MEGHALAYA
Class XI
Mathematics
Sample Paper Solution-1

Time allowed: 3 hours **Maximum Marks: 100**

Section A

1. $[\sin(x+1)]' = \cos(x+1) .1 = \cos(x+1)$

2. C = {January, July}

3. Given sequence is an A.P. with the first term 3 and the common difference 5.
Let a = 3, d = 5 and $a_n = a + (n - 1)d = 33$
$\Rightarrow a_n = 3 + (n - 1)5 = 33$
$\Rightarrow n = 7$
Number of terms will be 7.

4. The vertices are on the x-axis, so the equation will be of the form
$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ...(1)

Given that a = 13, ae = 5 $\Rightarrow e=\frac{5}{13}$

Now, we have, $b^2=a^2\left[1-e^2\right]$

Therefore, we get

$$b^2=13^2\left[1-\left(\frac{5}{13}\right)^2\right]$$

$$=169\left[1-\frac{25}{169}\right]$$

$$=169\left[\frac{169-25}{169}\right]$$

$$=169-25$$

$$=144$$

Substituting the values of a^2 and b^2 in equation (1), we have

$$\frac{x^2}{13^2}+\frac{y^2}{12^2}=1$$

$$\Rightarrow\frac{x^2}{169}+\frac{y^2}{144}=1$$

5. $^{n-1}p_3 : {}^{n}p_4 = 1:9$

$$\Rightarrow \frac{(n-1)!}{(n-4)!} \times \frac{(n-4)!}{n!} = 9$$

$$\Rightarrow \frac{1}{n} = \frac{1}{9}$$

$$\Rightarrow n = 9$$

6. f(x) = $\frac{x^2-4}{x^2-8x+12}$

For f(x) to be defined, $x^2 - 8x + 12$ must be non-zero i.e. $x^2 - 8x + 12 \neq 0$

$(x - 2)(x - 6) \neq 0$

i.e. $x \neq 2$ and $x \neq 6$

Therefore domain will be $R-\{2,6\}$

So domain of f = R - {2, 6}

7. $\frac{1}{1-i} = \frac{1}{1-i} \times \frac{1+i}{1+i} = \frac{1+i}{(1)^2-(i)^2} = \frac{1+i}{1+1} = \frac{1}{2} + \frac{1}{2}i$

Comparing with $x + iy$, $x = \frac{1}{2}, y = \frac{1}{2}$

Argument $= \theta = \tan^{-1}\frac{y}{x} = \tan^{-1}\left(\frac{\frac{1}{2}}{\frac{1}{2}}\right) = \tan^{-1} 1 = \frac{\pi}{4}$

8. secx = 2

$$\Rightarrow \cos x = \frac{1}{2} = \cos\frac{\pi}{3}$$

$$\Rightarrow x = 2n\pi \pm \frac{\pi}{3}$$

9. $\lim_{x\to 0}\frac{\sin 4x}{\sin 2x} = \lim_{x\to 0}\frac{2\sin 2x\cos 2x}{\sin 2x}$ = 2

$\lim_{x\to 0}\frac{\sin 4x}{\sin 2x} = \lim_{x\to 0}\frac{2\sin 2x\cos 2x}{\sin 2x}$ = 2

10. Relation R from P to Q is R = {(9, 3), (9, –3), (4, 2), (4, –2), (25, 5), (25, –5)}

11. $|x-1| \leq 4$

$-4 \leq x - 1 \leq 4$

$-3 \leq x \leq 5$

So solution set is [−3, 5]

12. If you are not a citizen of India, then you were not born in India.

13. Sample space S = {HH, HT, TH, TT} i.e. total number of cases = 4
Favourable cases for atleast one head are {HH, HT, TH}.
Required probability $= \frac{3}{4}$

14. Since the denominator of x^2 is greater than the denominator of y^2, the major axis is along the x-axis. Comparing the given equation with the standard equation of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

a = 5, b =3

$C = \sqrt{a^2 - b^2} = \sqrt{25 - 9} = 4$

So the foci are (4, 0) and (−4, 0)

