \left(\frac{x^3}{3} + x \right) \Big|_0^1 + \left(\frac{x^2}{2} + x \right) \Big|_1^2 \\ &= \left[\left(\frac{1}{3} + 1 \right) - 0 \right] + \left[(2 + 2) - \left(\frac{1}{2} + 1 \right) \right] = \frac{23}{6} \text{ sq. units} \end{aligned}$$

OR



(i) between the curves $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, and the x-axis between $x = 0$ to $x = a$

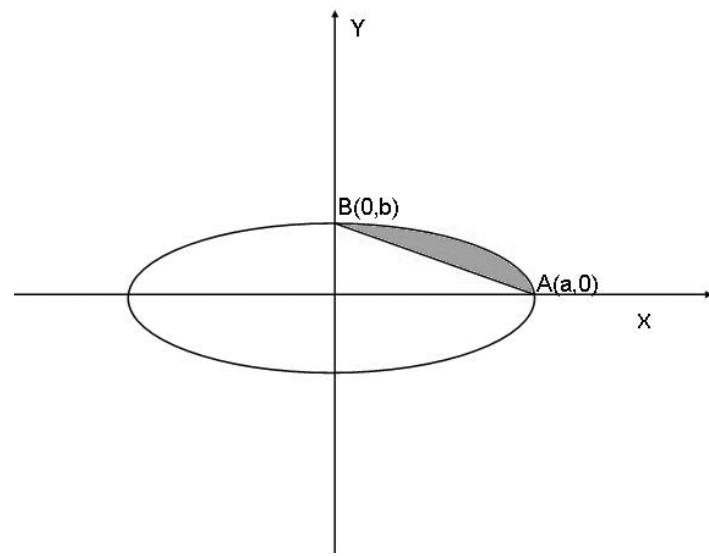
$$\int_0^a b \sqrt{1 - \frac{x^2}{a^2}} dx = \frac{b}{a} \int_0^a \sqrt{a^2 - x^2} dx$$

$$= \frac{b}{a} \left[\frac{x\sqrt{a^2 - x^2}}{2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} \right]_0^a$$

$$= \frac{b}{2a} \left[\left(0 + a^2 \sin^{-1}(1) \right) - \left(0 + a^2 \sin^{-1}(0) \right) \right]$$

$$= \frac{b}{2a} \left[a^2 \times \frac{\pi}{2} \right]$$

$$\therefore \int_0^a b \sqrt{1 - \frac{x^2}{a^2}} dx = \frac{1}{4} \pi ab$$



(ii) Area of triangle AOB is in the first quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$,

where OA = a and OB = b.

=the area enclosed between the chord AB and the arc AB of the ellipse .

$$\begin{aligned}
 &= \text{Area of Ellipse (In quadrant I)} - \text{Area of } \Delta AOB \\
 &= \int_0^a b \sqrt{1 - \frac{x^2}{a^2}} dx - \frac{1}{2} ab = \frac{1}{4} \pi ab - \frac{1}{2} ab \\
 &= \frac{(\pi - 2)ab}{4}
 \end{aligned}$$

$$\text{(iii) Ratio} = \frac{\frac{1}{4} \pi ab}{\frac{(\pi - 2)}{4} ab} = \frac{\pi}{\pi - 2}$$

27. Let the equation of plane be $ax + by + cz + d = 0 \dots (1)$

Since the plane passes through the point A (0, 0, 0) and B(3, -1, 2), we have
 $a \times 0 + b \times 0 + c \times 0 + d = 0$

$$\Rightarrow d = 0 \dots (2)$$

Similarly for point B (3, -1, 2), $a \times 3 + b \times (-1) + c \times 2 + d = 0$

$$3a - b + 2c = 0 \quad (\text{Using, } d = 0) \dots (3)$$

$$\text{Given equation of the line is } \frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$$

$$\text{We can also write the above equation as } \frac{x-4}{1} = \frac{y-(-3)}{-4} = \frac{x-(-1)}{7}$$

The required plane is parallel to the above line.

$$\text{Therefore, } a \times 1 + b \times (-4) + c \times 7 = 0$$

$$\Rightarrow a - 4b + 7c = 0 \dots (4)$$

Cross multiplying equations (3) and (4), we obtain:

$$\frac{a}{(-1) \times 7 - (-4) \times 2} = \frac{b}{2 \times 1 - 3 \times 7} = \frac{c}{3 \times (-4) - 1 \times (-1)}$$

$$\Rightarrow \frac{a}{-7+8} = \frac{b}{2-21} = \frac{c}{-12+1}$$

$$\Rightarrow \frac{a}{1} = \frac{b}{-19} = \frac{c}{-11} = k$$

$$\Rightarrow a = k, b = -19k, c = -11k$$

Substituting the values of a, b and c in equation (1), we obtain the equation of plane as:

$$kx - 19ky - 11kz + d = 0$$

$$\Rightarrow k(x - 19y - 11z) = 0 \quad (\text{From equation (2)})$$

$$\Rightarrow x - 19y - 11z = 0$$

So, the equation of the required plane is $x - 19y - 11z = 0$

OR

We know that, equation of a line passing through x_1, y_1, z_1 with direction ratios a, b, c is given by $\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$

