Meghalaya Board
Class X
Mathematics
Sample Paper – 2 Solution

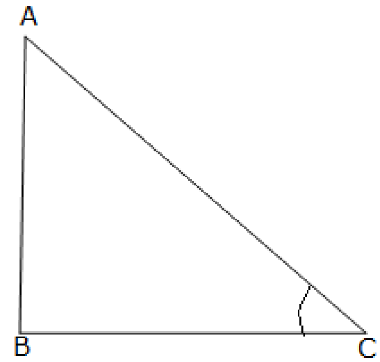
Time: 3 hrs **Total Marks: 80**

Section A

1. Correct answer: i
When a die is thrown once the outcomes are 1, 2, 3, 4, 5, and 6 out which 2, 3 and 5 are prime numbers.

Therefore, P (prime number) = $\frac{3}{6}=\frac{1}{2}$

2. Correct answer: iv
Consider the following figure.



Let AB be the pole and BC be its shadow.

Given that $\frac{AB}{BC}=\frac{\sqrt{3}}{1}$

Let $\theta=\angle ACB$ be the angle of elevation.

Now consider ΔABC:

$\tan\theta=\frac{AB}{BC}=\frac{\sqrt{3}}{1}$ ....(1)

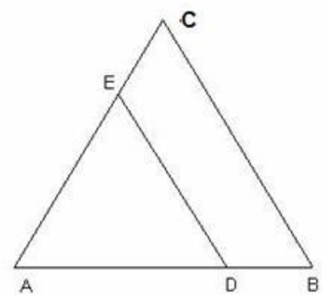
Also, we know that $\tan 60^\circ=\sqrt{3}$....(2)

From equations (1) and (2), we have,

$\tan\theta=\tan 60^\circ$

$\Rightarrow \theta=60^\circ$

3. Correct answer: iv



$\frac{AD}{AE}=\frac{DB}{EC}$

$\Rightarrow \frac{x}{x+2}=\frac{x-2}{x-1}$

$\Rightarrow x(x-1)=x^2-4$

$\Rightarrow x=4$

4. Correct answer: i

$x^2 + 2x + 1 = (x+1)^2$

$\Rightarrow$ -1, -1 are the roots of the given polynomial f(x).

$\therefore$ $\alpha = \beta = -1$

$$\therefore \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta+\alpha}{\alpha\beta} = \frac{-1-1}{(-1)(-1)} = -2$$

5. Correct answer: iii

A real number is an irrational number when it has a non-terminating non repeating decimal representation.

Thus, 0.101100101010……. is an irrational number.

6. Correct answer: iv

The angle between a pair of tangents to a circle which are inclined to each other at an angle is supplementary to the angle between the two radii of the circle.

Thus, the angle between the radii of the circle = $180^\circ - 35^\circ = 145^\circ$

7. Correct answer: ii

Centroid of $\Delta ABC = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

$$= \left(\frac{3-7+10}{3}, \frac{-5+4-2}{3}\right)$$

$$= (2, -1)$$

8. Correct answer: ii

We know that, the probability of an event E lies between 0 and 1, i.e., $0 \leq P(E) \leq 1$. Thus, −1.5 cannot be the probability of any event.

9. Correct answer: iv

The centre (2, −3) is mid-point of diameter AB.

Let the co-ordinates of A be (x, y).

$\therefore \left(\frac{x+1}{2}, \frac{y+4}{2}\right) = (2, -3)$

$\Rightarrow$ x + 1 = 4 and y + 4 = -6

$\Rightarrow$ x = 3 and y = −10

Therefore, the co-ordinates of A are (3, −10).

10. Correct answer: iii

The given quadratic equation is $kx^2 - 5x + k = 0$.

