Nagaland Board
Class XI
Mathematics
Sample Solution-1

Time allowed: 3 hours **Maximum Marks: 80**

Section A

1. Choose the correct answer from the given alternatives.

(a) Correct option: (i)

Let the G.P. be $a, ar, ar^2.....$

Then the new G.P. is $2a, 2ar, 2ar^2.....$ or $2a, (2a)r, (2a)r^2...$

So we see that the common ratio is same as that of the original G.P.

(b) Correct option: (iv)

A set which is empty or consists of a definite number of elements is called a finite set. The roots of the equation $x^2 - 25 = 0$, are -5, 5. Hence, the set of roots of the equation $x^2 - 25 = 0$ is finite.

(c) Correct option: (iv)

In a Δ ABC, we have $A + B + C = 180°$

So, $\sin 2A = 2\sin A \cos A$

$= 2\sin[180° - (B + C)]\cos[180° - (B + C)]$

$= 2\sin(B + C) \times [-\cos(B + C)]$

$= -2\sin(B + C)\cos(B + C)$

(d) Correct option: (iv)

Probability of occurrence of neither E nor F

$P(\overline{E} \cap \overline{F})$

Using De Morgan's law we can write

$\overline{E} \cap \overline{F} = \overline{E \cup F}$

$\Rightarrow P(\overline{E} \cap \overline{F}) = P(\overline{E \cup F})$

$\Rightarrow P(\overline{E} \cap \overline{F}) = 1 - P(E \cup F)$

(e) Correct option: (i)

$$\frac{7!}{5!\times 3!}=\frac{7\times 6\times 5!}{5!\times 3\times 2\times 1}$$

$$\frac{7!}{5!\times 3!}=7$$

(f) Correct option: (iii)

The least value of x^2 is zero and will be so when x = 0, when x = 0, y = 0

(g) Correct option: (iv)

$$\frac{x^2}{9}+\frac{y^2}{3}=1$$

$$a^2=9,\ b^2=3$$

$$\text{So, } b^2=a^2\left(1-e^2\right)$$

$$3=9\left(1-e^2\right)$$

$$e=\sqrt{\frac{2}{3}}$$

$$\text{Distance between foci}=2ae=2\times 3\times\sqrt{\frac{2}{3}}$$

$$=2\sqrt{3}\sqrt{2}$$

$$=2\sqrt{6}$$

(h) Correct option: (ii)

Equation of the line through the points $(2,3)$ and $(5,3)$ is:

$$y-3=\frac{3-3}{5-2}(x-2)$$

$$3(y-3)=0(x-2)$$

$$3y-9=0$$

$$y=3$$

(i) Correct option: (i)

$$\text{C.V.}=\frac{\sigma}{\bar{x}}\text{ if }\bar{x}=1\text{ then C.V.}=\sigma$$

(j) Correct option: (i)

$$\lim_{x\to 0}\frac{e^x+e^{-x}-2}{x^2}$$

$$=\lim_{x\to 0}\frac{e^{2x}+1-2e^{x}}{x^{2}e^{x}}$$
$$=\lim_{x\to 0}\frac{\left(e^{x}-1\right)^{2}}{x^{2}e^{x}}$$
$$=\lim_{x\to 0}\left(\frac{e^{x}-1}{x}\right)^{2}\times\frac{1}{e^{x}}$$
$$=1$$

Section B

2. $S_n = (pn + qn^2)$

$S_1 = (p.1 + q.1^2) = p + q = a_1$

$S_2 = (p.2 + q.2^2) = 2p + 4q = a_1 + a_2$

$a_2 = S_2 - S_1 = p + 3q$

$d = a_2 - a_1 = (p + 3q) - (p + q) = 2q$

3. Let the equation of the circle be $(x - h)^2 + (y - k)^2 = r^2$

Since the circle passes through (2, –2) and (3, 4), we have,

$(2 - h)^2 + (-2 - k)^2 = r^2$... (1)

and $(3 - h)^2 + (4 - k)^2 = r^2$... (2)

Also since the centre lies on the line x + y = 2, we have,

h + k = 2 ... (3).

Solving the equations (1), (2) and (3), we get,

h =0 .7, k = 1.3 and $r^2 = 12.58$

Hence, the equation of the required circle is

$(x - .7)^2 + (y - 1.3)^2 = 12.58$

4. Shankar has to select some or all of his 6 friends.

This can also be done as follows:

The friends may be invited singly, in two's, or in three's, ..., etc.

