ICSE Board
Class IX Mathematics
Sample Paper 2 – Solution

SECTION – A (40 Marks)

Q. 1.

(a) $$\frac{\sin 30° - \sin 90° + 2\cos 0°}{\tan 30° \times \tan 60°} = \frac{\frac{1}{2} - 1 + 2}{\frac{1}{\sqrt{3}} \times \sqrt{3}} = \frac{1}{2} + 1 = \frac{3}{2} = 1\frac{1}{2}$$

(b) $$\frac{3 \times 27^{n+1} + 9 \times 3^{n-1}}{8 \times 3^{3n} - 5 \times 27^{n}} = \frac{3 \times 3^{3n+3} + 9 \times 3^{3n-1}}{8 \times 3^{3n} - 5 \times 3^{3n}}$$

$$= \frac{3^{3n}\left(3 \times 3^{3} + 9 \times 3^{-1}\right)}{3^{3n}\left(8 - 5\right)}$$

$$= \frac{3 \times 27 + 9 \times \frac{1}{3}}{3}$$

$$= \frac{81 + 3}{3}$$

$$= \frac{84}{3}$$

$$= 28$$

(c) Given, $\frac{2 + \sqrt{3}}{2 - \sqrt{3}} = x + y\sqrt{3}$

Rationalize the denominator

$$\frac{\left(2 + \sqrt{3}\right)\left(2 + \sqrt{3}\right)}{\left(2 - \sqrt{3}\right)\left(2 + \sqrt{3}\right)} = x + y\sqrt{3}$$

$$\Rightarrow \frac{4 + 3 + 4\sqrt{3}}{4 - 3} = x + y\sqrt{3}$$

$$\Rightarrow 7 + 4\sqrt{3} = x + y\sqrt{3}$$

Comparing the real and irrational parts on both sides, we get

$x = 7, y = 4$

Q. 2.

(a)

Given that,

$$\text{arc } AXB = \frac{1}{2}\text{arc } BYC$$

$$\Rightarrow \angle AOB = \frac{1}{2}\angle BOC$$

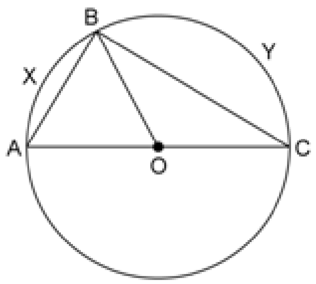
Since AOC is a straight line,

$$\angle AOB + \angle BOC = 180°$$

$$\therefore \frac{1}{2}\angle BOC + \angle BOC = 180°$$

$$\therefore \frac{3}{2}\angle BOC = 180°$$

$$\therefore \angle BOC = 180° \times \frac{2}{3} = 120°$$



(b) Given, x + y = 6; x – y = 4

We know that

$$(x+y)^2 = (x-y)^2 + 4xy$$

$$(6)^2 = (4)^2 + 4xy$$

$$\Rightarrow 36 - 16 = 4xy$$

$$\Rightarrow 20 = 4xy$$

$$\Rightarrow xy = 5$$

(c) Let, $\frac{\log a}{b-c} = \frac{\log b}{c-a} = \frac{\log c}{a-b} = k$

$\Rightarrow \log a = k(b - c), \log b = k(c - a)$ and $\log c = k(a - b)$

Now, $A = a^a.b^b.c^c$

Taking log both sides

$$\log A = a \log a + b \log b + c \log c$$
$$= a.k(b - c) + b.k(c - a) + c.k(a - b)$$
$$= k[ab - ac + bc - ab + ac - bc]$$
$$= k \times 0 = 0$$

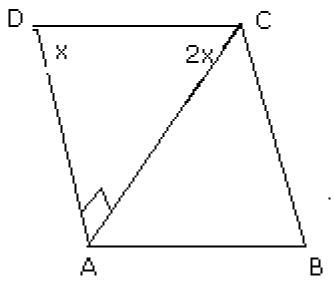
$\Rightarrow \log A = 0$

$\Rightarrow \log A = \log 1 \Rightarrow A = 1$

$\therefore a^a.b^b.c^c = 1$

Hence Proved.

