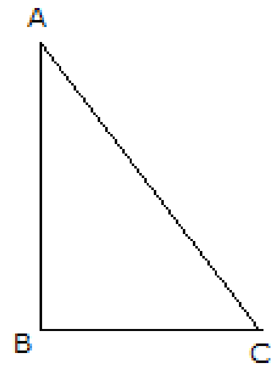
Mizoram Board
Class X
Mathematics
Sample Paper – 1 Solution

Time: 3 hrs **Total Marks: 80**

Section A

1. Correct answer: iv
In the word 'PROBABILITY', there are 11 letters out of which 4 are vowels (O, A, I, I).
P (getting a vowel) = $\frac{4}{11}$

2. Correct answer: ii
Let AB be the height h of the pole.
Length of the shadow = BC = $\sqrt{3}\,h$
If θ denotes the angle of elevation of the sun, then $\tan\theta$ =
$\frac{h}{\sqrt{3}h} = \frac{1}{\sqrt{3}}$
$\Rightarrow \theta$ = 30^oCorrect answer: C
$\angle$A = $\angle$R = 80°
$\angle$B = $\angle$Q = 60°
Therefore, using the angle sum property, we have:
$\angle$P = 180° - (80° + 60°) = 40°



3. Correct answer: iii
$\angle$A = $\angle$R = 80°
$\angle$B = $\angle$Q = 60°
Therefore, using the angle sum property, we have:
$\angle$P = 180° - (80° + 60°) = 40°

4. Correct answer: i
It is given that $\frac{1}{2}$ is a root of the equation $x^2 + kx - \frac{5}{4} = 0$.
$\therefore \left(\frac{1}{2}\right)^2 + kx\left(\frac{1}{2}\right) - \frac{5}{4} = 0$
1 + 2k − 5 = 0 or k = 2
Putting the value of k in the given equation we get,
$4x^2 + 8x - 5 = 0$
$\Rightarrow$ (2x + 5)(2x − 1) = 0
$\Rightarrow x = \frac{1}{2}$ or $-\frac{5}{2}$

Hence, the other root of the given equation is $-\frac{5}{2}$.

5. Correct answer: i
If the denominator of a rational number is of the form 2^n5^m, then it will terminate
After n places if n>m or m places if m>n.
Now, $\frac{2^3}{2^2 5}=\frac{2}{5}=\frac{2}{2^\circ 5}$ will ternminator after 1 decimal place.

6. Correct answer: ii
Given, AR = 4 cm, BR = 3 cm and AC = 11 cm
We know that the lengths of the tangents drawn to the circle from an external point are equal.
Therefore, AR = AQ = 4 cm, BR = BP = 3 cm and
PC = QC = AC – AQ = 11 cm – 4 cm = 7 cm
BC = BP + PC = 3 cm + 7 cm = 10 cm

7. Correct answer: iii

$$\frac{\text{Volume of cylinder}}{\text{Volume of cone}}=\frac{(\pi r^2 h)}{\frac{1}{3}\pi r^2 h}=\frac{3}{1}$$

Hence, the required ratio is 3:1.

8. Correct answer: ii
Consider the equation $-x^2 + 3x - 3 = 0$.
Here, a = −1, b = 3 and c = −3
Sum of the roots = $-\frac{b}{a} = -\frac{3}{(-1)} = 3$

9. Correct answer: iv
By distance formula,
Required distance = $\sqrt{(8-3)^2+(-6-4)^2}=\sqrt{25+100}=\sqrt{125}=5\sqrt{5}$ units

10. Correct answer: iii
As $\frac{4}{5}$, a and 2 are in A.P.,
Therefore, $a - \frac{4}{5} = 2 - a$
$2a = 2 + \frac{4}{5}$

a = $\frac{14}{5}\times\frac{1}{2}$

Hence, a=$\frac{7}{5}$

11. Correct answer:B

Using the Euclid's Division Algorithm to find the H.C.F, we get

$404 = 96 \times 4 + 20$

$96 = 20 \times 4 + 16$

$20 = 16 \times 1 + 4$

$16 = 4 \times 4 + 0$

12. Correct answer: A

Since, (x + 1) is a factor of f(x) = $2x^3 + ax^2 + 2bx + 1$,

f(-1) = 0

$\Rightarrow$ -2 + a - 2b + 1 =0

$\Rightarrow$ a - 2b = 1...(1)

Given, 2a - 3b = 4... (2)

Solving (1) and (2), we get,

a = 5

b = 2

13. Correct answer: B

The lines corresponding to the two equations are parallel. In other words, they do not meet at any point therefore there is no solution.