The number of such selections is

C (6, 1) + C (6, 2) + C (6, 3) + C (6, 4) + *C* (6, 5) + C (6, 6)

$$= 6 + \frac{6!}{2!.4!} + \frac{6!}{3!.3!} + \frac{6!}{2!.4!} + 6 + 1$$

= 6 + 15 + 20 + 15 + 6 + 1 = 63

5. $\sec^2 2x = 1 - \tan 2x$

$1 + \tan^2 2x = 1 - \tan 2x$

tan2x (tan2x +1) = 0

tan2x (tan 2x + 1) = 0

tan2x = 0 or tan2x = -1

$2x = m\pi$ or $\tan 2x = \tan\dfrac{3\pi}{4}$

$x = \dfrac{m\pi}{2}$ or $2x = n\pi + \dfrac{3\pi}{4}$

$x = \dfrac{n\pi}{2} + \dfrac{3\pi}{8}$

$m, n \notin I.$

6. $x - iy = \sqrt{\dfrac{a-ib}{c-id}} \Rightarrow (x - iy)^2 = \left[\sqrt{\dfrac{a-ib}{c-id}}\right]^2$

Now, $(x - iy)^2 = |x - iy|^2$

$\therefore (x - iy)^2 = |x - iy|^2 = \left[\left|\sqrt{\dfrac{a-ib}{c-id}}\right|\right]^2$

But $|x - iy| = \sqrt{x^2 + y^2}$

$\Rightarrow |x - iy|^2 = \left[\sqrt{x^2 + y^2}\right]^2 = x^2 + y^2$ ...(i)

$\left[\left|\sqrt{\dfrac{a-ib}{c-id}}\right|\right]^2 = \left|\dfrac{a-ib}{c-id}\right| = \dfrac{|a-ib|}{|c-id|} = \dfrac{\sqrt{a^2 + (-b)^2}}{\sqrt{c^2 + (-d)^2}} = \dfrac{\sqrt{a^2 + b^2}}{\sqrt{c^2 + d^2}}$ ...(ii)

From (i) and (ii), we have

$x^2 + y^2 = \dfrac{\sqrt{a^2 + b^2}}{\sqrt{c^2 + d^2}}$

$\Rightarrow (x^2 + y^2)^2 = \left[\dfrac{\sqrt{a^2 + b^2}}{\sqrt{c^2 + d^2}}\right]^2 = \dfrac{a^2 + b^2}{c^2 + d^2}$

Section C

7. $\text{L.H.L.} = \lim_{x\to 0} f(x) = \lim_{h\to 0} f(0-h)$

$$= \lim_{h\to 0} \frac{|1-h|}{-h}$$

$$= \lim_{h\to 0} \frac{h}{-h} = -1$$

$$\text{R.H.L.} = \lim_{h\to 0^+} f(x) = \lim_{h\to 0} f(0+h)$$

$$= \lim_{h\to 0} \frac{|h|}{h} = \lim_{h\to 0} \frac{h}{h} = 1$$

$\text{L.H.L.} \neq \text{R.H.L.}$

$\Rightarrow \lim_{x\to 0} f(x)$ does not exists

OR

$$y = \sqrt{\sin x}$$

$$y + \Delta y = \sqrt{\sin(x+\Delta x)}$$

$$\lim_{\Delta x\to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x\to 0} \frac{\sqrt{\sin(x+\Delta x)} - \sqrt{\sin x}}{\Delta x}$$

$$= \lim_{\Delta x\to 0} \frac{\sqrt{\sin(x+\Delta x)} - \sqrt{\sin x}}{\Delta x} \text{ x } \frac{\sqrt{\sin(x+\Delta x)} + \sqrt{\sin x}}{\sqrt{\sin(x+\Delta x)} + \sqrt{\sin x}}$$

$$= \lim_{\Delta x\to 0} \frac{\sin(x+\Delta x) - \sin x}{\Delta x} \text{ x } \lim_{\Delta x\to 0} \frac{1}{\sqrt{\sin(x+\Delta x)} + \sqrt{\sin x}}$$

$$= \lim_{\Delta x\to 0} \frac{2\cos\left(x + \frac{\Delta x}{2}\right)\sin\left(\frac{\Delta x}{2}\right)}{2\times\frac{\Delta x}{2}} \times \frac{1}{2\sqrt{\sin x}}$$

$$\cos x \lim_{\Delta x\to 0} \frac{\sin\frac{\Delta x}{2}}{\frac{\Delta x}{2}} \text{ x } \frac{1}{2\sqrt{\sin x}}$$

$$= \frac{\cos x}{2\sqrt{\sin x}}$$

8. Consider f(x) = $\frac{2}{x+1}-\frac{x^2}{3x-1}$

$$\frac{d}{dx}f(x)=2.\frac{d}{dx}\left(\frac{1}{x+1}\right)-\frac{d}{dx}\left(\frac{x^2}{3x-1}\right)$$

