**MEGHALAYA**
**Class XII Mathematics**
**Sample Paper-1**

**Time allowed: 3 hours** **Maximum Marks: 100**

**General Instructions:**

i. Write all the answers in the Answer Scripts.
ii. The question paper consists of four Sections – A, B,C and D.
iii. Section – A consists of 15 questions, carrying 2 marks each.
iv. Section – B consists of 5 questions, carrying 4 marks each, out of which 2 questions have internal choices.
v. Section – C consists of 10 questions, carrying 4 marks each.
Question No.s 21 to 25 are to be answered by both Regular and Private Candidates
Question No.s 26 to 30 are to be answered by Elementary School Teacher Candidates only.
N. B. : Regular and Private Candidates are not to attempt Question No.s 26 to 30.
Elementary School Teacher Candidates are not to attempt Question No.s 21 to 25.
vi. Section – D has 5 questions carrying 6 marks each, out of which two questions have Internal choices.

## Section A

**1.** **a)** If f is a function from A to B, where A = {p, q , r } and B = {4, 5, 6}, such that f = {(p, 6), (q , 5 ), (r, 4) }, find $f^{-1}$ 1

**b)** If A and B are symmetric matrices then prove that (AB – BA) is a skew symmetric matrix. 1

**2. a)** If $\sin^{-1}\left(\frac{3}{5}\right) = x$, find cosx 1

**b)** Find the angle between the curves $y^2 = x$ and $x^2 = y$ at (1, 1). 1

**3. a)** Find the projection of vector $2\hat{i} + \hat{j}$ on the vector $\hat{i} + 2\hat{j}$. 1

**b)** For what values of x is the following matrix singular

$$A = \begin{bmatrix} 3-2x & x+1 \\ 2 & 4 \end{bmatrix}$$ 1

**4. a)** Find the principal values of $\tan^{-1}(-1)$ 1

**b)** Show that the relation R, in set of real numbers defined as $R = \{(a,\ b) : a \le b\}$, is transitive. 2

**5.** Determine the value of constant k, so that the function

$$f(x) = kx^2 \qquad x \le 2$$
$$= 3 \qquad x > 2$$

Is continuous at x=2

**6.** Find the value of $\tan(\cos^{-1}\frac{4}{5} + \tan^{-1}\frac{2}{3})$ 2

**7.** If f, g: R → R be functions defined as f(x) = 2x – 6, g(x) = $x^2$ + 3. Find gof (4). 2

**8.** Find whether $y = \frac{a}{x} + b$, is a solution of $\frac{d^2y}{d^2x} + \frac{2}{x}\left(\frac{dy}{dx}\right) = 0$ 2

**9.** This 3 x 2 matrix gives information about the number of men and women workers in three factories I, II and III who lost their jobs in the last 2 months. What do you infer from the entry in the third row and second column of this matrix? 2

| | Men workers | Women workers |
|---|---|---|
| Factory I | 40 | 15 |
| Factory II | 35 | 40 |
| Factory III | 72 | 64 |

**10.** Find the equation of a line through (-2, 1, 3) and parallel to $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$. 2

**11.** If $A' = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$, then find (A + 2B)'. 2

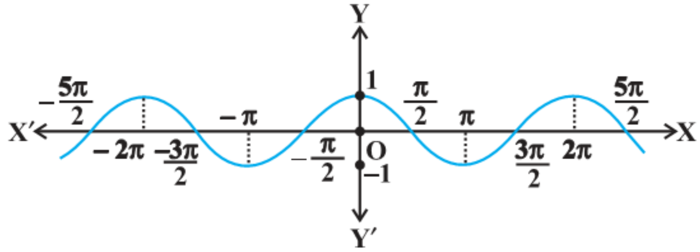
**12.** Without expanding, find the value of the following determinant

$$\begin{vmatrix} 0 & q-r & r-s \\ r-q & 0 & p-q \\ s-r & q-p & 0 \end{vmatrix}$$

2

**13.** Find the number of all possible matrices of order 3 × 3 with each entry 0 or 1. 2

**14.** From the graph of y = cos x, identify the intervals of x at which the function can be inverted. 2



y = cos x

**15.** Evaluate: $\int \tan^{-1}(\cot x)\, dx$ 2

**Section B**

**16.** If $\hat{a}$ and $\hat{b}$ are two unit vectors and $\theta$ is the angle between them, show that $\sin\frac{\theta}{2} = \frac{1}{2}|\hat{a} - \hat{b}|$.

**17.** Evaluate the integral $\int_0^1 \frac{dx}{\sqrt{1+x} - \sqrt{x}}$

**OR**

Evaluate: $\int_0^{\pi/2} \sin 2x \tan^{-1}(\sin x) dx$

**18.** Differentiate $\frac{x^3\sqrt{5+x}}{(7-3x)^5\sqrt[3]{8+5x}}$, wrt x

**19. a)** Find the value of '$\alpha$' for which $\alpha(\hat{i} + \hat{j} + \hat{k})$ is a unit vector. 2

**b)** Find the slope of the tangent to the curve $y = x^3 - x + 1$ at the point where the curve cuts the y-axis. 2

**20.** ABCD is a parallelogram with $\overrightarrow{AB} = 2\hat{i} - 4\hat{j} + 5\hat{k}; \overrightarrow{AD} = \hat{i} - 2\hat{j} - 3\hat{k}$

Find a unit vector parallel to its diagonal $\overrightarrow{AC}$. Also, find the area of the parallelogram ABCD

**OR**

Find the projection of $(\vec{b} + \vec{c})$ on $\vec{a}$, where $\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$.