14. Correct answer: A

If $ax^2 + bx + c$, $a \neq 0$ is factorizable into product of two linear factors, then roots of $ax^2 + bx + c = 0$ can be found by equating each factor to zero.

15. Correct answer: B

Here we need to find 3 + 6 + 9 + 12 +... to 50 terms

So we take a = 3, d = 3, n = 50

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$= \frac{50}{2}[6+(50-1)3]$$

= 3825

The sum of 50 multiples of 3 is 3825.

16. Correct answer: B

Let AX and DY be the bisectors of angles A and B respectively.

$$\frac{Ar(\Delta ABC)}{Ar(\Delta DEF)} = \frac{AX^2}{DY^2}$$

$$\Rightarrow \frac{AX}{DY} = \sqrt{\frac{169}{225}}$$

$$\Rightarrow AX : DY = 13 : 15$$

17. Correct answer: B

The centroid of a triangle is the point of intersection of its medians and it divides each median in the ratio 2: 1.

18. Correct answer: D

The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

So, ratio of the area of these triangles = $\frac{4^2}{9^2} = \frac{16}{81}$

19. Correct answer: B

Here we need to find 3 + 6 + 9 + 12 +... to 50 terms

So we take a = 3, d = 3, n = 50

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{50}{2}[6 + (50-1)3]$$

= 3825

The sum of 50 multiples of 3 is 3825.

20. Correct answer:A

If $ax^2 + bx + c$, $a \neq 0$ is factorizable into product of two linear factors, then roots of $ax^2 + bx + c = 0$ can be found by equating each factor to zero

21. Correct answer: C

Take each point and substitute the coordinates of the point in the given equation to identify if it satisfies the equation.

Substitute the coordinates (3,-2) into the equation 2x - 3y = 12 to check if it satisfies the given equation.

LHS = 2(3)-3(-2) =12 = RHS

Hence, the coordinates (3, -2) satisfies the equation of the line 2x - 3y = 12.

22. Correct answer: B

The coin is cylinder with volume $\pi r^2 h$

Let there be n coins.

Thus, Volume of n coins = Volume of cuboid

$$\Rightarrow n \times \frac{22}{7} \times \frac{1.75}{2} \times \frac{1.75}{2} \times \frac{2}{10} = 11 \times 10 \times 7$$

$$\Rightarrow n \times \frac{22}{7} \times 0.875 \times 0.875 \times 0.2 = 11 \times 10 \times 7$$

$$\Rightarrow n = \frac{11 \times 10 \times 7 \times 7}{0.875 \times 0.875 \times 0.2 \times 22}$$

$$n = 1600$$

23. Correct answer: A

Arithmetic mean is the only measure of central tendency which takes into account all the observations and is most suitable when the data does not have extreme values.

24. Correct answer: D

When a coin is tossed, either a head or tail is obtained. So it is a certain event.

Section B

25. α, β are roots of x^2 - (k + 6)x + 2(2k - 1)

$\alpha+\beta=k+6, \alpha\beta=2(2k-1)$

Now, $\alpha+\beta=\frac{1}{2}\alpha\beta$

$\Rightarrow k+6=\frac{1}{2}\times 2(2k-1)$

$\Rightarrow k+6=2k-1$

$\Rightarrow k=7$

26. 870 = 225 $\times$3 + 195
225 = 195 $\times$1 + 30
195 = 30 $\times$6 + 15
30 = 15 $\times$2 + 0
$\therefore$HCF (870,225) = 15

27. It is known that radius is $\perp$ to the tangent at the point of contact.
Therefore, m$\angle$OAT = 90°.
In ΔOAT,

$\frac{AT}{OT}$ = cos30° = $\frac{\sqrt{3}}{2}$

$\Rightarrow AT=\frac{\sqrt{3}}{2}(OT)=\frac{\sqrt{3}}{2}(4)=2\sqrt{3}$ cm

28. There are 10 ribs in an umbrella.

The area between two consecutive ribs subtends an angle of $\frac{360°}{10}=36°$ at the centre of the assumed flat circle.