Q. 3.



(a) In ΔADC,

$x + 2x + 90° = 180°$ (sum of all angles in a Δ is 180°)

$\Rightarrow 3x = 90°$

$\Rightarrow x = 30°$

$\Rightarrow m\angle D = 30°$

$\therefore$ $m\angle B = m\angle D = 30°$ [Opposite angles of ||gm are equal]

And $m\angle A = m\angle C = 180° - 30°$

[sum of co- interior angles = 180° in a ||gm]

$\Rightarrow m\angle A = m\angle C = 150°$

Thus, the angles of a parallelogram are 150°, 30°, 150° and 30°.

(b) Let $x = 5.3\overline{47} = 5.34747.....$ ....(i)

Multiplying (i) by 10, we get

$10x = 53.4747....$ ....(ii)

Multiplying (ii) by 100, we get

$1000x = 5347.47.....$ ....(iii)

Subtracting (ii) from (iii), we get

$1000x - 10x = 5347.47.... - 53.47...$

$\Rightarrow 990x = 5294$

$$\Rightarrow x = \frac{5294}{990}$$

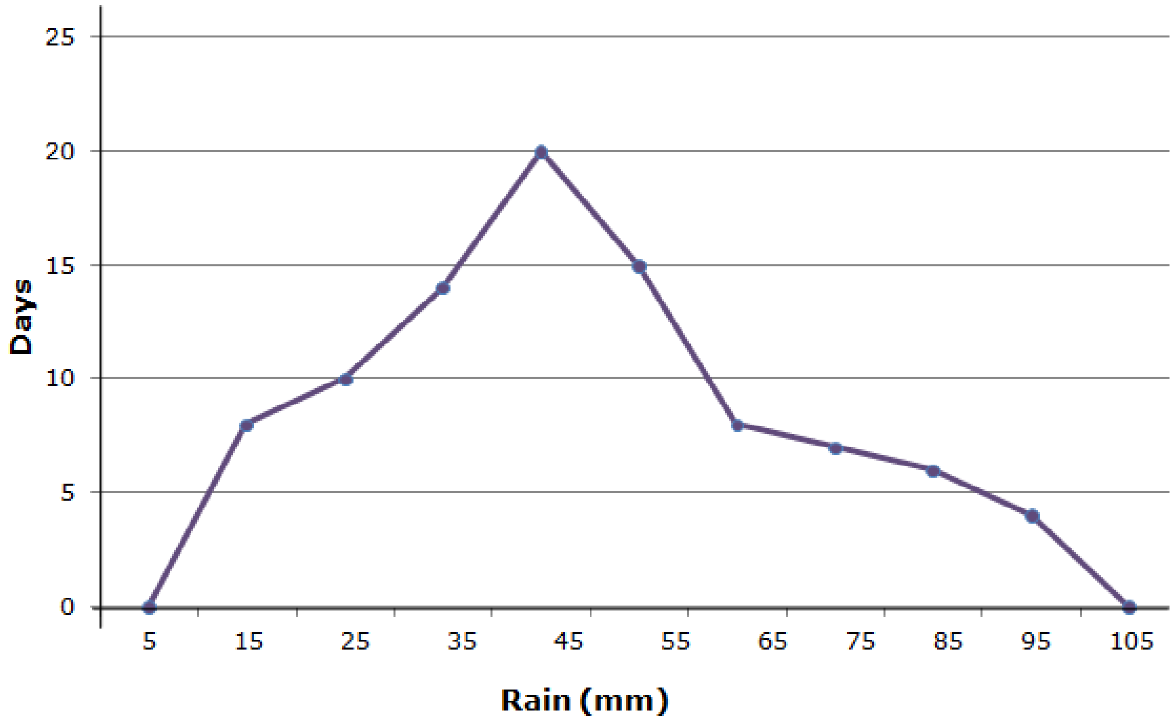
$$\Rightarrow x = \frac{2647}{495}$$

$$\therefore 5.3\overline{47} = \frac{2647}{495}$$

(c) While drawing frequency polygon, we represent each class by its mid-value.