For repeated roots, we have

$b^2 - 4ac = 0$

$\Rightarrow (-5)^2 - 4 \times k \times k = 0$

$\Rightarrow 25 - 4k^2 = 0$

$\Rightarrow 4k^2 = 25$

$\Rightarrow k = \pm\frac{5}{2}$

Section B

11. Since a – b, a and a + b are the zeros of f(x).

$\therefore (a-b)+a+(a+b) = -\frac{\text{Coeff. of } x^2}{\text{Coeff. of } x^3}$

$\Rightarrow 3a = -\frac{-3}{1}$

$\Rightarrow 3a = 3$

$\Rightarrow a = 1$

And, $(a-b)a(a+b) = -\frac{\text{Coeff. term}}{\text{Coeff. of } x^3}$

$\Rightarrow a(a^2 - b^2) = -\frac{1}{1}$

$\Rightarrow 1(1-b^2) = -1$

$\Rightarrow b^2 = 2$

$\Rightarrow b = \pm\sqrt{2}$

12. $455 = 84 \times 5 + 35$

$\Rightarrow 84 = 35 \times 2 + 14$

$\Rightarrow 35 = 14 \times 2 + 7$

$\Rightarrow 14 = 7 \times 2 + 0$

Therefore, HCF = 7

13. Since the lengths of tangents from an exterior point to a circle are equal.

Therefore, XP = XQ (from X) ….(i)

AP = AR (from A) ….(ii)

BQ = BR (from B) ….(iii)

Now, XP = XQ

$\Rightarrow$ XA + AP = XB + BQ

$\Rightarrow$ XA + AR = XB + BR (Using (ii) and (iii))

14. Let r be the radius of the wheel.

Distance covered in 1 revolution = $2\pi r$

Distance covered in 5000 revolutions = $5000 \times 2\pi r$ = 11 km

$5000 \times \frac{22}{7}(2r) = 11 \times 1000$ metres

$2r = \frac{7}{10}$ metres = 70 cm

Thus, the diameter of the wheel is 70 cm.

15. In right triangle ABC, we have

$\sin\ 45^\circ = \frac{BC}{AC}$

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{BC}{150}$

$\Rightarrow BC = \frac{150}{\sqrt{2}}$

$\Rightarrow BC = 75\sqrt{2}$ m

Thus, the width of the river is $75\sqrt{2}$ metres.

16.

$$\text{LHS} = \frac{\sec A + \tan A}{\sec A - \tan A}$$

$$= \frac{\sec A + \tan A}{\sec A - \tan A} \times \frac{\sec A + \tan A}{\sec A + \tan A}$$

$$= \frac{(\sec A + \tan A)^2}{\sec^2 A - \tan^2 A}$$

$$= (\sec A + \tan A) \qquad \left(\because \sec^2\theta = 1 + \tan^2\theta \therefore \sec^2\theta - \tan^2\theta = 1\right)$$

$$= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)^2$$

$$= \left(\frac{1 + \sin A}{\cos A}\right)^2$$

Hence L.H.S. = R.H.S

Or

Consider,

$7\sin^2\theta + 3\cos^2\theta = 4$

$\Rightarrow 7\sin^2\theta + 3(1 - \sin^2\theta) = 4$

$\Rightarrow 7\sin^2\theta + 3 - 3\sin^2\theta = 4$

$\Rightarrow 4\sin^2\theta = 1$

$\Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = 30^\circ$

Thus, $\sec 30^\circ + \operatorname{cosec} 30^\circ = \frac{2}{\sqrt{3}} + 2$

Section C

17. Area of quadrilateral ABCD = Area of ΔABC + Area of ΔACD

Area of triangle $=\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$

Area of ΔABC = $\frac{1}{2}$ [1(−3 − 2) + 7(2 − 1) + 12(1 + 3)]

= $\frac{1}{2}$ [−5 + 7 + 48]

= 25 sq. units

Area of ΔACD = $\frac{1}{2}$ [1(2 − 21) + 12(21 − 1) + 7(1 − 2)]

= $\frac{1}{2}$ [−19 + 240 − 7]

= 107 sq. units

Therefore, area of quadrilateral ABCD = 25 + 107 = 132 sq. units.

18.

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{x}=\frac{1}{a+b+x}$$

$$\Rightarrow \frac{1}{a}+\frac{1}{b}=\frac{1}{a+b+x}-\frac{1}{x}$$

$$\Rightarrow \frac{a+b}{ab}=\frac{x-a-b-x}{(a+b+x)x}$$

$$\Rightarrow \frac{a+b}{ab}=\frac{-(a+b)}{(a+b+x)x}$$

$\Rightarrow ax + bx + x^2 = -ab$

$\Rightarrow x^2 + ax + bx + ab = 0$

$\Rightarrow x(x + a) + b(x + a) = 0$

$\Rightarrow (x + a)(x + b) = 0$

$\Rightarrow x + a = 0$ or $x + b = 0$

$\Rightarrow x = -a, -b$

19.