Area between two consecutive ribs of circle $=\frac{36°}{360°}\text{ x } \pi r^2$

$=\frac{36°}{360°}\text{ x }\frac{22}{7}\text{ x }(40)^2$

$=\frac{1}{10}\text{ x }\frac{22}{7}\text{ x }40\text{ x }40=502.86\text{ cm}^2$

29. Let AB be the tower and BC be distance between tower and car. Let θ be the angle of depression of the car.

In ΔABC,

$$\tan\theta = \frac{BC}{AB} \Rightarrow \tan 30° = \frac{BC}{AB} \Rightarrow \frac{1}{\sqrt{3}} = \frac{BC}{150}$$

$$\therefore BC = \frac{150}{\sqrt{3}} = \frac{150\sqrt{3}}{3} = 50\sqrt{3}$$

Hence, distance between the tower and car is $50\sqrt{3}$.

30. $$\text{LHS} = \sqrt{\frac{\sec\theta - 1}{\sec\theta + 1}} + \sqrt{\frac{\sec\theta + 1}{\sec\theta - 1}}$$

$$= \frac{\left(\sqrt{\sec\theta - 1}\right)^2 + \left(\sqrt{\sec\theta + 1}\right)^2}{\left(\sqrt{\sec\theta + 1}\right)\left(\sqrt{\sec\theta - 1}\right)}$$

$$= \frac{\sec\theta - 1 + \sec\theta + 1}{\sqrt{\sec^2\theta - 1}}$$

$$= \frac{2\sec\theta}{\sqrt{\tan^2\theta}}$$

$$= \frac{2\sec\theta}{\tan\theta} = 2 \times \frac{1}{\cos\theta} \times \frac{\cos\theta}{\sin\theta} = 2\operatorname{cosec}\theta = \text{RHS}$$

Hence, LHS = RHS.

Or

$$\frac{5\sin^2 30° + \cos^2 45° - 4\tan^2 30°}{2\sin 30° \cos^2 30° + \tan 45°}$$

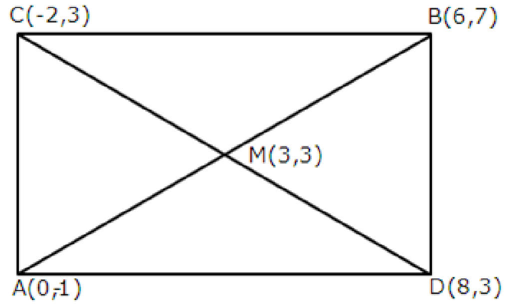
$$= \frac{5\left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 - 4\left(\frac{1}{\sqrt{3}}\right)^2}{2 \times \frac{1}{2} \times \frac{\sqrt{3}}{2} + 1}$$

$$= \frac{5\left(\frac{1}{4}\right) + \left(\frac{1}{2}\right) - 4\left(\frac{1}{3}\right)}{\frac{\sqrt{3}}{2} + 1}$$

$$= \frac{5}{6(2+\sqrt{3})}$$

31. The co-ordinates of the mid-point of AB is given by

$$\left(\frac{0+6}{2},\frac{-1+7}{2}\right)= 3,3$$



The co-ordinates of the mid-point of CD are given by

$$\left(\frac{-2+8}{2},\frac{3+3}{2}\right)= 3,3$$

$\therefore$ Diagonals AB and CD bisect each other at the point M (3, 3).