Using quotient rule,

$$=2\left[\frac{(x+1)\frac{d}{dx}(1)-1.\frac{d}{dx}(x+1)}{(x+1)^2}\right]-\left[\frac{(3x-1)\frac{d}{dx}(x^2)-x^2\frac{d}{dx}(3x-1)}{(3x-1)^2}\right]$$

$$=-\frac{2}{(x+1)^2}-\left[\frac{2x(3x-1)-3x^2}{(3x-1)^2}\right]$$

$$=-\frac{2}{(x+1)^2}-\left[\frac{6x^2-2x-3x^2}{(3x-1)^2}\right]$$

$$=-\frac{2}{(x+1)^2}-\frac{3x^2-2x}{(3x-1)^2}$$

9. $\cos^2x+\cos^2(x+\frac{\pi}{3})+\cos^2(x-\frac{\pi}{3})$

$$=\cos^2x+\cos^2(x+\frac{\pi}{3})+1-\sin^2(x-\frac{\pi}{3})$$

$$=1+\cos^2x+\left[\cos^2(x+\frac{\pi}{3})-\sin^2(x-\frac{\pi}{3})\right]$$

$$=1+\cos^2x+\left[\cos(x+\frac{\pi}{3}+x-\frac{\pi}{3})\cos(x+\frac{\pi}{3}-x+\frac{\pi}{3})\right] \quad \left[\because \cos^2A-\sin^2B=\cos(A+B)\cos(A-B)\right]$$

$$=1+\cos^2x+\cos(2x)\cos\left(\frac{2\pi}{3}\right)$$

$$=1+\cos^2x+\cos(2x)\left(-\frac{1}{2}\right)$$

$$=1+\cos^2x+\left(2\cos^2x-1\right)\left(-\frac{1}{2}\right)$$

$$=1+\cos^2x+\left(-\cos^2x+\frac{1}{2}\right)$$

$$=1+\frac{1}{2}=\frac{3}{2}$$

10. L.H.S. = $\frac{1}{2}\left[2\cos 2\theta \cos\frac{\theta}{2} - 2\cos 3\theta \cos\frac{9\theta}{2}\right]$

$= \frac{1}{2}\left[\left(\cos\frac{5\theta}{2} + \cos\frac{3\theta}{2}\right) - \left(\cos\frac{15\theta}{2} + \cos\frac{3\theta}{2}\right)\right]$

$= \frac{1}{2}\left[\cos\frac{5\theta}{2} - \cos\frac{15\theta}{2}\right]$

$= \frac{1}{2} \times 2\sin 5\theta \sin\left(\frac{5\theta}{2}\right)$

$= \sin(5\theta)\sin\left(\frac{5\theta}{2}\right)$ = R.H.S.

11. Let $y = f(x) = \sqrt{x-5}$

Square root is defined only for non – negative real numbers so

Domain of f is the set of numbers for which $x - 5 \geq 0$

i.e $x \geq 5$

$\Rightarrow$ Domain of f=$[5,\infty)$

Now, $\sqrt{x-5} = y$

$\Rightarrow (x-5) = y^2$

$\Rightarrow x = y^2 + 5$

since x is taking values greater than or equal to 5 so

Range of f = $[0,\infty)$

OR

$f(x) = \frac{3}{2-x^2}$

For f to be defined $2 - x^2 \neq 0$, i.e. $2 \neq x^2$

So Domain = {x: x is a real number and $x \neq \pm\sqrt{2}$}

For range,

$\frac{3}{2-x^2} = y$

$2y - x^2y = 3$

$x^2 = \frac{2y-3}{y}$

$x = \pm\sqrt{\frac{2y-3}{y}}$

For x to be defined $\frac{2y-3}{y}$ must be positive, i.e. either both Numerator and Denominator should be positive, in which case $y > 3/2$ or both should be negative, in which case $y < 0$.

Range = $(-\infty,0)\cup\left(\frac{3}{2},\infty\right)$

12. Centre of the circle is the point of intersection of diameters x – y – 9 = 0 and x - 2y – 7 = 0,

Solving these equations for x and y we get x = 11, y = 2

$\therefore$ Centre of circle is at (11, 2)

Area of this circle = 154 sq. units

$\pi r^2 = 154$

$r^2 = 154 \times \frac{7}{22} = 7\times 7$

r = 7 units

Hence, the equation of circle is

$(x - 11)^2 + (y - 2)^2 = 7^2$

OR

Co-ordinates of foci = $(\pm c, 0) = (\pm 3\sqrt{5}, 0)$

$\Rightarrow c = 3\sqrt{5}$

Latus Rectum = $\frac{2b^2}{a} = 8$

$\Rightarrow b^2 = 4a$

Also $c^2 = a^2 + b^2$

$\Rightarrow 45 = 4a + a^2$

$\Rightarrow a^2 + 4a - 45 = 0$

$\Rightarrow (a + 9)(a - 5) = 0$

a = 5, a = -9 (rejected)