15. $x^2 - 4x + 13 = 0$

Roots of equation are given by $\frac{4 \pm \sqrt{16 - 4 \times 13}}{2} = \frac{4 \pm 6i}{2}$

$x = 2 \pm 3i$

Section B

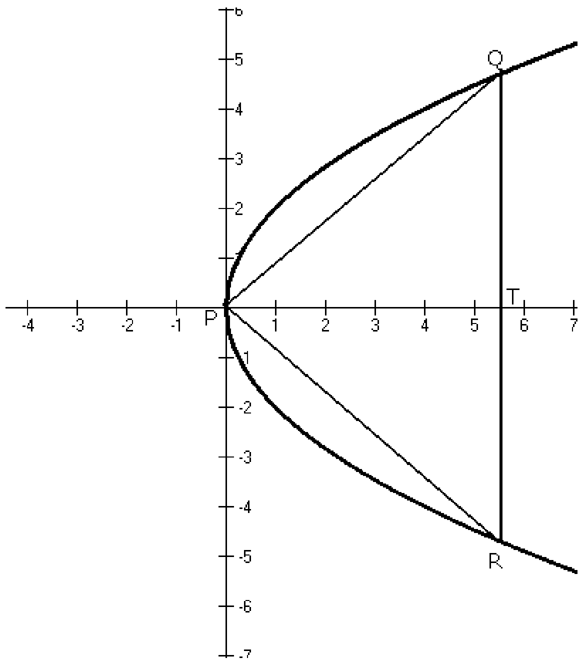
16. We have following cases
(i) If the group of three particular students joins, we have to choose 7 students from remaining 22 students.
So number of ways = $^{22}C_7$
(ii) Three particular students do not join. Here we have to choose 10 students from remaining 22 students.
So number of ways = $^{22}C_{10}$
Hence, total number of ways = $^{22}C_7 + {}^{22}C_{10}$
= 817190

17. Let the two vertices of the triangle be Q and R
Points Q and R will have the same x-coordinate = k(say)
Now in the right ΔPRT, right angled at T.

$$\tan 60^\circ = \frac{k}{RT} \Rightarrow \sqrt{3} = \frac{k}{RT} \Rightarrow RT = \frac{k}{\sqrt{3}}$$

$$\Rightarrow R\left(k, \frac{k}{\sqrt{3}}\right)$$

Now R lies on the parabola : $y^2 = 4\,ax$



$$\Rightarrow \left(\frac{k}{\sqrt{3}}\right)^2 = 4\,a(k)$$

$$\Rightarrow \frac{k}{3} = 4a$$

$$\Rightarrow k = 12a$$

Length of side of the triangle = $2(RT) = 2.\frac{k}{\sqrt{3}} = 2.\frac{(12a)}{\sqrt{3}} = 8\sqrt{3}a$

18. $a_n = \frac{(1+2+3+....n)}{n} = \frac{n(n+1)}{2n}$

$S_n = \sum_n a_n$

$$= \frac{1}{2}\sum_{i=1}^{n}(n+1)$$

$$= \frac{1}{2}\frac{n(n+1)}{2} + \frac{n}{2}$$

$$= \frac{(n^2+n)}{4} + \frac{n}{2}$$

$$= \frac{n(n+3)}{4}$$

OR

Given that the first term is 1.

Also given that each term is the sum of all the terms which follow it.

Let 1, r, r^2,... be an infinite G.P., where r is the common ratio.

Sum of terms of an infinite G.P., $S = \dfrac{a}{1-r}$

Here, a = r

Thus, $S = \dfrac{r}{1-r}$

From the given statement of the problem, we have,

$1 = \dfrac{r}{1-r}$

$\Rightarrow 1 - r = r$

$\Rightarrow r + r = 1$

$\Rightarrow 2r = 1$

$\Rightarrow r = \dfrac{1}{2}$

Thus the required G.P. is:

$1, \dfrac{1}{2}, \dfrac{1}{4}, \dfrac{1}{8}, \dfrac{1}{16}, ...$

19. Let A, B and C represent the set of students who received medals for Soccer, Basketball and Cricket.

$n(A) = 38$, $n(B) = 15$, $n(C) = 20$, $n(A \cup B \cup C) = 58$, $n(A \cap B \cap C) = 3$

Using counting theorems,

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

Substituting the values we get,

$\Rightarrow 58 = 38 + 15 + 20 - n(A \cap B) - n(B \cap C) - n(A \cap C) + 3$

$\Rightarrow n(A \cap B) + n(B \cap C) + n(A \cap C) = 76 - 58 = 18$

Now, each of $n(A \cap B)$, $n(B \cap C)$, $n(A \cap C)$ include the 3 students who received medals for all three sports.