$$\frac{x}{a}+\frac{y}{b} = 2$$

$\Rightarrow$ bx + ay = 2ab… (1)

ax − by = $a^2 - b^2$… (2)

Multiplying (1) with a and (2) with b, we get

$$\cancel{abx} + a^2y = 2a^2b$$
$$\cancel{abx} - b^2y = a^2b - b^3$$
$$- \quad + \quad - \quad +$$
$$y(a^2 + b^2) = a^2b + b^3$$
$$\Rightarrow y(a^2 + b^2) = b(a^2 + b^2)$$
$$\Rightarrow y = b$$

From (1), bx + ab = 2ab
$\Rightarrow$ bx = ab
$\Rightarrow$ x = a
Hence, x = a and y = b.

20. Let $\frac{3}{2\sqrt{5}}$ be a rational number. So, $\frac{3}{2\sqrt{5}} = \frac{a}{b}$ Where a and b are co-prime integers and b $\neq 0$.

$\sqrt{5} = \frac{3b}{2a}$

Now, a, b, 2 and 3 are integers Therefore, $\frac{3b}{2a}$ is a rational number.

$\Rightarrow \sqrt{5}$ is a rational number.

This is a contradiction as we know that $\sqrt{5}$ is irrational.

Therefore, our assumption is wrong. Hence, $\frac{3}{2\sqrt{5}}$ is an irrational number.

21. Let us assume that Prema invests Rs. x @10% and Rs. y @8% in the first year.
We know that

Interest = $\frac{P \times R \times T}{100}$

According to given conditions,

$\frac{x \times 10 \times 1}{100} + \frac{y \times 8 \times 1}{100} = 1640$

$\Rightarrow$ 10x + 8y = 164000 ...(i)

After interchanging the rates, we have

$\frac{y \times 10 \times 1}{100} + \frac{x \times 8 \times 1}{100} = 1600$

$\Rightarrow$ 10y + 8x=160000

Or, 8x + 10y=160000 ...(ii)

Adding (i) and (ii), we get
18x + 18y=324000
$\Rightarrow$ x + y = 18000 ...(iii)

Subtracting (ii) from (i), we get

$2x - 2y = 4000$
$\Rightarrow x - y = 2000$ ...(iv)

Adding (iii) and (iv), we get
$2x = 20000$
$\Rightarrow x = 10000$
Substituting this value of x in (iii), we get
$y = 8000$

Thus, the sums invested in the first year at the rate 10% and 8% are Rs. 10000 and Rs. 8000, respectively.

22. Let P(x, y), Q(a + b, b − a) and R(a − b, a + b) be the given points.
It is given that $PQ = PR \Rightarrow PQ^2 = PR^2$

$\{x - (a + b)\}^2 + \{y - (b - a)\}^2 = \{x - (a - b)\}^2 + \{y - (a + b)\}^2$

$\Rightarrow x^2 - 2x(a + b) + (a + b)^2 + y^2 - 2y(b - a) + (b - a)^2$
$= x^2 + (a - b)^2 - 2x(a - b) + y^2 - 2y(a + b) + (a + b)^2$
$\Rightarrow -2x(a + b) - 2y(b - a) = -2x(a - b) - 2y(a + b)$
$\Rightarrow -ax - bx - by + ay = -ax + bx - ay - by$
$\Rightarrow 2bx = 2ay$
$\Rightarrow bx = ay$

Section D

23. Steps of construction:-

(1) Draw a line segment AB of 4.6 cm.

(2) At A draw an angle of 60°.

(3) With centre B and radius 5.1 cm draw an arc which intersects line AC of angle at C.

(4) Join BC.

(5)At A draw an acute angle BAX of any measure.