$\Rightarrow a = 5 \Rightarrow b^2 = 20$

Hence, the equation of the hyperbola is

$$\frac{x^2}{25} - \frac{y^2}{20} = 1$$

13. We have

$$\frac{1}{3}+\frac{1}{5^2}+\frac{1}{3^3}+\frac{1}{5^4}+\frac{1}{3^5}+\frac{1}{5^6}+----=\left[\frac{1}{3}+\frac{1}{3^3}+\frac{1}{3^5}+-----\right]+\left[\frac{1}{5^2}+\frac{1}{5^4}+\frac{1}{5^6}+-----\right]$$

$$=\frac{1/3}{1-\frac{1}{3^2}}+\frac{\frac{1}{5^2}}{1-\frac{1}{5^2}}=\frac{1}{3}\times\frac{9}{8}+\frac{1}{25}\times\frac{25}{24}$$

$$=\frac{3}{8}+\frac{1}{24}=\frac{10}{24}=\frac{5}{12}$$

14. $(99)^5$

To be able to use binomial theorem, let us express 99 as a binomial: 99 = 100 – 1

$(99)^5 = (100 - 1)^5 = {}^5C_0(100)^5(-1)^0 + {}^5C_1(100)^4(-1)^1 + {}^5C_2(100)^3(-1)^2 + {}^5C_3(100)^2(-1)^3$
$+ {}^5C_4(100)^1(-1)^4 + {}^5C_5(100)^0(-1)^5$

$= 1.(100)^5 - 5(100)^4 + 10.(100)^3 - 10(100)^2 + 5.(100) - 1$

$= (10000000000) - 5(100000000) + 10.(1000000) - 10(10000) + 5.(100) - 1$

$= (10000000000) - (500000000) + (10000000) - (100000) + (500) - 1$

$= 10010000500 - 500100001$

$= 9509900499$

OR

Given: $(3 + ax)^9$

General term in the expansion of $(3 + ax)^9$

$t_{r+1} = {}^9C_r(ax)^r(3)^{9-r}$

Coefficient of $x^r = {}^9C_r(a)^r(3)^{9-r}$

Coefficient of $x^2 = {}^9C_2(a)^2(3)^{9-2} = {}^9C_2a^23^7$

Coefficient of $x^3 = {}^9C_3(a)^3(3)^{9-3} = {}^9C_3a^33^6$

$$\frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3} = \frac{{}^9C_2a^23^7}{{}^9C_3a^33^6} = \frac{3.{}^9C_2}{a.{}^9C_3} = \frac{3.3}{7a} = \frac{9}{7a}$$

15. Let the point in the XY plane be P(x, y, 0).

$PA^2 = (x - 4)^2 + (y - 0)^2 + (0 - 5)^2$
$= (x - 4)^2 + y^2 + 25$

$PB^2 = (x - 0)^2 + (y - 5)^2 + (0 - 4)^2$
$= x^2 + (y - 5)^2 + 16$

$PC^2 = (x - 0)^2 + (y - 0)^2 + (0 - 1)^2$
$= x^2 + y^2 + 1$

As $PA^2 = PC^2$

so $(x - 4)^2 + y^2 + 25 = x^2 + y^2 + 1$
$16 - 8x + 24 = 0$
$-8x + 40 = 0$

So x = 5

Also, $PB^2 = PC^2$

$x^2 + (y - 5)^2 + 16 = x^2 + y^2 + 1$

$y^2 + 25 - 10y + 16 = y^2 + 1$

$-10y = -40$

$\Rightarrow y = 4$

So point P has co-ordinates (5, 4, 0)

16. $\lim\limits_{x\to\sqrt{2}} \dfrac{x^4-4}{x^2+3\sqrt{2}x-8} = \lim\limits_{x\to\sqrt{2}} \dfrac{\left(x-\sqrt{2}\right)\left(x+\sqrt{2}\right)\left(x^2+2\right)}{x^2+4\sqrt{2}x-\sqrt{2}x-8}$

$\lim\limits_{x\to\sqrt{2}} \dfrac{\left(x-\sqrt{2}\right)\left(x+\sqrt{2}\right)\left(x^2+2\right)}{\left(x-\sqrt{2}\right)\left(x+4\sqrt{2}\right)}$

Using the limit we get

$\dfrac{\left(\sqrt{2}+\sqrt{2}\right)(2+2)}{\sqrt{2}+4\sqrt{2}} = \dfrac{8\sqrt{2}}{5\sqrt{2}} = \dfrac{8}{5}$