Classes Rain (mm)	Class Marks	Days
0 – 10	5	0
10 – 20	15	8
20 – 30	25	10
30 – 40	35	14
40 – 50	45	20
50 – 60	55	15
60 – 70	65	8
70 – 80	75	7
80 – 90	85	6
90 – 100	95	4
100 – 110	105	0

The frequency polygon is as follows:



(d)

Q. 4.

(a) Each exterior angle of first polygon = $\frac{360°}{n-1}$

Each exterior angle of second polygon = $\frac{360°}{n+2}$

$$\frac{360°}{n-1}-\frac{360°}{n+2}=6$$

$$\Rightarrow \frac{1}{n-1}-\frac{1}{n+2}=\frac{1}{60}$$

$$\Rightarrow \frac{n+2-n+1}{(n-1)(n+2)}=\frac{1}{60}$$

$$\Rightarrow \frac{3}{n^2+2n-n-2}=\frac{1}{60}$$

$$\Rightarrow n^2+n-2=180$$

$$\Rightarrow n^2+n-182=0$$

$$\Rightarrow n^2+14n-13n-182=0$$

$$\Rightarrow n(n+14)-13(n+14)=0$$

$$\Rightarrow (n+14)(n-13)=0$$

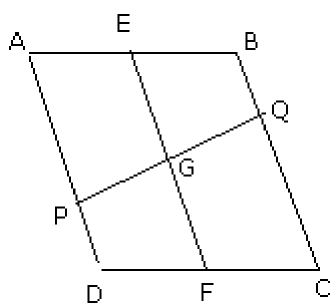
$n=-14$ is not applicable

$\therefore n-13=0$

$\therefore n=13$

(b)

Given: A ||gm ABCD, E and F are the midpoints of AB and DC. Line PQ meets AD, EF and BC at P, G and Q respectively.



To prove: PG = GQ

Proof: $AE=\frac{1}{2}AB$ and $DF=\frac{1}{2}DC$

[E and F are the mid-points]

$\Rightarrow$ AE = DF

Also, AE || DF [As AB || DC]

AE and DF are the parts of AB and DC.

$\therefore$ AEFD is a parallelogram

[$\because$ Opposite sides of parallelogram are equal and parallel]

$\therefore$ AD || EF || BC

$\therefore$ PG = GQ

[By intercept theorem AD, EF and BC are parallel, PQ cuts them]

Hence proved.

(c) P = Rs. 5000, r = 12%, T = 1 year

Amount at the end of first year

$$= 5000 + \left(\frac{5000 \times 12 \times 1}{100}\right)$$

= Rs. 5000 + 600 = Rs. 5600

Amount at the end of first year after payment of Rs. 2000

= Rs. 5600 – Rs. 2000 = Rs. 3600

Amount at the end of second year

$$= 3600 + \left(\frac{3600 \times 12 \times 1}{100}\right)$$

$$= \text{Rs.}3600 + 432 = \text{Rs.}4032$$

Amount at the end of second year after payment of Rs. 2000 = Rs. 4032 – Rs. 2000 = Rs. 2032

Amount at the end of third year

$$= 2032 + \left(\frac{2032 \times 12 \times 1}{100}\right)$$

$$= \text{Rs. } 2032 + \text{Rs. } 243.84$$

$$= \text{Rs. } 2275.84$$

SECTION – B (40 Marks)

Q. 5.

(a) Area of a square formed = 484 m^2

$\Rightarrow$ $(\text{Side})^2$ = 484

$\Rightarrow$ Side = 22 m

Thus, perimeter of a square = 4 × Side = 4 × 22 = 88 m

Let r be the radius of the circle formed.

Now,

Circumference of a circle = Perimeter of a square

$$\Rightarrow 2 \times \frac{22}{7} \times r = 88$$

$$\Rightarrow r = \frac{88 \times 7}{2 \times 22}$$

$$\Rightarrow r = 14 \text{ m}$$

$$\therefore \text{ Area of a circle} = \pi r^2 = \frac{22}{7} \times 14 \times 14 = 616 \text{ m}^2$$

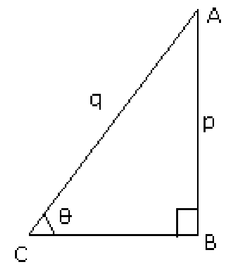
(b) $\sin\theta = \frac{p}{q} = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{AB}{AC}$

By Pythagoras theorem,

$BC^2 = AC^2 - AB^2 = q^2 - p^2$

$BC = \sqrt{q^2 - p^2}$

$\cos\theta = \frac{BC}{AC} = \frac{\sqrt{q^2 - p^2}}{q}$



Now,

$\sin\theta + \cos\theta = \frac{p}{q} + \frac{\sqrt{q^2 - p^2}}{q} = \frac{\sqrt{q^2 - p^2} + p}{q}$

(c) Points (a, 0), (0, b) and (1, 1) are collinear. So, area of triangle formed by these points will be 0.