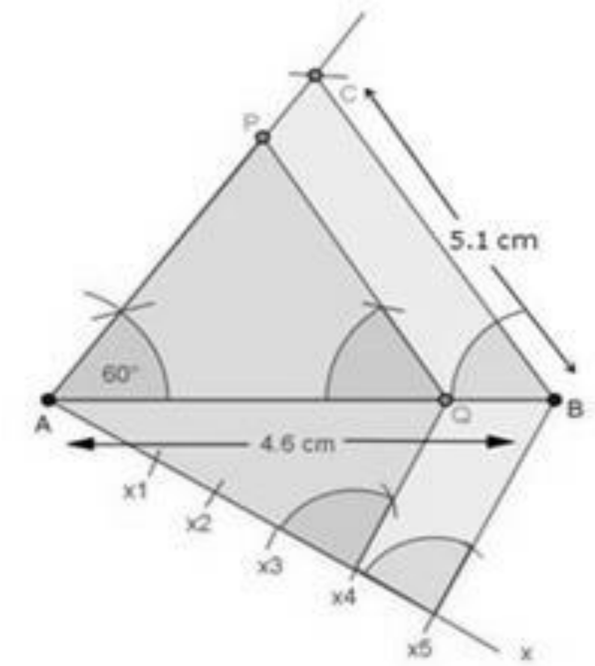
(6) Starting from A, cut 5 equal parts on Ax_5.

(7) Join X_5B

(8) Through X_4, Draw $X_4Q \parallel X_5B$

(9) Through Q, Draw QP || BC

$\therefore \Delta PAQ \sim \Delta CAB$



24. There are three sections of each class and it is given that the number of trees planted by any class is equal to class number.

The number of trees planted by class I = number of sections $\times$ 1 = 3 $\times$ 1 = 3

The number of trees planted by class II = number of sections $\times$ 2 = 3 $\times$ 2 = 6

The number of trees planted by class III = number of sections $\times$ 3 = 3 $\times$ 3 = 9

Therefore, we have the sequence: 3, 6, 9, ..., (12 terms)

To find total number of trees planted by all the students, we need to find sum of the 12 terms of the sequence.

First term = a = 3

Common difference = d = 6 - 3 = 3

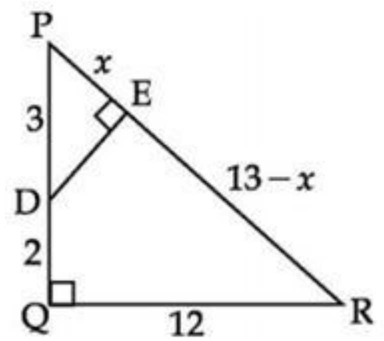
n = 12

$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$\Rightarrow S_{12} = \frac{12}{2}[2\times3 + (12-1)3]$$
$$= 6[6+33]$$
$$= 6\times39$$
$$= 234$$

Thus, in total 234 trees will be planted by the students.

Values inferred are environmental friendly and social.

25. In right ΔPQR,
$PR^2 = PQ^2 + QR^2 = 25 + 144 = 169$
∴ PR = 13 cm
Let PE = x, then ER = 13 – x
In ΔPQR and ΔPED,
∠PQR = ∠PED
∠QPR = ∠EPD
∴ ΔPQR ~ ΔPED[AA similarity]



$$\therefore \frac{PQ}{PE} = \frac{QR}{ED} = \frac{PR}{PD}$$
$$\frac{5}{x} = \frac{12}{ED} = \frac{13}{3}$$
$$\therefore PE = x = \frac{5\times3}{13} = \frac{15}{13} = 1\frac{2}{13}$$
$$ED = \frac{12\times3}{13} = \frac{36}{13} = 2\frac{10}{13}\text{cm}$$