OR

$y = \sin(x^2 + 3)$

$y + \Delta y = \sin((x + \Delta x)^2 + 3)$

$$\lim_{x\to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x\to 0} \frac{\sin\left((x+\Delta x)^2+3\right) - \sin\left(x^2+3\right)}{\Delta x}$$

$$= \lim_{\Delta x\to 0} \frac{2\cos\left(\dfrac{\left((x+\Delta x)^2+3\right)+\left(x^2+3\right)}{2}\right)\sin\left(\dfrac{\left((x+\Delta x)^2+3-x^2-3\right)}{2}\right)}{\Delta x}$$

$$= \lim_{\Delta x\to 0} \frac{2\cos\left(x^2+3+\dfrac{(\Delta x)^2}{2}+x\Delta x\right)\sin\left(\Delta x\left(\dfrac{\Delta x}{2}+x\right)\right)}{\Delta x}$$

$$= 2\lim_{\Delta x\to 0} \cos\left(x^2+3+\frac{\Delta x^2}{2}+x\,\Delta x\right) \times \lim_{\Delta x\to 0} \frac{\sin \Delta x\left(\dfrac{\Delta x}{2}+x\right)}{\Delta x}$$

$$= 2\cos(x^2+3) \lim_{\Delta x\to 0} \frac{\sin \Delta x\left(\dfrac{\Delta x}{2}+x\right)}{\Delta x\left(\dfrac{\Delta x}{2}+x\right)} \times \left(\frac{\Delta x}{2}+x\right)$$

$= 2x\cos(x^2 + 3)$

Section-D

17. System of inequations

$x \geq 0, y \geq 0, 5x + 3y \leq 500; x \leq 70$ and $y \leq 125$.

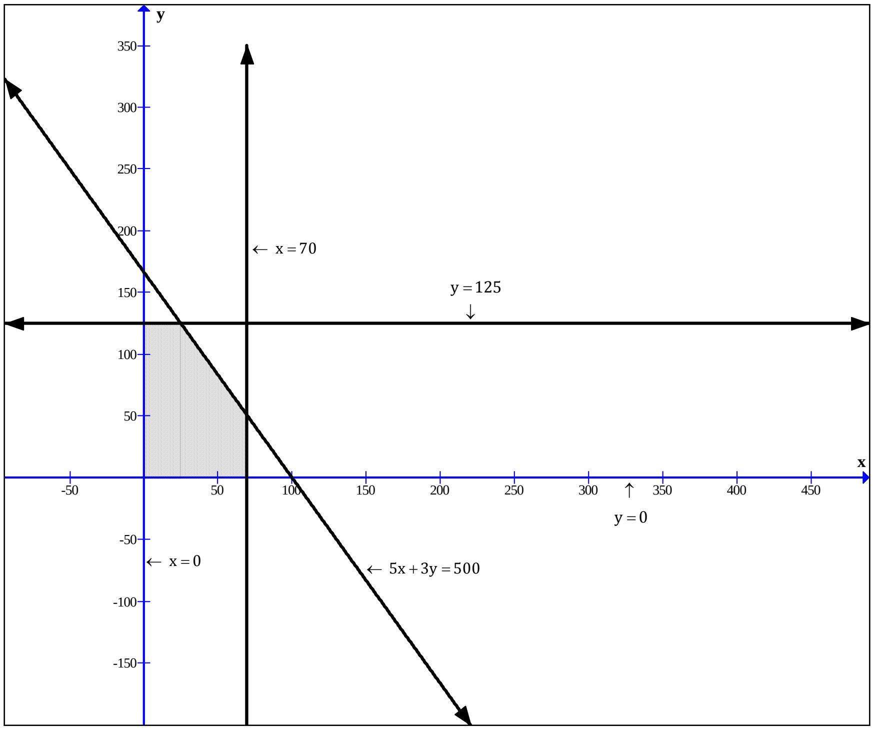
Converting inequations to equations

$$5x + 3y = 500 \quad \Rightarrow y = \frac{500-5x}{3}$$

x	100	40	-80
y	0	100	300

$x \leq 70$ is $x = 70$ and $y \leq 125$ is $y = 125$.

Plotting these lines and determining the area of each line we get



OR

$$\frac{1}{2}\left(\frac{3x+20}{5}\right) \geq \frac{1}{3}(x-6)$$

$$\Rightarrow \frac{1}{10}(3x+20) \geq \frac{1}{3}(x-6)$$

$$\Rightarrow \frac{30}{10}(3x+20) \geq \frac{30}{3}(x-6)$$

$$\Rightarrow 3(3x+20) \geq 10(x-6)$$

$$\Rightarrow 9x+60 \geq 10x-60$$

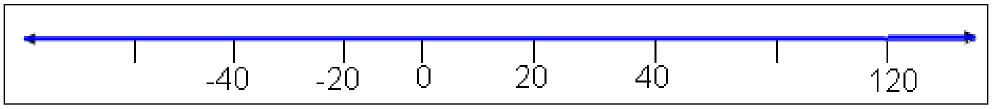
$$\Rightarrow 60+60 \geq 10x-9x$$

$$\Rightarrow 120 \geq x$$

$$\Rightarrow x \leq 120$$

$$\Rightarrow x \in (-\infty, 120]$$

Thus, all real numbers less than or equal to 120 are the solution of the given inequality. The solution set can be graphed on a real line as shown.



18. The successive differences between adjacent numbers in the sequence 2, 4, 7, 11, 16 ... are 2, 3, 4, 5 ... respectively.