**Section C**

**21.** Discuss the applicability of Lagrange's mean value theorem for the function:

$f(x) = |\sin x|$ in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

**22.** Solve the differential equation

$$(xdy - ydx)y \sin\left(\frac{y}{x}\right) = (ydx + xdy)x\cos\left(\frac{y}{x}\right)$$

**23.** If $u = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ and $v = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, where $-1 < x < 1$, then write the value of $\frac{du}{dv}$.

**24.** Using properties of determinants, prove that

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = \left(1+a^2+b^2\right)^3$$

**25.** The scalar product of the vector $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ with a unit vector along the sum of vectors $\vec{b}=2\hat{i}+4\hat{j}-5\hat{k}$ and $\vec{c}=\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to one. Find the value of $\lambda$ and hence find the unit vector along $\vec{b}+\vec{c}$.

**(Only for Elementary School Teacher Candidates in lieu of Question No.s 21 to 25)**

**26.** State if whether the following are True or False? 1×4

**a)** Cot x is not continuous at $x=n\pi$ where $n\in Z$.

**b)** If $\vec{a}$ and $\vec{b}$ are equal vectors then $|\vec{a}|=|\vec{b}|$

**c)** x = 0 is the point of minima of the function $f(x)=x^3$

**d)** Sin x + $|x|$ is not continuous at x = 0

**27.**

**a)** Find the value of x : $\sin\left\{\sin^{-1}\frac{3}{5}+\cos^{-1}x\right\}=1$ 1×4

**b)** Integrate $\int x.5^x\,dx$

**c)** Cartesian equation of a line is $\frac{x-5}{2}=\frac{y-(-2)}{1}=\frac{z-4}{3}$. Find its vector equation.

**d)** Find the principal value of $\cot^{-1}\left(\sin\left(-\frac{\pi}{2}\right)\right)$.

**28. a)** Give an example of two non-zero matrices A and B such that AB = 0 but BA ≠ 0.

**b)** A line makes angles $30^\circ, 60^\circ$ and $90^\circ$ with the positive directions of the x, y and z axes respectively. Find its direction cosines.

**c)** Find the derivative of $y=\frac{x}{x+1}$.

**d)** Find the area of the parallelogram having adjacent sides $\vec{a}$ and $\vec{b}$ given by $2\hat{i}+\hat{j}+k$ and $3\hat{i}+\hat{j}+4k$ respectively.

**29. a)** What type of function is the sin function in R?

**(i)** One-one

**(ii)** Many-one

**(iii)** One-one and onto

**(iv)** Neither onto nor one-one

**b)** What is the range of the function, $f(x)=\frac{|x-1|}{x-1}$ is

**(i)** Range of f = {-2, -1, 1, 2}
**(ii)** Range of f = {-1, 0, 1}
**(iii)** Range of f = {-1, 1}
**(iv)** Range of f = {-2, -1, 0, 1, 2}

**c)** Let R be the relation on the set {1, 2, 3, 4} given by
R = {(1, 2), (2, 2), (1, 1), (4, 4), (1, 3),(3, 3), (3, 2)} then R is
**(i)** R is reflexive and symmetric but not transitive.
**(ii)** R is reflexive and transitive but not symmetric.
**(iii)** R is symmetric and transitive but not reflexive.
**(iv)** R is an equivalence relation

**d)** The number of elements in a 3 × 2 matrix is not the same as____
**(i)** The number of elements in a 3 × 3 matrix
**(ii)** The number of elements in a 2 × 3 matrix
**(iii)** The number of elements in a 6 × 1 matrix
**(iv)** The number of elements in a 1 × 6 matrix

**30.** Fill in the blanks with correct answer: 1 × 4 = 4

**(a)** $\int \sin x dx$ _____

**(b)** $\frac{d \sin^{-1} x}{dx}$ is ____

**(c)** $\lim_{x \to a} \frac{x^n - a^n}{x - a}$ is ___

**(d)** $\int_{-1}^{1} x dx$ is ___1

**31.** A doctor is to visit a patient. From past experience, it is known that the probabilities that he will come by train, bus, scooter or by other means of transport are respectively $\frac{3}{10}, \frac{1}{5}, \frac{1}{10}$ and $\frac{2}{5}$. The probabilities that he will be late are $\frac{1}{4}, \frac{1}{3}$ and $\frac{1}{12}$ if he comes by train, bus and scooter respectively. But if he comes by other means of transport, then he will not be late. When he arrives, he is late. What is the probability that the doctor came by train?

**32.** (i) If f: N→ Z s.t f(x) = x and g: Z→ Z s.t g(x) = |x|. Show that g of is injective but g is not.

(ii) If f: N→ N s.t f(x) = x+1 and g: N→ N s.t $g(x) = \begin{cases} x-1 & \text{if } x > 1 \\ 1 & \text{if } x = 1 \end{cases}$.

Show that g of is subjective but f is not.

**33.** Show that the semi vertical angle of the right circular cone of maximum volume and the given slant height is tan√2.

**OR**

ΔABC is right angled at C. P is a point on AB at a distance of a and b from sides AC and BC respectively. Show that the minimum length of the hypotenuse AB is given by:

$$\ell = \sqrt[\frac{3}{2}]{\left[a^{\frac{3}{2}} + b^{\frac{2}{3}}\right]}$$

**34.** Solve the given differential equation: $(x + 1)\dfrac{dy}{dx} = 2e^{-y} - 1$ if $y(0) = 0$

**35.** Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\dfrac{8}{27}$ of the volume of the sphere.

**OR**

Find the point on the curve $x^2 = 8y$ which is nearest to the point (2, 4)