$\Rightarrow \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$

$\Rightarrow [a(b - 1) + 0(1 - 0) + 1(0 - b)] = 0$

$\Rightarrow ab - a - b = 0$

$\Rightarrow ab = a + b$

On dividing throughout by ab, we get

$1 = \frac{1}{a} + \frac{1}{b}$

Q. 6.

(a)

$\left(x^2 + y^2 - z^2\right)^2 - (2xy)^2$

$= \left(x^2 + y^2 - z^2 + 2xy\right)\left(x^2 + y^2 - z^2 - 2xy\right)$

$= \left(x^2 + y^2 + 2xy - z^2\right)\left(x^2 + y^2 - 2xy - z^2\right)$

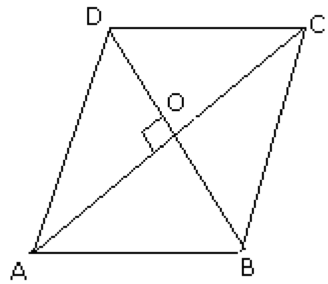
$= \left\{(x+y)^2 - (z)^2\right\}\left\{(x-y)^2 - (z)^2\right\}$

$= (x + y + z)(x + y - z)(x - y - z)(x - y + z)$

(b) Given: A parallelogram ABCD, such that BD ⊥ AC.

To prove: ABCD is a rhombus

Proof:



In ΔOAB and ΔOBC

OA = OC [Diagonals of a ||gm bisect each]

$\angle AOB = \angle BOC$ [Each 90°]

OB = OB

$\therefore \Delta OAB \cong \Delta OBC$ [SAS axioms of congruency]

AB = BC [C.P.C.T]

BC = AD and AB = DC [Given]

$\Rightarrow$ AB = DC = BC = AD

Hence ABCD is a rhombus.