By distance formula:

$AD^2 = (8 - 0)^2 + (3 + 1)^2 = 64 + 16 = 80$

$DB^2 = (6 - 8)^2 + (7 - 3)^2 = 4 + 16 = 20$

Also, $AB^2 = (6 - 0)^2 + (7 + 1)^2 = 36 + 64 = 100$

Clearly $AD^2 + DB^2 = AB^2$

Hence the park is rectangular. Its area

$$= AD \times DB = \sqrt{80} \times \sqrt{20} = \sqrt{1600} = 40 \text{ sq.km}$$

32. Discriminant = $b^2 - 4ac = 49 - 4 \times 6 \times 2 = 1 > 0$

So, the given equation has two distinct real roots.

Now, $6x^2 - 7x + 2 = 0$

$\Rightarrow 36x^2 - 42x + 12 = 0$

$$\Rightarrow \left(6x-\frac{7}{2}\right)^2 + 12 - \left(\frac{7}{2}\right)^2 = 0$$

$$\Rightarrow \left(6x-\frac{7}{2}\right)^2 - \left(\frac{1}{2}\right)^2 = 0$$

$$\Rightarrow \left(6x-\frac{7}{2}\right)^2 = \left(\frac{1}{2}\right)^2$$

The roots are given by $6x-\frac{7}{2} = \pm \frac{1}{2}$

$\Rightarrow 6x = 4, 3$

$\Rightarrow x = \frac{2}{3},\frac{1}{2}$

Or

Let $f(x) = 2x^2 - 3x + p$

If f(a) = 0, then it is said that a is a zero of f(x).

Given, 3 is a zero of f(x).

$\therefore f(3) = 0$

$2(3)^2 - 3(3) + p = 0$

$18 - 9 + p = 0$

$p = -9$

$\therefore f(x) = 2x^2 - 3x - 9$

$= 2x^2 - 6x + 3x - 9$
$= 2x(x - 3) + 3(x - 3)$
$= (x - 3)(2x + 3)$

Thus, the other zero of f(x) is $\frac{-3}{2}$.

33. The system has infinitely many solutions. Therefore,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{2}{a-b} = \frac{3}{a+b} = \frac{7}{3a+b-2}$$

Equating (1) and (2), we get:
2a + 2b = 3a - 3b
or, a = 5b ... (4)
Equating (2) and (3), we get:
9a + 3b - 6 = 7a + 7b
or, 2a - 4b = 6 ... (5)
On solving equations (4) and (5), we get,
10b - 4b = 6 or **b = 1**
Thus, from (4), we get, **a = 5**

34. To obtain minimum number of rooms, we need to accommodate maximum number of participants.
In each room we have to have same number of participants in the same subject.

$60 = 2^2 \times 3 \times 5$

$84 = 2^2 \times 3 \times 7$

$108 = 2^2 \times 3^3$

$\text{HCF}(60,\ 84,\ 108) = 2^2 \times 3 = 12$

So, in each room 12 participants can be seated.

Number of rooms required

$= \frac{\text{Total number of participants}}{12}$

$= \frac{60+84+108}{12} = 21$

Section C

35. Assume the fixed charge = Rs. x
and the subsequent charge = Rs. y
According to the question, we have,
$x + 4y = 27$ ... (i)
and $x + 2y = 21$ ... (ii)
Subtracting (ii) from (i), we have,
$2y = 6$ or $y = 3$
So, from (i),
$x = 27 - 12 = 15$
Thus, the fixed charge is Rs. 15 and the charge for each extra day is Rs. 3.

