

**NAGALAND
Class XII
Mathematics**

Time allowed: 3 hours

Maximum Marks: 80

General Instructions:

- i. Approximately 15 minutes is allotted to read the question paper and revise the answers.
- ii. The question paper consists of 26 questions. All questions are compulsory.
- iii. Marks are indicated against each question.
- iv. Internal choice has been provided in some questions.
- v. Use of simple calculators (non-scientific and non-programmable) only is permitted.

N.B: Check that all pages of the question paper is complete as indicated on the top left side.

Section A

1. Choose the correct answer from the given alternatives.

10×1

(a) If $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ then $(A^2)'$ is equal to

(i) $\begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$

(ii) $\log\sqrt{5}$

(iii) $\begin{bmatrix} 1 & 5 \\ 5 & 1 \end{bmatrix}$

(iv) None

(b) $\int_1^3 \frac{x}{x^2+1} dx$

(i) $\log\sqrt{5}$

(ii) 1

(iii) -1

(iv) $2\log\sqrt{5}$

(c) The simplest form of $\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right)$ for $x > 0$

(i) x

(ii) $-\frac{x}{2}$

(iii) $\frac{x}{2}$

- (iv) Both (ii) and (iii)
- (d) Which of the following statements is correct?
- (i) Mean of the Binomial distribution = np
- (ii) Variance of a Binomial distribution is npq
- (iii) Standard Deviation = $\sqrt{\text{Variance}}$
- (iv) All of the above are correct
- (e) A firm produces x type A and y type B bags. Demand for bag B is at most one fourth of A. The corresponding constraint is
- (i) $\frac{x}{4} \leq y$
- (ii) $x - 4y \geq 0$
- (iii) $x < 4y$
- (iv) $4y - x \geq 0$
- (f) Identify the type of differential equation $(x - y) \frac{dy}{dx} = x + 2y$
- (i) Variable separable
- (ii) Homogeneous
- (iii) Linear
- (iv) Both variable separable and linear
- (g) The function $f(x) = 3x + 1$ is
- (i) Injective
- (ii) Not injective
- (iii) Insufficient information
- (iv) It is injective for some intervals of the domain
- (h) The equation of line joining the points (1, 2) and (4, 7) is
- (i) $5x - 3y - 1 = 0$
- (ii) $3x + 5y + 1 = 0$
- (iii) $3x - 5y + 1 = 0$
- (iv) $5x - 3y + 1 = 0$
- (i) A unit vector parallel to $i + j$ and perpendicular to $i - j$ is
- (i) $\frac{1}{\sqrt{2}}i + \frac{1}{\sqrt{2}}j$
- (ii) $\frac{3}{2}i + \frac{1}{2}j$
- (iii) $\frac{1}{\sqrt{2}}i - \frac{1}{\sqrt{2}}j$
- (iv) $\frac{3}{\sqrt{2}}i + \frac{1}{\sqrt{2}}j$

- (j) If $f(x) = |x - 2|$ then at $x = 2$ $f'(x)$ is
- Continuous but not differentiable
 - Differentiable but not continuous
 - Continuous and differentiable both
 - Neither continuous nor differentiable

Section B

10×2

- Find the probability distribution of the number of Kings drawn when two cards are drawn one by one, without replacement, from a pack of 52 playing cards.
- Find the area of the parallelogram having adjacent sides \vec{a} and \vec{b} given by $2\hat{i} + \hat{j} + k$ and $3\hat{i} + \hat{j} + 4k$ respectively.
- Find $\frac{dy}{dx}$, if $\log(x^2 + y^2) = \tan^{-1}\left(\frac{y}{x}\right)$
- Find $\int \frac{dx}{\sqrt{9 - 25x^2}}$
- Let $f(x) = x + 3$, $g(x) = x - 3$; $x \in \mathbb{N}$,
Show that (i) f is not an onto function (ii) $g \circ f$ is an onto function
- Find the value of $\tan(\cos^{-1}\frac{4}{5} + \tan^{-1}\frac{2}{3})$
- Verify Rolle's theorem for the function: $f(x) = x^2 - 5x + 4$ on $[1, 4]$
- Evaluate: $\int \frac{1 + \cot x}{x + \log \sin x} dx$
- For what values of ' λ ', are the vectors $(2\hat{i} - 3\hat{j})$ and $(\lambda\hat{i} - 6\hat{j})$ parallel?
- For what values of x is the following matrix singular

$$A = \begin{bmatrix} 3 - 2x & x + 1 \\ 2 & 4 \end{bmatrix}$$

Section C

10×4

12. Solve the following system of equations, using Matrices $4x + 2y + 3z = 2$; $x + y + z = 1$; $3x + y - 2z = 5$
13. A random variable X has the following probability distribution :