That is, 2, 3, 4, 5, ... form an AP

If t_n is the n^{th} term and s_n the sum of n terms, then,

$s_n = 2 + 4 + 7 + 11 + 16 + \ldots + t_{n-1} + t_n$ ...(i)

$s_n = \quad 2 + 4 + 7 + 11 + \ldots + t_{n-2} + t_{n-1} + t_n$ ...(ii)

Subtracting (ii) from (i), we get,

$0 = 2 + 2 + 3 + 4 + 5 + \ldots + (t_n - t_n - 1) - t_n$

$\Rightarrow t_n = 2 + [2 + 3 + 4 + 5 + \ldots + (t_n - t_{n-1})$

$$= 2 + \left(\frac{n-1}{2}\right)[4 + (n-2)] \quad \text{[By using } s_n = \frac{n}{2}[2a + (n-1)d]$$

$$= 2 + \left(\frac{n-1}{2}\right)[n+2]$$

$$= 2 + \frac{(n-1)(n+2)}{2}$$

$$t_n = \frac{4 + (n^2 + n - 2)}{2} = \frac{n^2 + n + 2}{2}$$

$$s_n = \sum_{k=1}^{n} t_k$$

$$\sum_{k=1}^{n} t_k = \sum_{k=1}^{n}\left(\frac{k^2 + k + 2}{2}\right)$$

$$= \frac{1}{2}\left(\sum_{k=1}^{n} k^2\right) + \frac{1}{2}\left(\sum_{k=1}^{n} k\right) + \frac{1}{2} \times 2n$$

$$= \frac{1}{2} \times \frac{n(n+1)(2n+1)}{6} + \frac{1}{2} \times \frac{n(n+1)}{2} + n$$

$$= \frac{n(n+1)(2n+1)}{12} + \frac{n(n+1)}{4} + n$$

$$=\frac{n(n+1)}{4}\left[\frac{2n+1}{3}+1\right]+n$$

$$=\frac{n(n+1)}{4}\left[\frac{2n+1+3}{3}\right]+n$$

$$=\frac{n(n+1)}{4}\left(\frac{2n+4}{3}\right)+n$$

$$=\frac{n(n+1)(n+2)}{6}+n$$

$$=\frac{n}{6}[(n+1)(n+2)+6]$$

$$=\frac{n}{6}[n^2+3n+8]$$

$$\text{Therefore,} s_n=\frac{n}{6}[n^2+3n+8]$$

OR

First, we need to find:

(1) the first element of the n^{th} row

(2) common difference 'd' of the sequence in the n^{th} row

(3) number of terms in the n^{th} row.

The first element of the n^{th} row is the n^{th} term of the sequence 1, 3, 7, 13, 21... Let T_n be the n^{th} term and S_n be the sum to n terms.

Therefore, $S_n = 1 + 3 + 7 + 13 + 21 + ... + T_{n-1} + T_n$ ...(1)

$S_n = 1 + 3 + 7 + 13 + \quad ... \quad T_{n-1} + T_n$ ...(2)

Subtracting (2) from (1), we get,

$0 = 1 + 2 + 4 + 6 + 8 + ...$ to $n - 1$ terms $- T_n$

$\Rightarrow T_n = 1 + [2 + 4 + 6 + 8 + ...$ to $(n - 1)$ terms$]$

$$= 1 + \left(\frac{n-1}{2}\right)[4 + (n - 1 - 1)2]$$

$$= 1 + \left(\frac{n-1}{2}\right)2n$$

$= n^2 - n + 1$

Also, the common difference d of the sequence in the n^{th} row is 1.

If P is the number of terms in the n^{th} row, then P is the n^{th} term of the sequence 2, 4, 6, 8, (because 1st row has 2 items, 2nd row has 4 items and so on)

The n^{th} term of the sequence 2, 4, 6, 8, ... is $2 + (n - 1)2 = 2n$

Hence, the number of terms in the n^{th} row is $2n$.

Hence, the sum of all the terms in the n^{th} row is $(n)[2(n^2 - n + 1) + (2n - 1)1]$

$= n(2n^2 - 2n + 2 + 2n - 1)$

$= n(2n^2 + 1)$

19. P(n): $10^n + 3 . 4^{n+2} + 5$ is divisible by 9.

Then we show

P(1): $10^1 + 3 . 4^{1+2} + 5$ is divisible by 9 is true.

That is, 207 is divisible by 9, which is true.

Now suppose, P(k) is true for some natural number k = 1.

That is, $10^k + 3 . 4^{k+2} + 5$ is divisible by 9.

We must show that,

P(k + 1): $10^{k+1} + 3 . 4^{k+3} + 5$ is divisible by 9, is true.