36. Let the required point $P = (x, 0)$
and the required ratio = $k : 1$
Here $m_1 = k$ and $m_2 = 1$
$x_1 = 3, x_2 = -2, y_1 = -3$ and $y_2 = 7$
By the Section formula,

$$(x, 0) = \left[\frac{k(-2) + 1(3)}{k+1}, \frac{k(7) + 1(-3)}{k+1}\right]$$

$$\Rightarrow (x, 0) = \left[\frac{-2k+3}{k+1}, \frac{7k-3}{k+1}\right]$$

So, $x = \frac{-2k+3}{k+1}$ and $0 = \frac{7k-3}{k+1}$

From (2), $7k - 3 = 0 \Rightarrow k = \frac{3}{7}$

Substituting in $x = \frac{-2k+3}{k+1}$, we get

$$x = \frac{-2\left(\frac{3}{7}\right)+3}{\frac{3}{7}+1} \Rightarrow x = 1.5$$

Ratio $(k:1) = \frac{3}{7} : 1 = 3 : 7$

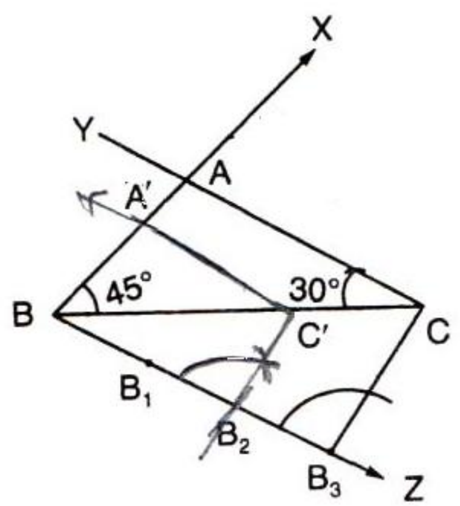
Point of division on x-axis = $(1.5, 0)$

37. It is given that ∠A = 105°, ∠C = 30°.
Using angle sum property of triangle, we get, ∠B = 45°

The steps of construction are as follows:

1. Draw a line segment BC = 6 cm.
2. At B, draw a ray making an angle of 45° with BC.
3. At C, draw a ray making an angle of 30° with BC. Let the two rays meet at point A.
4. Below BC, make an acute angle ∠CBX.
5. Along BX mark off three points B1, B2 , B3 such that BB1 = B1B2 = B2B3.
6. Join B3C.
7. From B2, draw B2C′ || B3C.
8. From C′, draw C′A′ || CA, meeting BA at the point A′.

Then A′BC′ is the required triangle.



Or

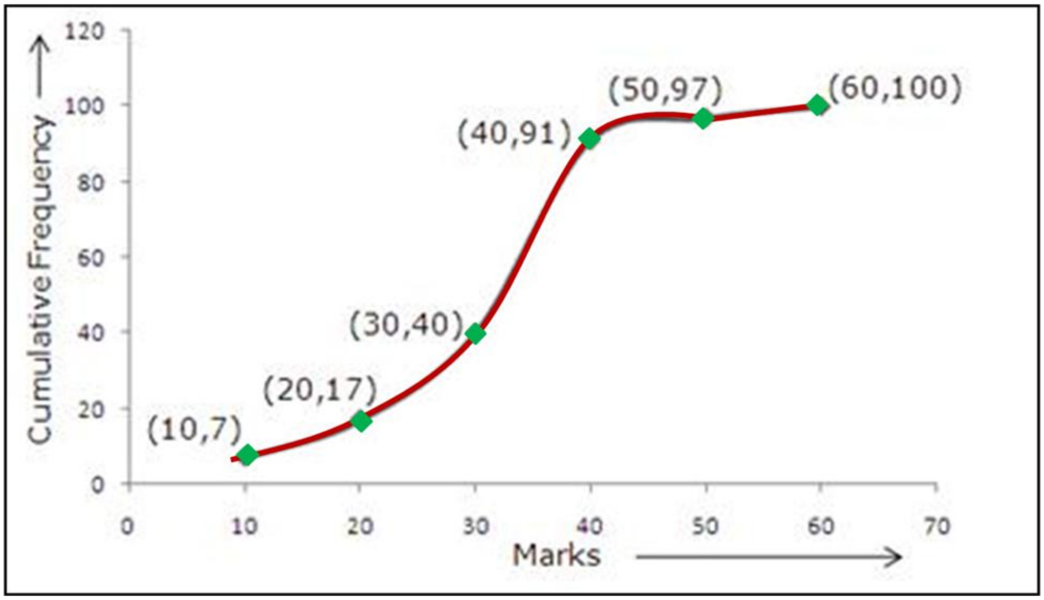
We first prepare the cumulative frequency distribution table as given below:

Marks	No. of students	Marks less than	Cumulative frequency
0-10	7	10	7
10-20	10	20	17
20-30	23	30	40
30-40	51	40	91

40-50	6	50	97
50-60	3	60	100

Now, we mark the upper class limits along x-axis by taking a suitable scale and the cumulative frequencies along the y-axis by taking a suitable scale. Thus, we plot the points (10,7), (20,17), (30,40), (40,91), (50,97) and (60,100).