So, the required equation of a line passing through $(-1, 3, -2)$ is:

$$\frac{x+1}{a} = \frac{y-3}{b} = \frac{z+2}{c} \quad \dots \dots \dots (1)$$

Given that line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ is perpendicular to line (1), so

$$a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$$

$$a \cdot 1 + b \cdot 2 + c \cdot 3 = 0$$

$$a + 2b + 3c = 0 \quad \dots \dots \dots 2$$

And line $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$ is perpendicular to line 1, so

$$a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$$

$$a \cdot -3 + b \cdot 2 + c \cdot 5 = 0$$

$$-3a + 2b + 5c = 0 \quad \dots \dots \dots 3$$

Solving equation 2 and 3 by cross multiplication,

$$\frac{a}{(2)(5) - (2)(3)} = \frac{b}{(-3)(3) - (1)(5)} = \frac{c}{(1)(2) - (2)(-3)}$$

$$\Rightarrow \frac{a}{10 - 6} = \frac{b}{-9 - 5} = \frac{c}{2 + 6}$$

$$\Rightarrow \frac{a}{4} = \frac{b}{-14} = \frac{c}{8}$$

$$\Rightarrow \frac{a}{2} = \frac{b}{-7} = \frac{c}{4} = \lambda \text{ (Say)}$$

$$\Rightarrow a = 2\lambda, b = -7\lambda, c = 4\lambda$$

Putting the value of a, b, and c in (1) gives

$$\frac{x+1}{2\lambda} = \frac{y-3}{-7\lambda} = \frac{z+2}{4\lambda}$$

$$\Rightarrow \frac{x+1}{2} = \frac{y-3}{-7} = \frac{z+2}{4}$$

- 28.** Suppose x is the number of pieces of Model A and y is the number of pieces of Model B. Then

Total profit (in Rs.) = $8000x + 12000y$

Let $Z = 8000x + 12000y$

Mathematical model for the given problem is as follows:

Maximise $Z = 8000x + 12000y \dots (1)$

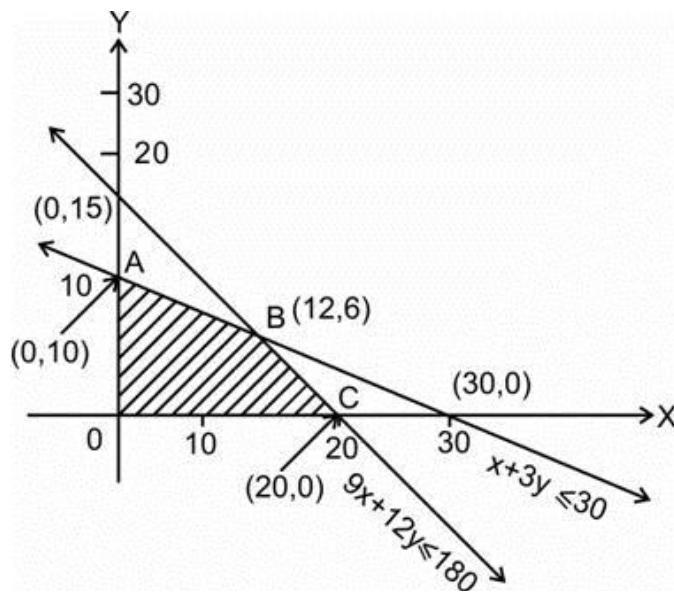
subject to the constraints,

$9x + 12y \leq 180$ (Fabrication constraint) i.e. $3x + 4y \leq 60 \dots (2)$

$x + 3y \leq 30$ (Finishing constraint) $\dots (3)$

$x \geq 0, y \geq 0 \dots (4)$

The feasible region (shaded) OABC determined by the linear inequalities (2) to (4) is shown below.



Corner Point	$Z = 8000x + 12000y$
Corner Point	$Z = 8000x + 12000y$
A(0, 10)	120000
B(12, 6)	168000 Maximum
C(20, 0)	16000

The company should produce 12 pieces of Model A and 6 pieces of Model B to realise maximum profit and maximum profit then will be Rs. 1,68,000.

29.

Let E_1 be the event that a red ball is transferred from bag A to bag B

Let E_2 be the event that a black ball is transferred from bag A to bag B

$\therefore E_1$ and E_2 are mutually exclusive and exhaustive.

$$P(E_1) = 3/7 ; P(E_2) = 4/7$$

Let E be the event that a red ball is drawn from bag B

$$P(E|E_1) = \frac{4+1}{(4+1)+5} = \frac{5}{10} = \frac{1}{2}$$

$$P(E|E_2) = \frac{3+1}{(5+1)+4} = \frac{4}{10} = \frac{2}{5}$$

$$\therefore \text{Required probability} = P(E_2|E) = \frac{P(E|E_2)P(E_2)}{P(E|E_1)P(E_1) + P(E|E_2)P(E_2)}$$

$$= \frac{\frac{4}{10} \times \frac{4}{7}}{\frac{1}{2} \times \frac{3}{7} + \frac{4}{10} \times \frac{4}{7}} = \frac{\frac{16}{70}}{\frac{3}{14} + \frac{16}{70}} = \frac{\frac{16}{70}}{\frac{31}{70}} = \frac{16}{31}$$

$$\therefore \text{Required probability} = P(E_1|E) = \frac{P(E|E_1)P(E_1)}{P(E|E_1)P(E_1) + P(E|E_2)P(E_2)}$$

$$= \frac{\frac{1}{2} \times \frac{3}{7}}{\frac{1}{2} \times \frac{3}{7} + \frac{4}{10} \times \frac{4}{7}} = \frac{\frac{3}{14}}{\frac{3}{14} + \frac{16}{70}} = \frac{\frac{3}{14}}{\frac{31}{70}} = \frac{15}{31}$$