(c) $3a = p\left(\frac{x}{2} - y\right)$

$$\Rightarrow \frac{x}{2} - y = \frac{3a}{p}$$

$$\Rightarrow y = \left(\frac{x}{2} - \frac{3a}{p}\right)$$

$$\Rightarrow y = \left(\frac{px - 6a}{2p}\right)$$

Given a = 32, x = 4, p = 5

$$\Rightarrow y = \frac{5 \times 4 - 6 \times 32}{2 \times 5}$$

$$\Rightarrow y = \frac{20 - 192}{10}$$

$$\Rightarrow y = -17.2$$

Q. 7.

(a) $2x - 3y = 7 \Rightarrow 2x = 7 + 3y$

$$\Rightarrow x = \frac{7+3y}{2}$$

Taking convenient values of y, we get

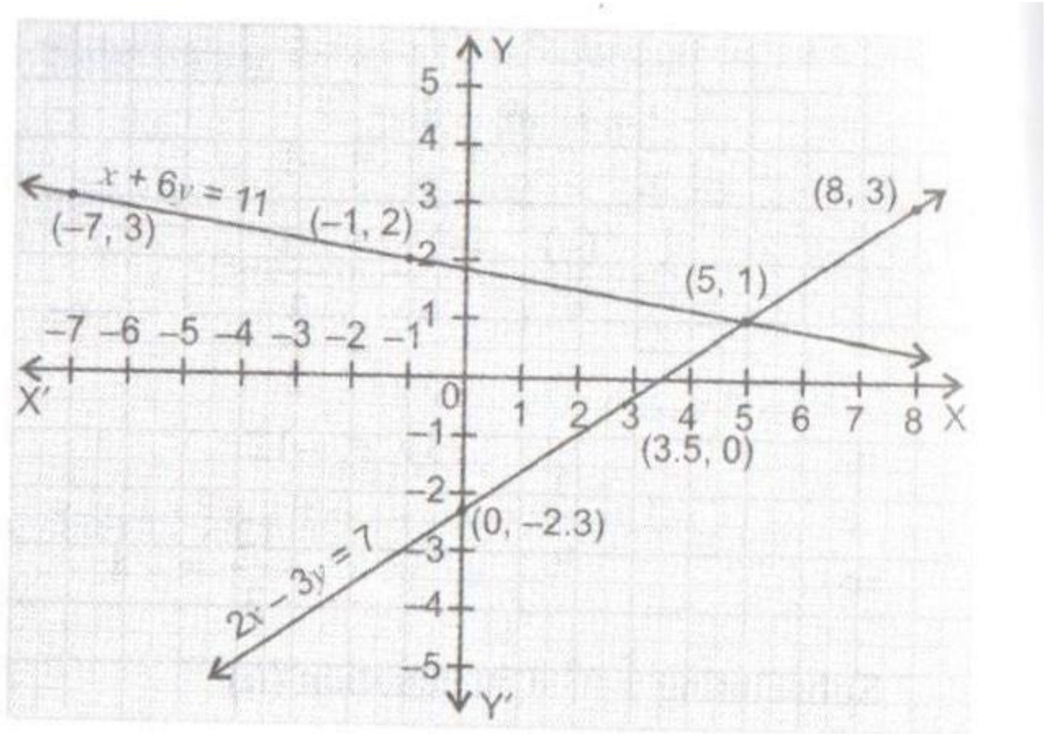
x	3.5	5	8
y	0	1	3

And $x + 6y = 11 \Rightarrow x = 11 - 6y$

Taking convenient values of y, we get

x	5	-1	-7
y	1	2	3

Now we plot these points on graph paper as follows:



The point of intersection of the two lines is (5, 1).

Hence the solution set is x = 5, y = 1.

(b) (i) Parallelograms BFED and AFEC are on the same base FE and between the same parallels AD || EF. Thus, they are equal in area.

$\therefore$ ar (||gm BFED) = ar (||gm AFEC) = 140 cm^2

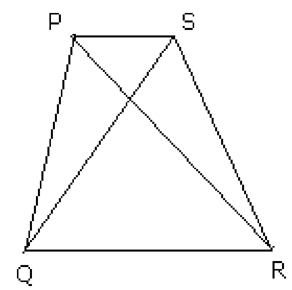
(ii) ΔBFD and ||gm BFED are on the same base BD and between the same parallels BD and FE.

$\therefore$ A(ΔBFD) = $\frac{1}{2} \times$ A(||gm BFED) = $\frac{1}{2} \times 140 = 70$ cm^2

Q. 8.

(a) Given: PQRS is a quadrilateral, and PR and QS are its diagonals.

To Prove: (PQ + QR + RS + SP) > (PR + QS)



Proof:

1. In ΔPQS, SP + PQ > QS
2. In ΔPQR, PQ + QR > PR
3. In ΔQRS, (QR + RS) > QS
4. In ΔRSP, (RS + SP) > PR

[Sum of the two sides of a Δ is greater than the third side]

Adding 1, 2, 3 and 4

2(PQ + QR + RS + SP) > 2(PR + QS)

$\Rightarrow$ PQ + QR + RS + SP > PR + QS [Hence Proved]

(b) Let the two places P_1 and P_2 be 30 km apart and let A start from P_1 and B from P_2.

Let A's speed of walking be x km/ hr and B's speed be y km/ hr.

According to the question, when they walk in the same direction

$10(x - y) = 30$ ....(i)

When they walk in opposite directions

$2(x + y) = 30$ ....(ii)

On dividing (i) by (ii), we get

$\frac{5(x-y)}{x+y} = 1 \Rightarrow 5x - 5y = x + y$

$\Rightarrow 4x - 6y = 0$ ....(iii)

On multiplying (ii) by 2, we get

$4x + 4y = 60$ ....(iv)