Number of students who received medals in exactly two sports,

$n(A \cap B) - 3 + n(B \cap C) - 3 + n(A \cap C) - 3 = 18 - 3 - 3 - 3 = 9$

20.

$2\cos^2 x + 3\sin x = 0$

Using $\cos^2 x = 1 - \sin^2 x$

$\Rightarrow 2\sin^2 x - 3\sin x - 2 = 0$

$\Rightarrow (\sin x - 2)(2\sin x + 1) = 0$

$\Rightarrow \sin x = 2$ (not possible) and $\sin x = -\frac{1}{2}$

$\Rightarrow \sin x = -\sin\frac{\pi}{6} = \sin(\pi + \frac{\pi}{6})$

$\Rightarrow \sin x = \sin\frac{7\pi}{6} \Rightarrow x = n\pi + (-1)^n \frac{7\pi}{6}$

21. $f(x) = 2x - 1, g(x) = 2x + 3; x \in R$

$(f+g)(x) = f(x) + g(x) = (2x-1) + (2x+3) = 4x + 2; x \in R$

$(f-g)(x) = f(x) - g(x) = (2x-1) - (2x+3) = -4$

$(fg)(x) = f(x)g(x) = (2x-1)(2x+3) = 4x^2 - 2x + 6x - 3 = 4x^2 + 4x - 3$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{2x-1}{2x+3}; x \in R - \left\{-\frac{3}{2}\right\}$

22. Let the equation of the circle be given by:

$x^2 + y^2 + 2gx + 2fy + c = 0$ ...... (1)

As center (-g, -f) lies on the y-axis

Therefore, x-coordinate = −g = 0

Therefore, g = 0

Therefore, equation (1) reduces to

$x^2 + y^2 + 2fy + c = 0$ ....(2)

Now, point A (1, 2) lies on the circle,

Therefore, $1^2 + 2^2 + 2f(2) + c = 0$

Therefore, 1 + 4 + 4f + c = 0

Therefore, 4f + c = −5 ....(3)

Also, point B (-1, 1) lies on the circle.

Therefore, $(-1)^2 + 1^2 + 2.1.f + c = 0$

1 + 1 + 2f + c = 0

Therefore, 2f + c = −2 ......(4)

Solving equation (3) and (4), we get,

$f = -\frac{3}{2}, c = 1$

Putting the values of 'f' and 'c' in equation (2), we get,

$x^2 + y^2 - 3y + 1 = 0$

OR

Since the major axis is on the x-axis,

So equation of ellipse is

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ....(i)