As $10^{k+1} + 3 . 4^{k+1+2} + 5 = 10 . 10^k + 3 . 4 . 4^{k+2} + 5$

$= 10 . 10^k + 12 . 4^{k+2} + 5$

$= 10 . 10^k + 30 . 4^{k+2} - 18 . 4^{k+2} + 50 - 45$

$= 10 . 10^k + 30 . 4^{k+2} + 50 - 18 . 4^{k+2} - 45$

$= 10(10^k + 3 . 4^{k+2} + 5) - 18 . 4^{k+2} - 45$

$= 10(10^k + 3.4^{k+2} + 5) - 9(2 . 4^{k+2} + 5)$

Note that $9(2 . 4^{k+2} + 5)$ is divisible by 9 and $10^k + 3 . 4^{k+2} + 5$ is divisible by 9 (by the nduction hypothesis)

Therefore, both terms are divisible by 9 implying that $10^{k+1} + 3 . 4^{k+1+2} + 5$ is divisible by 9

Hence, P(k + 1) is true.

Hence , by PMI, P(n) is true for every natural number n.

OR

Let $P(n): 1 + \frac{1+2}{2} + \frac{1+2+3}{3} + ... + \frac{1+2+...+n}{n} = \frac{n(n+3)}{4}$

P(1): $1 = \frac{(1+3)}{4}$

$\therefore$ 1 = 1

LHS = RHS

Let P(k) be true.

Therefore, $1 + \frac{1+2}{2} + \frac{1+2+3}{3} + ... + \frac{1+2+...+k}{k} = \frac{k(k+3)}{4}$ ....(i)

To prove P(k + 1): $1 + \frac{1+2}{2} + \frac{1+2+3}{3} + ... + \frac{1+2+...+(k+1)}{k+1} = \frac{(k+1)(k+4)}{4}$

Consider $1 + \frac{1+2}{2} + ... \frac{1+2+...+k}{k} + \frac{1+2+...+(k+1)}{k+1}$

$= \frac{k(k+3)}{4} + \frac{1+2+...+(k+1)}{k+1}$ (from equation (i))

$= \frac{k(k+3)}{4} + \frac{(k+1)(k+2)}{2(k+1)}$

$$=\frac{1}{2}\left[\frac{k(k+3)}{2}+(k+2)\right]$$

On solving, we get,

$$\frac{1}{2}\left[\frac{k^2+5k+4}{2}\right]$$

$$=\frac{1}{4}(k+1)(k+4)$$

= R.H.S.

20. Let assumed mean be A = 105

Classes	f_i	x_i	$u_i = \frac{x_i - 105}{30}$	f_iu_i	$f_iu_i^2$
0 – 30	2	15	-3	-6	18
30 – 60	3	45	-2	-6	12
60 – 90	5	75	-1	-5	5
90 – 120	10	105	0	0	0
120 – 150	3	135	1	3	3
150 – 180	5	165	2	10	20
180 - 210	2	195	3	6	18
	$\sum f_i = 30$			$\sum f_iu_i = 2$	$\sum f_iu_i^2 = 76$

$$\text{Mean } \overline{X} = A + \frac{\sum f_iu_i}{\sum f_i} \times h$$

$$= 105 + \frac{2}{30} \times 30$$

$$= 107$$

$$\text{Variance } \sigma^2 = h^2 \times \left[\frac{1}{N}\sum f_iu_i^2 - \left(\frac{1}{N}\sum f_iu_i\right)^2\right]$$

$$= (30)^2\left[\frac{1}{30} \times 76 - \left(\frac{2}{30}\right)^2\right]$$

$$= 900\left[\frac{76}{30} - \frac{4}{900}\right] = 2276$$

OR

Number of observations = 100 = n

Incorrect mean $(\bar{x})$ = 40

$$\Rightarrow \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$40 = \frac{1}{100}\sum x_i$$

Incorrect sum = $\sum x_i$ = 4000

Correct sum = 4000 – 50 + 40 = 3990

Correct mean $\bar{x}' = \frac{3990}{100}$ = 39.9

Now $\sigma^2 = \frac{1}{n}\sum_{i=1}^{n} x_i^2 - (\bar{x})^2$

$n(\sigma^2 + (\bar{x})^2) = \sum x_i^2$

Incorrect $\sum x_i^2$ = 100 $\left[(5.1)^2 + (40)^2\right]$

= 100 × 1626.01

= 162601

Corrected $\sum x_i^2$ = 162601 – $(50)^2$ + $(40)^2$ = 161701

Corrected Variance = $\frac{1}{100}$ (corrected $\sum x_i^2$) – (corrected mean)^2

= $\frac{1}{100}$ (161701) – $(39.9)^2$

= 1617.01 – 1592.01 = 25

Corrected standard deviation = 5

21. (i) Here n = 18, $\bar{X} = 7$

$\therefore \sum x = 18 \times 7 = 126 \left(\because \sum x = n\bar{x}\right)$

Correct $\sum x = 126 - 21 + 12 = 117$

$\therefore$ Correct arithmetic mean = $\frac{117}{18} = 6.5$

$\therefore$ (ii) Calculation of correct standard deviation

$\therefore$ We know that $\sigma = \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2}$ or $4 = \sqrt{\frac{\sum x^2}{18} - (7)^2}$