Join the plotted points by a free hand to obtain the required ogive.



38. Here, a = 52° and d = 8°

Let the polygon have n sides. Then, the sum of interior angles of the polygon is

$(n - 2)180°$

$\Rightarrow S_n = (n - 2)180$

$\Rightarrow \frac{n}{2}[2a + (n - 1)d] = (n - 2)180$

$\Rightarrow \frac{n}{2}[104 + (n-1)8] = (n - 2)180$

$\Rightarrow \frac{n}{2}[8n + 96] = (n - 2)180$

$\Rightarrow 8n^2 + 96n = 360n - 720$

$\Rightarrow 8n^2 - 264n + 720 = 0$

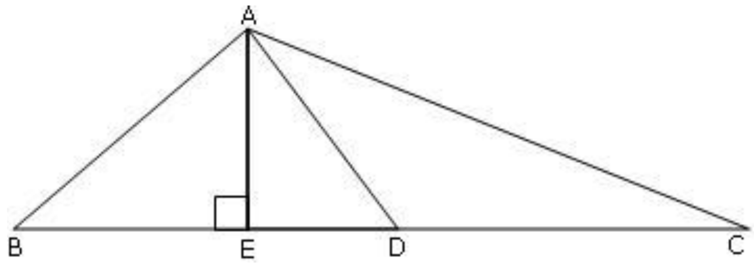
$\Rightarrow n^2 - 33n + 90 = 0$

$\Rightarrow (n - 30)(n - 3) = 0$

$\Rightarrow n = 3, 30$

Hence, the number of sides of the polygon can be either 3 or 30.

39. AD is the median of triangle ABC since D is the mid-point of BC.



$$\Rightarrow BD = DC = \frac{BC}{2} \text{....(i)}$$

In right ΔAEB,

$$AB^2 = AE^2 + BE^2 \text{ (Pythagoras theorem)}$$

$$AB^2 = (AD^2 - DE^2) + (BD - DE)^2$$

Using the Pythagoras theorem in right triangle AED and $BE = BD - DE$, we get

$$AB^2 = AD^2 - DE^2 + \left(\frac{BC}{2} - DE\right) \text{....(From (i))}$$

$$AB^2 = AD^2 - DE^2 + \frac{BC^2}{4} + DE^2 - 2\left(\frac{BC \times DE}{2}\right)$$

$$AB^2 = AD^2 - BC \times DE + \frac{BC^2}{4}$$

Hence proved.