X	0	1	2	3	4	5	6	7
P(X)	0	k	2k	2k	3k	k ²	2k ²	7k ² + k

Determine: (i) k (ii) $P(X < 3)$ (iii) $P(X > 6)$ (iv) $P(1 \leq X < 3)$

OR

In answering a question on a MCQ test with 4 choices per question, a student knows the answer, guesses it or copies the answer. Let $\frac{1}{2}$ be the probability that he knows the answer, $\frac{1}{4}$ be the probability that he guesses and $\frac{1}{4}$ be the probability that he copies it. Assuming that a student, who copies the answer, will be correct with the probability $\frac{3}{4}$, what is the probability that student knows the answer, given that he answered it correctly?

Arjun does not know that answer to one of the questions in the test. The evaluation process has negative marking. Which value would Arjun violate if he resorts of unfair means? How would an act like the above hamper his character development in the coming years?

14. Find the co-ordinates of the point where the line through (3, -4, -5) and (2, -3, 1) crosses the plane $2x + y + z = 7$.

OR

Find the equation of perpendicular drawn from (1, 0, 0) to the line.

$$\frac{x-1}{2} = \frac{y-1}{-3} = \frac{z+10}{8}$$

15. Give the intervals in which the function $f(x) = \frac{4 \sin x - 2x - x \cos x}{2 + \cos x}$ is increasing or decreasing.

16. Evaluate: $\int_0^{\frac{\pi}{2}} \log(\sin x) dx$

OR

Evaluate: $\int_0^2 (x^2 + e^x) dx$ using integral as limit of sums.

17. Find the angle between the lines whose direction cosines are given by the equations: $3l + m + 5n = 0$; $6mn - 2nl + 5lm = 0$

18. Find the angle between \vec{a} and \vec{b} . If $\vec{a} + \vec{b} + \vec{c} = 0$ and $|\vec{a}| = 3$, $|\vec{b}| = 5$ & $|\vec{c}| = 7$

OR

Find λ if the vectors $\vec{a} = \hat{i} - \lambda\hat{j} + 3\hat{k}$ and $\vec{b} = 4\hat{i} - 5\hat{j} + 2\hat{k}$ are perpendicular to each other.

19. If $f: \mathbb{R} - \{2\} \rightarrow \mathbb{R} - \{3\}$ is defined by $f(x) = \frac{3x+1}{x-2}$, where \mathbb{R} is the set of real number show that f is invertible and hence find f^{-1} .

20. Evaluate: $\int \frac{\cos^2 x}{1 + \tan x} dx$

21. If $a, b,$ and c are in A.P., find the value of the determinant

$$\Delta = \begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$$

OR

Let $a, b,$ and c be positive numbers, but not equal

Show that the value of the determinant $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is negative

$$\begin{vmatrix} 0 & q-r & r-s \\ r-q & 0 & p-q \\ s-r & q-r & 0 \end{vmatrix}$$

Section D

5×6

22. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

OR

Find the point on the curve $x^2 = 8y$ which is nearest to the point $(2, 4)$

23. Calculate the area

(i) between the curves $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, and the x -axis between $x = 0$ to $x = a$

(ii) Triangle AOB is in the first quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $OA = a$ and $OB = b$.

Find the area enclosed between the chord AB and the arc AB of the ellipse

(iii) Find the ratio of the two areas found.

OR

24. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$, find A^{-1}

Using A^{-1} , solve the following system of equation

$$x + y + 2z = 0$$

$$x + 2y - z = 9$$

$$x - 3y + 3z = -14$$

OR

Find A^{-1} using elementary transformations

$$\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$$

- 25.** A square piece of tin with side 18 cm is to be made into a box without a top by cutting a square piece from each corner and folding up the flaps. What should be the side of the square to be cut off, so that the volume of the box is the largest? Also find the maximum volume of the box.

OR

A window is in the form of a rectangle surmounted by a semi circular opening. The total perimeter of the window is 10 m. Find the dimensions of the window to admit maximum light through the whole opening.

- 26.** Find the vector and Cartesian equation of the plane through $3\hat{i} - \hat{j} + 2\hat{k}$ and parallel to the lines

$$\vec{r} = -\hat{j} + 3\hat{k} + \lambda(2\hat{i} - 5\hat{j} - \hat{k})$$

$$\vec{r} = \hat{i} - 3\hat{j} + \hat{k} + \mu(-5\hat{i} + 4\hat{j})$$

OR

Find the distance of the point A (-2, 3, -4) from the line $\frac{x+2}{3} = \frac{2y+3}{4} = \frac{3z+4}{5}$ measured parallel to the plane $4x + 12y - 3z + 1 = 0$.