Subtracting (iii) from (iv), we get

$10y = 60 \Rightarrow y = 6$

Substituting value of y in (iv),

$4x + 4(6) = 60$

$\Rightarrow 4x + 24 = 60 \Rightarrow 4x = 36$

$\Rightarrow x = \frac{36}{4} = 9$

$\therefore$ A's speed = 9 km/ hr and B's speed = 6 km/ hr

Q. 9.

(a) Mean height of the 10 girls = 1.38 m

Sum of heights of 10 girls = $1.38 \times 10 = 13.8$ m

Mean height of 40 boys = 1.44 m

Sum of heights of 40 boys = $1.44 \times 40 = 57.6$ m

Sum of heights of 50 students (10 girls + 40 boys) = 13.8 + 57.6 = 71.4 m

$\therefore$ Mean height of 50 students = $\frac{71.4}{50} = 1.428$ m

(b) Given: $m\angle D = 90°$, AB = 8 cm, BC = 6 cm and CA = 3 cm

To find: length of CD

Let CD = x cm

In ΔADC,

$AD^2 + CD^2 = AC^2$ [By Pythagoras theorem]

$AD^2 = AC^2 - DC^2 = 3^2 - x^2$

$AD^2 = 9 - x^2$

In ΔADB,

$AD^2 + BD^2 = AB^2$ [By Pythagoras theorem]

$9 - x^2 + (6+x)^2 = 8^2$ [$\because AD^2 = 9 - x^2$ and $BD = 6 + x$]

$\Rightarrow 9 - x^2 + 36 + x^2 + 12x = 64$

$\Rightarrow 12x = 64 - 45 = 19$

$\therefore \ x = \frac{19}{12}$

$CD = \frac{19}{12}\text{cm} = 1\frac{7}{12}$

(c) Volume of rectangular tank = 80 × 60 × 60 cm³ = 288000 cm³

One liter = 1000 cm³

Volume of water flowing in per sec

$= 1.5\ \text{cm}^2 \times 3.2\ \frac{\text{m}}{\text{s}}$

$= 15.\ \text{cm}^2 \times \frac{(3.2 \times 100)\ \text{cm}}{\text{s}}$

$= 480\ \frac{\text{cm}^3}{\text{s}}$

Volume of water flowing in 1 min = 480 × 60 = 28800 cm³

Hence,

28800 cm³ of water can be filled in 1 min

$\Rightarrow$ Time required to fill 288000 cm³ of water $= \left(\frac{1}{28800} \times 288000\right)$ min $= 10$ min

Q. 10.

(a) Using sec (90° – θ) = cosec θ, tan (90° – θ) = cot θ

And cos (90° – θ) = sin θ

$$\frac{\sec(90^0-\theta).\operatorname{cosec}\theta-\tan(90^0-\theta)\cot\theta+\cos^2 25^0+\cos^2 65^0}{3\tan 27^0\tan 63^0}$$

$$=\frac{\operatorname{cosec}\theta.\operatorname{cosec}\theta-\cot\theta.\cot\theta+\cos^2(90^0-65^0)+\cos^2 65^0}{3\tan(90^0-63^0)\tan 63^0}$$

$$=\frac{\operatorname{cosec}^2\theta-\cot^2\theta+\sin^2 65^0+\cos^2 65^0}{3\cot 63^0\tan 63^0}$$

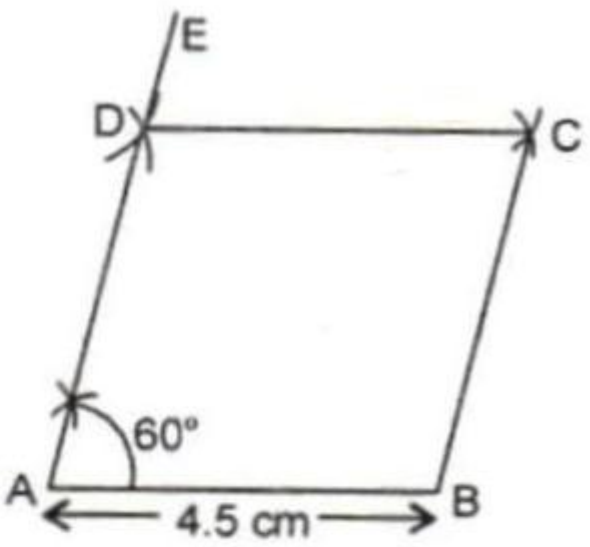
$$=\frac{1+1}{3}$$

$$=\frac{2}{3}$$

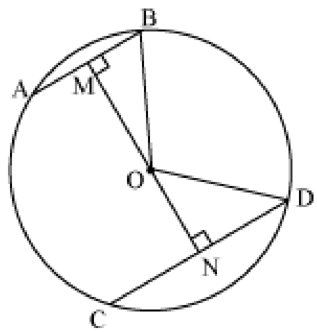
(b) Steps of construction:

1. We draw AB = 4.5 cm
2. Now we construct m∠BAE = 60°
3. Then we cut off AD = 4.5 cm from AE
4. We draw two arcs of radius 4.5 cm, one with centre at D and other at B.
5. Let them cut at C.
6. Join DC and BC.