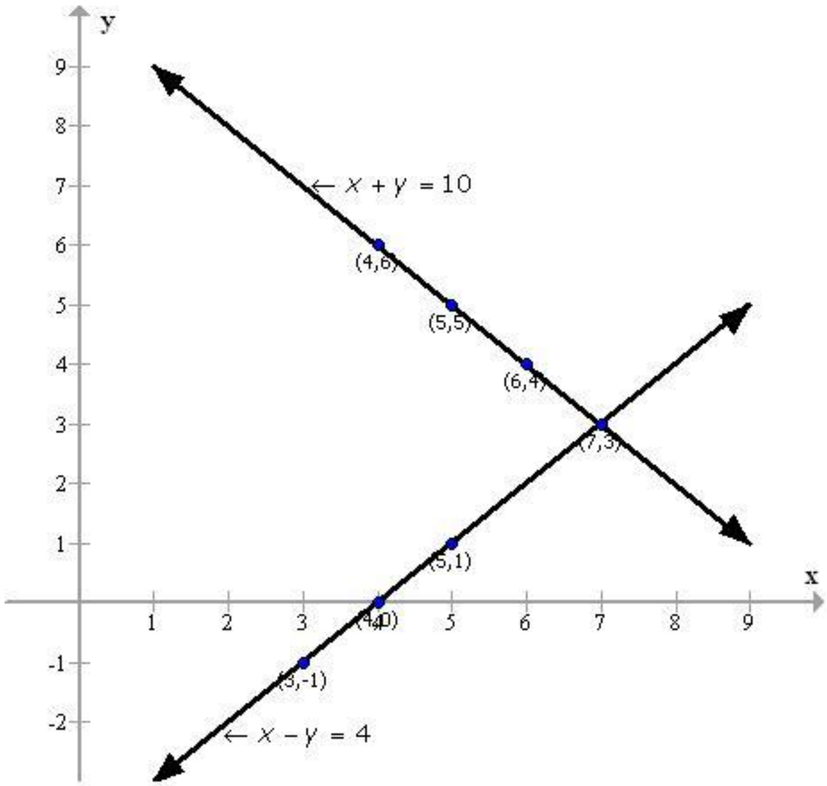
26. Let the number of girls and boys in the class be x and y, respectively.
According to the given conditions, we have:
x + y = 10
x - y = 4
x + y = 10 ⇒ x = 10 - y
Three solutions of this equation can be written in a table as follows:

x	5	4	6
y	5	6	4

x - y = 4 ⇒ x = 4 + y
Three solutions of this equation can be written in a table as follows:

x	5	4	3
y	1	0	-

The graphs of the two equations can be drawn as follows:



From the graph, it can be observed that the two lines intersect each other at the point (7, 3).

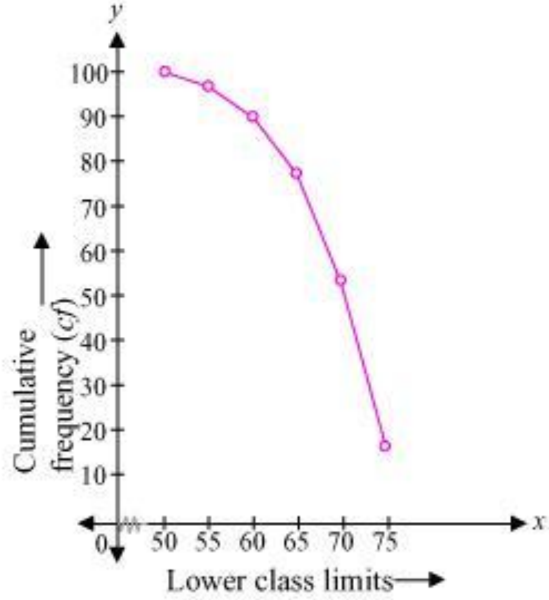
So, x = 7 and y = 3 is the required solution of the given pair of equations.

Or

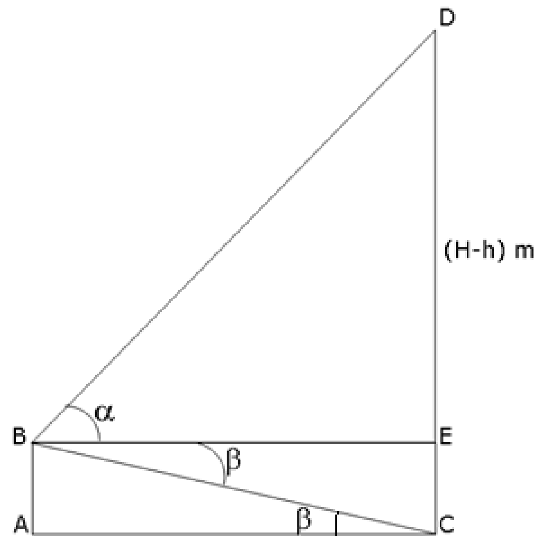
We can obtain cumulative frequency distribution of more than type as following:

Production yield (lower class limits)	Cumulative frequency
More than or equal to 50	100
More than or equal to 55	100 - 2 = 98
More than or equal to 60	98 - 8 = 90
More than or equal to 65	90 - 12 = 78
More than or equal to 70	78 - 24 = 54
More than or equal to 75	54 - 38 = 16