Now (i) passes through (4, 3) and (6, 2) so,

$\frac{16}{a^2}+\frac{9}{b^2}=1$ and

$\frac{36}{a^2}+\frac{4}{b^2}=1$

Solving we get,

$\frac{64}{a^2}+\frac{36}{b^2}=4$

$\frac{324}{a^2}+\frac{36}{b^2}=9$

So $a^2=52$

and $b^2=13$

Required equations is,

$\frac{x^2}{52}+\frac{y^2}{13}=1$

23. Number of hours for the first job is x.

So, wages earned in scheme I = 500 + 50x

wages earned in scheme II = 250x

Scheme I will be beneficial if the wages earned are more in scheme I

$500 + 50x > 250x$

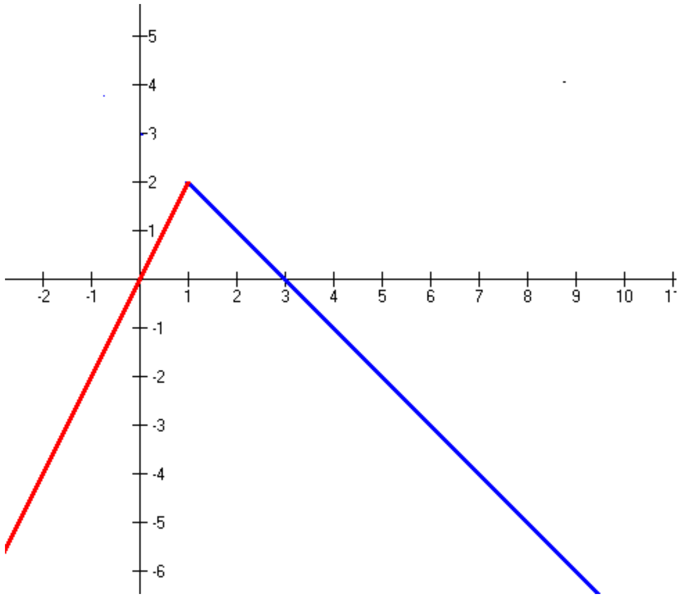
$\Rightarrow 500 > 200x$

$\Rightarrow x<\frac{500}{200}$

$x < 2.5$

So, the carpenter should work less than 2.5 hours so that scheme I is beneficial to him.

24. Range of f = $(-\infty, 2)$



25. There are 3 ways in which he can answer 8 questions, taking at least 3 from each section, is as follows:

Case I: 3 from the first section and 5 from the second

Case II: 4 from the first section and 4 from the second

Case III: 5 from the first section and 3 from the second

The number of ways for doing this is

Case I: ${}^5C_3 \times {}^7C_5$; Case II: ${}^5C_4 \times {}^7C_4$; Case III: ${}^5C_5 \times {}^7C_3$

Total number of ways

$= {}^5C_3 \times {}^7C_5 + {}^5C_4 \times {}^7C_4 + {}^5C_5 \times {}^7C_3 = 210 + 175 + 35 = 420$

Section C

26. The given differential equation is

$$(xdy - ydx)y\sin\left(\frac{y}{x}\right) = (ydx + xdy)x\cos\left(\frac{y}{x}\right)$$

This can be written as

$$xy.\sin\left(\frac{y}{x}\right)dy - y^2.\sin\left(\frac{y}{x}\right)dx = xydx.\cos\left(\frac{y}{x}\right) + x^2dy.\cos\left(\frac{y}{x}\right)$$

$$\frac{dy}{dx} = \frac{xy\cos\frac{y}{x} + y^2\sin\frac{y}{x}}{xy\sin\frac{y}{x} - x^2\cos\frac{y}{x}}$$

Put y = vx, we get $\frac{dy}{dx} = v + x.\frac{dv}{dx}$

$$v + x\frac{dv}{dx} = \frac{v\cos v + v^2\sin v}{v\sin v - \cos v}$$

$$x\frac{dv}{dx} = \frac{v\cos v + v^2\sin v}{v\sin v - \cos v} - v$$

$$= \frac{2v\cos v}{v\sin v - \cos v}$$

$$\Rightarrow \left(\frac{v\sin v - \cos v}{v\cos v}\right)dv = 2\frac{dx}{x}$$

$$\Rightarrow \tan v\,dv - \frac{1}{v}dv = 2.\frac{dx}{x}$$

Integrating, we get

-log|cosv| - log|v| = 2logx – log c

log c = log|x^2. cosv.v|

$$c = x^2 \times \frac{y}{x}\cos\frac{y}{x}$$

$$\Rightarrow \text{solution is } xy\cos\left(\frac{y}{x}\right) = c$$

27. Let the number of items of type A and B produced be x and y respectively.

The L. P. P. is Maximise:

z = 300x + 160y

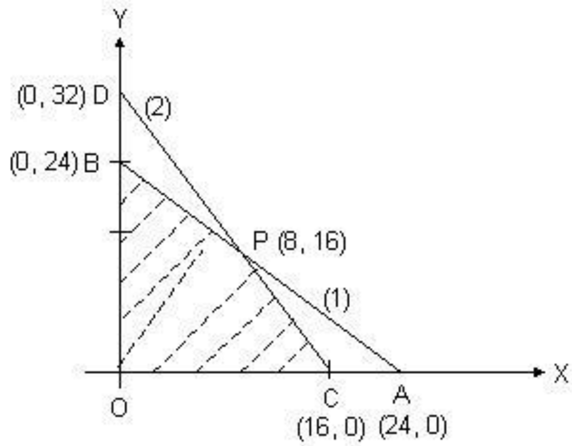
Subject to the constraints

$$x + y \le 24$$

$$x.1 + y.\frac{1}{2} \le 16$$

$$x \ge 0,\ y \ge 0.$$

Graphically



These meet at P (8, 16).

The feasible region is OCPB.

The value of z = 300x + 160y

at O(0, 0) is zero

at C(16, 0) is 4800

at B(0 ,24) is 3840

at P(8, 16) is 4960

Clearly value is max. at P(8, 16) $\Rightarrow$ 8 items of type A and 16 of type B should be produced for maximum profit.

28.

$$\text{Given } A=\begin{bmatrix}2 & -3 & 3\\ 2 & 2 & 3\\ 3 & -2 & 2\end{bmatrix}$$

A=IA

$$\begin{bmatrix}2 & -3 & 3\\ 2 & 2 & 3\\ 3 & -2 & 2\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}A$$

$R_1 \to R_1 + R_2 - R_3$

$$\begin{bmatrix}1 & 1 & 4\\ 2 & 2 & 3\\ 3 & -2 & 2\end{bmatrix}=\begin{bmatrix}1 & 1 & -1\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}A$$

$R_2 \to R_2 - 2R_1, R_3 \to R_3 - 3R_1$

$$\begin{bmatrix}1 & 1 & 4\\ 0 & 0 & -5\\ 0 & -5 & -10\end{bmatrix}=\begin{bmatrix}1 & 1 & -1\\ -2 & -1 & 2\\ -3 & -3 & 4\end{bmatrix}A$$

$R_2 \leftrightarrow R_3$

$$\begin{bmatrix}1 & 1 & 4\\ 0 & -5 & -10\\ 0 & 0 & -5\end{bmatrix}=\begin{bmatrix}1 & 1 & -1\\ -3 & -3 & 4\\ -2 & -1 & 2\end{bmatrix}A$$

$R_2 \to \frac{1}{-5}R_2, R_3 \to \frac{1}{-5}R_3$

$$\begin{bmatrix} 1 & 1 & 4 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 \\ \frac{3}{5} & \frac{3}{5} & \frac{-4}{5} \\ \frac{2}{5} & \frac{1}{5} & \frac{-2}{5} \end{bmatrix} A$$

$R_2 \to R_2 - 2R_3, R_1 \to R_1 - 4R_3$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{-3}{5} & \frac{1}{5} & \frac{3}{5} \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & \frac{-2}{5} \end{bmatrix} A$$

$R_1 \to R_1 - R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{2}{5} & 0 & \frac{3}{5} \\ \frac{-1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & \frac{-2}{5} \end{bmatrix} A$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = -\frac{1}{5}\begin{bmatrix} 2 & 0 & -3 \\ 1 & -1 & 0 \\ -2 & -1 & 2 \end{bmatrix} A$$

Therefore, $A^{-1} = -\frac{1}{5}\begin{bmatrix} 2 & 0 & -3 \\ 1 & -1 & 0 \\ -2 & -1 & 2 \end{bmatrix}$