ABCD is the required rhombus.



(c) Construction: Draw $OM \perp AB$ and $ON \perp CD$. Join OB and OD.



$BM = \frac{AB}{2} = \frac{5}{2}$ and $ND = \frac{CD}{2} = \frac{11}{2}$ (Perpendicular from centre bisects the chord)

Let ON be x.

Then, OM will be $6 - x$.

In ΔMOB,

$OM^2 + MB^2 = OB^2$

$\therefore (6-x)^2 + \left(\frac{5}{2}\right)^2 = OB^2$

$\therefore 36 + x^2 - 12x + \frac{25}{4} = OB^2$ ....(1)

In ΔNOD, $ON^2 + ND^2 = OD^2$

$\therefore OD^2 = x^2 + \left(\frac{11}{2}\right)^2 = x^2 + \frac{121}{4}$ ....(2)

We have $OB = OD$ ....(radii of same circle)

$36 + x^2 - 12x + \frac{25}{4} = x^2 + \frac{121}{4}$ ....[From (1) and (2)]

$\therefore 12x = 36 + \frac{25}{4} - \frac{121}{4} = \frac{144 + 25 - 121}{4} = \frac{48}{4} = 12$

$\therefore 12x = 12$

$\Rightarrow x = 1$

From equation (2),

$OD^2 = (1)^2 + \left(\frac{121}{4}\right) = 1 + \frac{121}{4} = \frac{125}{4}$

$\Rightarrow OD = \frac{5}{2}\sqrt{5}$

Hence, the radius of the circle is $\frac{5}{2}\sqrt{5}$ cm.

Q. 11.

(a) Given : $a+\frac{1}{a}=p$,

Cubing on both the sides, we get

$$\left(a+\frac{1}{a}\right)^3=p^3$$

$$\Rightarrow a^3+\frac{1}{a^3}+3\left(a+\frac{1}{a}\right)=p^3$$

$$\Rightarrow a^3+\frac{1}{a^3}+3p=p^3$$

$$\Rightarrow a^3+\frac{1}{a^3}=p^3-3p=p\left(p^2-3\right)$$

Hence Proved.

(b) Let $\frac{1}{x+y}=a,\ \frac{1}{x-y}=b$

Then, we have

$5a + 3b = 4$ ....(i)

$2a + 5b = \frac{27}{5}$ ....(ii)

Multiplying equation (i) by 2 and equation (ii) by 5, we get

$10a + 6b = 8$ ....(iii)

$10a + 25b = 27$ ....(iv)

Subtracting (iv) from (iii), we get $-19b = -19$

$b = \frac{-19}{-19}=1$

Substituting value of b in equation (i), we get

$5a + 3(1) = 4$

$\Rightarrow 5a + 3 = 4 \Rightarrow 5a = 1 \Rightarrow a = \frac{1}{5}$

Now $\frac{1}{x+y}=a,\ \frac{1}{x-y}=b$

$\therefore \frac{1}{x+y}=\frac{1}{5},\ \frac{1}{x-y}=1$

$\Rightarrow x+y=5$ ....(v)

$x-y=1$ ....(vi)

Adding (v) and (vi), we get $2y = 4 \Rightarrow y = 2$

Substituting the value of y in equation (vi), we get $x - 2 = 1 \Rightarrow x = 3$

Hence, $x = 3$, $y = 2$.

(c)

x	25	35	45	55	65	75	Total
f	10	6	8	12	5	9	50
fx	250	210	360	660	325	675	2,480

Here, $\sum f = 50, \sum fx = 2480$

$\therefore$ Mean = $\dfrac{\sum fx}{\sum f} = \dfrac{2480}{50} = 49.6$