Now, taking lower class limits on x-axis and their respective cumulative frequencies on y-axis, we can obtain the ogive as follows:



27. Let B be the window of a house AB and let CD be the other house.
Then, AB = EC = h metres.
Let CD = H metres. Then, ED = (H − h) m



In ΔBED,

$\cot \alpha = \dfrac{BE}{ED}$

BE = (H − h) cot α ... (a)

In ΔACB,

$\dfrac{AC}{AB} = \cot \beta$

AC = h.cot β .... (b)

But BE = AC

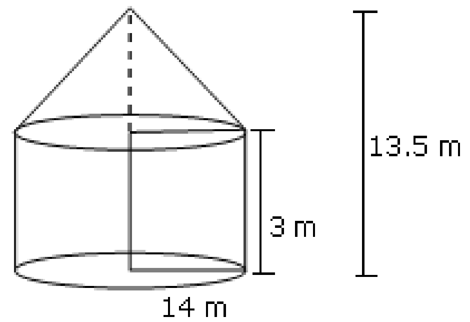
∴(H − h) cot α = hcot β ....[From (a) and (b)]

$H = h\,\dfrac{(\cot\alpha + \cot\beta)}{\cot\alpha}$

H = h(1 + tanα cotβ)

Thus, the height of the opposite house is h(1 + tanα.cotβ) metres.

28. Radius of conical portion = Radius of cylindrical portion = 14 m
Height of cylindrical portion = 3 m
Height of conical portion = 13.5 m − 3 m = 10.5 m



C.S.A. of tent = C.S.A. of cylinder + C.S.A. of cone

$= 2\pi rh + \pi rl$

$= 2\pi\ (14)\ (3) + \pi(14)$ $\sqrt{14^2 + 10.5^2}$

$= 264 + 14\pi\ \sqrt{306.25}$

$= 264 + 14\ \pi\ (17.5)$

$= 264 + 770$

$= 1034\ m^2$

Cost of painting the inside of tent,
i.e. 1034 m^2 at the rate of Rs. 2 per sq. m = Rs. 1034 x 2 = Rs. 2068.

Or

Given, AC = BD = 7 cm and AB = CD = 1.75 cm = $\frac{7}{4}$ cm

Area of shaded region = 2 (area of semi-circle of radius $\frac{7}{2}$ cm) − 2(area of semi-circle of radius $\frac{7}{8}$ cm)

$$=2\left[\frac{1}{2}\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\right]-2\left[\frac{1}{2}\times\frac{22}{7}\times\frac{7}{8}\times\frac{7}{8}\right]$$

$$=\left(\frac{77}{2}-\frac{77}{32}\right)$$

$$=\frac{77}{2}\left[1-\frac{1}{16}\right]=\frac{77}{2}\times\frac{15}{16}\text{ cm}^2=\frac{1155}{32}\text{ cm}^2\ =36.1\text{ cm}^2$$

29. Consider the following table:

C.I	f_i	x_i	d_i	$f_i d_i$
100 - 150	4	125	-2	-8
150 - 200	5	175	-1	-5
200 - 250	12	225	0	0
250 - 300	2	275	1	2
300 - 350	2	325	2	4
Total	25			-7

Let A = 225

$$d_i = \frac{x_i - 225}{50}$$

$$\bar{x} = A + \frac{\sum f_i d_i}{\sum f_i} \times h = 225 - \frac{7}{\cancel{25}} \times \cancel{50}^{2} = 225 - 14 = 211$$

30. Given : In ΔXYZ and ΔDEF

$$\frac{XY}{DE} = \frac{YZ}{EF} = \frac{XA}{DB} \quad ...(1)$$

To prove: $\Delta XYZ \sim \Delta DEF$

Proof: Since XA and DB are medians

2YA = YZ

2EB = EF ...(2)

From (1) and (2)

$$\frac{XY}{DE} = \frac{2YA}{2EB} = \frac{XA}{DB}$$

$\Rightarrow \Delta XYA \sim \Delta DEB$ (BY SAS rule)

$\Rightarrow \angle Y = \angle E$ ...(3)

Now, in ΔXYZ and ΔDEF,

$\frac{XY}{DE} = \frac{YZ}{EF}$ from (1)

$\angle Y = \angle E$ from (3)

$\Rightarrow \Delta XYZ \sim \Delta DEF$ (BY SAS rule)