29. $\int \frac{\sqrt{1+x^2}}{1-x^2}dx = \int \frac{\sqrt{1+x^2} \times \sqrt{1+x^2}}{\left(1-x^2\right) \times \sqrt{1+x^2}} dx$

$$= \int \frac{1+x^2}{\left(1-x^2\right) \times \sqrt{1+x^2}}dx = -\int \frac{-1-x^2}{\left(1-x^2\right) \times \sqrt{1+x^2}} dx$$

$$= -\int \frac{2-1-x^2-2}{\left(1-x^2\right) \times \sqrt{1+x^2}}dx = -\int \frac{1-x^2-2}{\left(1-x^2\right) \times \sqrt{1+x^2}} dx$$

$$=-\int\frac{1-x^2}{\left(1-x^2\right)\times\sqrt{1+x^2}}dx+\int\frac{2}{\left(1-x^2\right)\times\sqrt{1+x^2}}dx$$

$$=-\int\frac{1}{\sqrt{1+x^2}}dx+\int\frac{2}{\left(1-x^2\right)\times\sqrt{1+x^2}}\ dx=I_1+I_2(\text{say})$$

$$I_1=-\log(x+\sqrt{1+x^2})+C_1$$

$$I_2:\text{Let }\ t=\frac{1}{x}\Rightarrow x=\frac{1}{t}\Rightarrow dx=-\frac{1}{t^2}dt$$

$$I_2=\int\frac{2}{\left(1-x^2\right)\times\sqrt{1+x^2}}dx=2\int\frac{-\frac{1}{t^2}dt}{\left(1-\frac{1}{t^2}\right)\times\sqrt{1+\frac{1}{t^2}}}$$

$$=-2\int\frac{tdt}{\left(t^2-1\right)\times\sqrt{t^2+1}}$$

$$\text{Let}\sqrt{t^2+1}=z\Rightarrow t^2=z^2-1\Rightarrow 2tdt=2zdz$$

$$I_2=-2\int\frac{zdz}{\left(z^2-1-1\right)\times z}=-2\int\frac{dz}{\left(z^2-2\right)}=-\frac{1}{2\sqrt{2}}\log\left|\frac{z-\sqrt{2}}{z+\sqrt{2}}\right|+C_2$$

$$=-\frac{1}{2\sqrt{2}}\log\left|\frac{\sqrt{t^2+1}-\sqrt{2}}{\sqrt{t^2+1}+\sqrt{2}}\right|+C_2=-\frac{1}{2\sqrt{2}}\log\left|\frac{\sqrt{\left(\frac{1}{x}\right)^2+1}-\sqrt{2}}{\sqrt{\left(\frac{1}{x}\right)^2+1}+\sqrt{2}}\right|+C_2$$

$$=-\frac{1}{2\sqrt{2}}\log\left|\frac{\sqrt{x^2+1}-x\sqrt{2}}{\sqrt{x^2+1}+x\sqrt{2}}\right|+C_2$$

$$I=-\log(x+\sqrt{1+x^2})+C_1-\frac{1}{2\sqrt{2}}\log\left|\frac{\sqrt{x^2+1}-x\sqrt{2}}{\sqrt{x^2+1}+x\sqrt{2}}\right|+C_2$$

$$I=-\log(x+\sqrt{1+x^2})-\frac{1}{2\sqrt{2}}\log\left|\frac{\sqrt{x^2+1}-x\sqrt{2}}{\sqrt{x^2+1}+x\sqrt{2}}\right|+C,\text{where, }C=C_1+C_2$$

OR

$$A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$$A^2 - 5A = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$A^2 - 5A + 7I = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

For the given Matrix A,

$A^2 - 5A + 7I = 0$

To find A^4

$A^2 - 5A + 7I = 0 \Rightarrow A^2 = 5A - 7I$

$\Rightarrow A^4 = A^2A^2 = (5A - 7I)(5A - 7I)$

$= 25A^2 - 35AI - 35IA + 49I^2 = 25A^2 - 35A - 35A + 49I$

$\left[\because AI = IA = A; I^2 = I\right]$

$\Rightarrow A^4 = 25A^2 - 70A + 49I$

$$\Rightarrow A^4 = 25\begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 70\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 49\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow A^4 = \begin{bmatrix} 200 & 125 \\ -125 & 75 \end{bmatrix} - \begin{bmatrix} 210 & 70 \\ -70 & 140 \end{bmatrix} + \begin{bmatrix} 49 & 0 \\ 0 & 49 \end{bmatrix} = \begin{bmatrix} 39 & 55 \\ -55 & -16 \end{bmatrix}$$

30.