40. To solve the equations, make the table corresponding to each equation.

2x – y +6 = 0

y = 2x+6

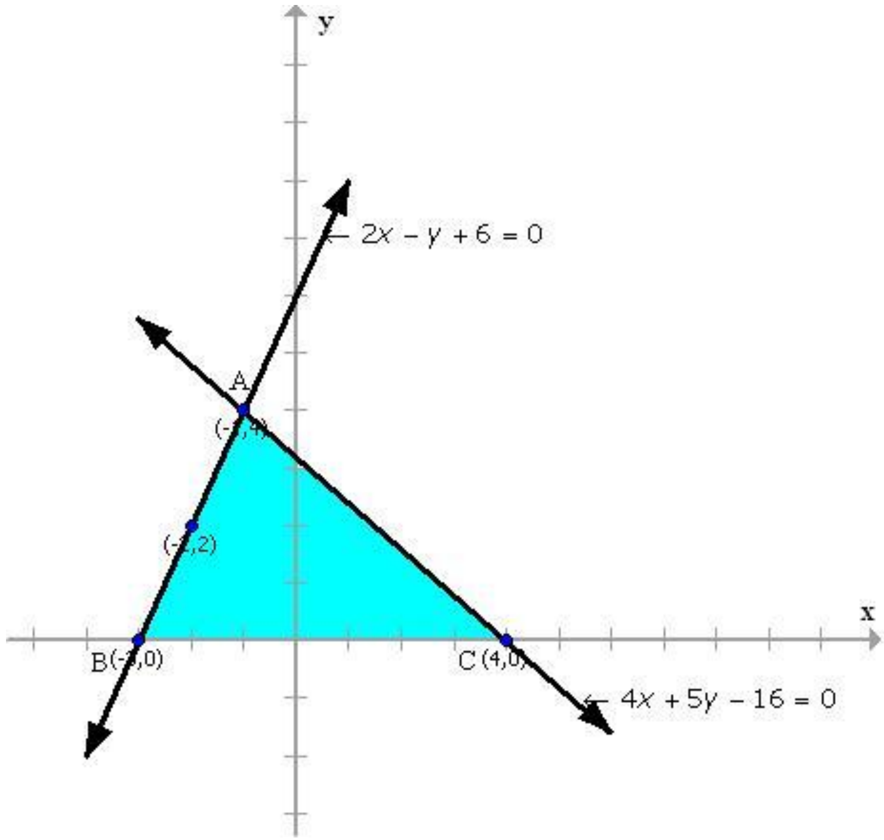
x	-1	-2	-3
y	4	2	0

4x + 5y - 16=0

$y = \frac{16 - 4x}{5}$

x	4	-1
y	0	4

Now plot the points and draw the graph.



Because the lines intersect at the point (-1, 4), x = -1 and y = 4 is the solution. Also, by observation, vertices of triangle formed by lines and x-axis are A (-1, 4), B (-3, 0) and C (4, 0).

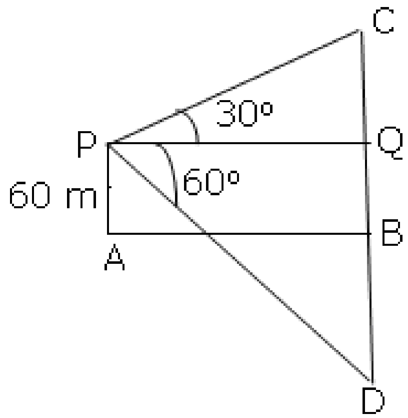
41. Let C be the cloud and D be its reflection. Let the height of the cloud be H metres.

BC = BD = H

BQ = AP = 60 m. Therefore CQ = H − 60 and DQ = H + 60

In ΔCQP,

$\frac{PQ}{CQ} = \cot 30^\circ$



⇒ $\frac{PQ}{H-60} = \sqrt{3}$ ⇒ PQ = (H - 60)$\sqrt{3}$ m ....(i)

In ΔDQP,

$\frac{PQ}{DQ} = \cot 60°$

$\Rightarrow \frac{PQ}{H+60} = \frac{1}{\sqrt{3}} \Rightarrow PQ = \frac{(H+60)}{\sqrt{3}}$ ....(ii)

From (i) and (ii),

$(H-60)\sqrt{3} = \frac{(H+60)}{\sqrt{3}}$

$\Rightarrow 3H - 180 = H + 60 \qquad \Rightarrow H = 120$

Thus, the height of the cloud is 120 m.