$4x + 2y + 3z = 2$; $x + y + z = 1$; $3x + y - 2z = 5$

Given system of equations is $AX = B$

$$A = \begin{pmatrix} 4 & 2 & 3 \\ 1 & 1 & 1 \\ 3 & 1 & -2 \end{pmatrix} \quad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad B = \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}$$

$$X = A^{-1}B, \text{where } A^{-1} = \frac{\text{adj}A}{|A|}$$

$$|A|=\begin{vmatrix}4&2&3\\1&1&1\\3&1&-2\end{vmatrix}=4(-2-1)-2(-2-3)+3(1-3)$$

$$=-12+10-6=-8\neq 0$$

So A^{-1} exists

$$\text{AdjA}=\begin{pmatrix}-3&7&-1\\5&-17&-1\\-2&2&2\end{pmatrix}$$

$$\Rightarrow A^{-1}=\frac{-1}{8}\begin{pmatrix}-3&7&-1\\5&-17&-1\\-2&2&2\end{pmatrix}$$

$$\Rightarrow X=A^{-1}B=\frac{-1}{8}\begin{pmatrix}-3&7&-1\\5&-17&-1\\-2&2&2\end{pmatrix}\begin{pmatrix}2\\1\\5\end{pmatrix}$$

$$=-\frac{1}{8}\begin{pmatrix}-4\\-12\\8\end{pmatrix}$$

$$\Rightarrow\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}\frac{1}{2}\\\frac{3}{2}\\-1\end{pmatrix}$$

$$\Rightarrow x=\frac{1}{2};y=\frac{3}{2};z=-1$$

OR

$$f(x)=\frac{x}{1+|x|},x\in R;-1<x<1$$

$$\Rightarrow f(x)=\begin{cases}\frac{x}{1+x}, & x\geq 0\\ \frac{x}{1-x}, & x<0\end{cases}\quad ;\text{using definition of modulus function}$$

(i)To show that f is a one-one function, we shall show that

$f(x_1)=f(x_2)\Rightarrow x_1=x_2$

Case I: Let x_1 be positive and x_2 be negative so $(x_1)\neq(x_2)$

Then $f(x_1)=\frac{x_1}{1+x_1}$

$f(x_2) = \frac{x_2}{1 - x_2}$

So $f(x_1) \neq f(x_2)$ and hence $(x_1) \neq (x_2) \Rightarrow f(x_1) \neq f(x_2)$

CaseII :Let both the numbers x_1 and x_2 be positive .

$$f(x_1) = f(x_2) \Rightarrow \frac{x_1}{1+x_1} = \frac{x_2}{1+x_2} \Rightarrow x_1 + x_1x_2 = x_2 + x_1x_2 \Rightarrow x_1 = x_2$$

CaseIII :Let both the numbers x_1 and x_2 be negative .

$$f(x_1) = f(x_2) \Rightarrow \frac{x_1}{1-x_1} = \frac{x_2}{1-x_2} \Rightarrow x_1 - x_1x_2 = x_2 - x_1x_2 \Rightarrow x_1 = x_2$$

Case I ,case II and case III together implies f is one –one

(ii) Now, we shall prove that f(x) is an onto function.

$$f(x) = \begin{cases} \frac{x}{1+x}, & x \geq 0 \\ \frac{x}{1-x}, & x < 0 \end{cases}$$

Case I: If x is non negative then

$$f(x) = y = \frac{x}{1+x}$$

$$\Rightarrow y + xy = x$$

$$\Rightarrow x = \frac{y}{1-y}$$

$$\text{Now}, f\left(\frac{y}{1-y}\right) = \frac{\frac{y}{1-y}}{1 + \frac{y}{1-y}} = y$$

Case II:If x is negative then

$$f(x) = y = \frac{x}{1-x}$$

$$\Rightarrow y - xy = x$$

$$\Rightarrow x = \frac{y}{1+y}$$

$$\text{Now}, f\left(\frac{y}{1+y}\right) = \frac{\frac{y}{1+y}}{1 - \frac{y}{1+y}} = y$$

Hence, for each x in A, there exists $y \in R$ such that f(y) = x

$\Rightarrow$ f is an onto function

Hence, f is a bijective function.