Section D

42. Diameter of graphite = 1mm = 0.1cm

Therefore, radius of graphite = $\frac{0.1}{2}$ = 0.05 cm

Length of pencil = 10 cm

Volume of graphite = $\pi r^2 h = \frac{22}{7} \times (.05)^2 \times 10 = 0.0785$ cm^3

Therefore, weight of graphite = volume × density
= 0.0785 × 2.3
= 0.180 gm
Diameter of the pencil = 0.7 cm
Therefore, radius of the pencil = 0.35 cm

Therefore, volume of the pencil = $\pi R^2 h = \frac{22}{7} \times (0.35)^2 \times 10$

Therefore, volume of wood = Volume of pencil – Volume of graphite
Therefore, Volume of wood = $\pi R^2 h - \pi r^2 h$
= $\pi h(R^2 - r^2)$

= $\frac{22}{7} \times 10[(0.35)^2 - (0.05)^2]$

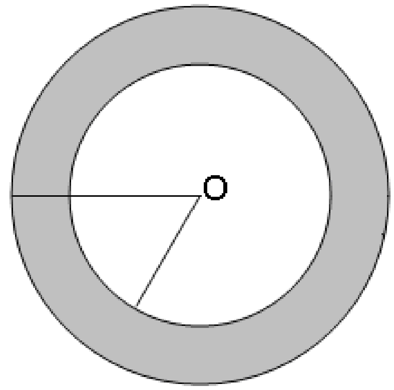
= 3.771 cm^3
Weight of wood = Volume × density
= 3.771 × .6
= 2.2626 gm

Or

Let the radius of inner circle be r cm. Then, its circumference = $(2\pi r)$ cm

$\therefore 2\pi r = 88 \Rightarrow 2 \times \frac{22}{7} \times r = 88$

$\therefore r = \frac{88 \times 7}{44} = 14$ cm



So, radius of the inner circle, i.e., $r_1 = 14$ cm

Let r_2 be the radius of the outer circle.

$\therefore$ Area of the ring $= \pi r_2^2 - \pi r_1^2 = \pi(r_2^2 - r_1^2)\text{cm}^2$

$$= \frac{22}{7}(r_2^2 - 14^2)\text{cm}^2 = \frac{22}{7}(r_2^2 - 196)\text{ cm}^2$$

By the given condition,

$$\frac{22}{7}r_2^2 - \frac{22}{7} \times 196 = 346.5$$

$$\Rightarrow \quad \frac{22}{7}r_2^2 - 616 = 346.5 \Rightarrow \frac{22}{7}r_2^2 = 616 + 346.5 = 962.5$$

$$\Rightarrow \quad r_2^2 = \frac{962.5 \times 7}{22} = 306.25 \Rightarrow r_2 = \sqrt{306.25} = 17.5\text{ cm}$$

Hence, the radius of the outer circle is 17.5 cm.

43.

C	50-60	60-70	70-80	80-90	90-100	100-110	Total
f_i	5	3	4	p	2	13	27 + p
x	55	65	75	85	95	105	
f_i	275	195	300	85p	190	1365	2325+ 85p

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i}$$

Substituting the values,

$$86 = \frac{2325 + 85p}{27 + p}$$

$$\Rightarrow 86p + 2322 = 2325 + 85p$$

$$\Rightarrow p = 3$$

44. Given, XY || QR

By using the Basic Proportionality Theorem,

$$\frac{PX}{XQ}=\frac{PY}{YR}$$

$$\Rightarrow \frac{PX}{XQ}+1=\frac{PY}{YR}+1$$

$$\Rightarrow \frac{PX+XQ}{XQ}=\frac{PY+YR}{YR}$$

$$\Rightarrow \frac{PQ}{XQ}=\frac{PR}{YR}$$

$$\Rightarrow \frac{7}{3}=\frac{6.3}{YR}$$

$$\Rightarrow YR=\frac{6.3\times 3}{7}=2.7\,c.m$$

Or

Let ABCD be a square of side a.

Therefore, its diagonal $=\sqrt{2}a$

Two desired equilateral triangles are formed as ΔABE and ΔDBF

Length of one side of ΔABE = a

Length of one side of ΔDBF $=\sqrt{2}a$

We know that equilateral triangles have all angles as 60°. So, all equilateral triangles are similar to each other.

So, the ratio between areas of these triangles will be equal to the square of the ratio of their corresponding sides of these triangles.

$$\therefore \frac{\text{area of }\Delta\,ABE}{\text{area of }\Delta\,DBF}=\left(\frac{a}{\sqrt{2}a}\right)^2=\frac{1}{2}$$

Hence, proved.