$\therefore$ Simplifying, we get $\sum x^2 = 1170$

But this is the incorrect value of $\sum x^2$

$\therefore$ Correct $\sum x^2 = 1170 - (21)^2 + (12)^2$

= 1170 - 441 + 144 = 873

$\therefore$ Correct S.D. $= \sqrt{\frac{\text{Correct } \sum x^2}{n} - (\text{Correct mean})^2}$

$= \sqrt{\frac{873}{18} - (6.5)^2}$

= 2.5

OR [5]

Total Students – 100

Let M, S, and P denote the students opting for Mathematics, Statistics and Physics respectively.

Number of students for Mathematics only = 15

Number of students for Statistics only = 12

Number of students for Physics only = 8

n $(P \cap M)$ = 40

n $(P \cap S)$ = 20

n $(M \cap S)$ = 10

n (P) = 65

Let x number of students opt for all the three subjects i.e., n $(P \cap M \cap S)$ = x

n (P) = n $(P \cap M)$ + n $(P \cap S)$ + students in only Physics - n $(P \cap M \cap S)$

65 = 40 + 20 + 8 – x

$\Rightarrow$ x = 3

i) n (M) = n $(M \cap S)$ + n $(P \cap M)$ + students in only Mathematics – n$(P \cap M \cap S)$

n (M) = 10 + 40 + 15 – 3 = 62

ii) n (S) = n $(M \cap S)$ + n $(P \cap S)$ + students in only Statistics – n$(P \cap M \cap S)$

n (S) = 10 + 20 + 12 – 3 = 39

iii) Number of students who opt for one of the three subjects,

n $(P \cap S \cap M)$ = n(P) + n(S) + n(M) – n$(P \cap M)$ – n$(P \cap S)$ – n$(M \cap S)$ + n$(P \cap S \cap M)$

= 65 + 39 + 62 – 40 – 20 – 10 + 3 = 99

Total students = 100

So number of students opting for none of subjects = 100 – 99 = 1

22. Let S be the sample space. Then n(S) = 36

Let E_1 = event that a doublet appears

Let E_2 = event of getting a total of 10.

Then, $E_1 = [(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)]$,

and $E_2 = [(4,6),(5,5),(6,4)]$

$\therefore E_1 \cap E_2 = [(5,5)]$

So, $n(E_1) = 6, n(E_2) = 3$ and $n(E_1 \cap E_2) = 1$.

Thus,

$$P(E_1)=\frac{n(E_1)}{n(S)}=\frac{6}{36}=\frac{1}{6},$$

$$P(E_2)=\frac{n(E_2)}{n(S)}=\frac{3}{36}=\frac{1}{12}$$

and

$$P(E_1\cap E_2)=\frac{n(E_1\cap E_2)}{n(S)}=\frac{1}{36}$$

Therefore, the probability of getting a doublet or a total of 10

$$=P(E_1 \text{ or } E_2)$$

$$=P(E_1\cup E_2)$$

$$=P(E_1)+P(E_2)-P(E_1\cap E_2)$$

$$=\frac{1}{6}+\frac{1}{12}-\frac{1}{36}$$

$$=\frac{8}{36}$$

$$=\frac{2}{9}$$

Thus P(getting neither a doublet nor a total of 10)

$$=P(\overline{E_1} \text{ and } \overline{E_2})$$

$$=P(\overline{E_1}\cap\overline{E_2})$$

$$=P(\overline{E_1\cup E_2})$$

$$=1-P(E_1\cup E_2)$$

$$=1-\frac{2}{9}$$

$$=\frac{7}{9}$$

OR

(i) There are a total of 60 marbles out of which 5 marbles are to be selected

Number of ways in which 5 marbles are to be selected out of 60 = $^{60}C_5$

(a)

Out of 20 blue marbles , 5 can be selected in= $^{20}C_5$

$$\text{P(all 5 blue marbles)} = \frac{^{20}C_5}{^{60}C_5} = \frac{\frac{20!}{5!15!}}{\frac{60!}{5!55!}} = \frac{20!5!55!}{5!15!60!}$$

$$= \frac{20.19.18.17.16}{60.59.58.57.56} = \frac{34}{11977}$$

(b) Number of ways in which 5 marbles are to be selected out of 60= $^{60}C_5$.

Out of 30 non-green marbles , 5 can be selected in $^{30}C_5$

$$\text{P(all 5 non-green marbles)} = \frac{^{30}C_5}{^{60}C_5} = \frac{\frac{30!}{5!25!}}{\frac{60!}{5!55!}} = \frac{30!5!55!}{5!25!60!}$$

$$= \frac{30.29.28.27.26.}{60.59.58.57.56} = \frac{117}{4484}$$

$$\text{P(atleast one green marble)} = 1 - \text{P(all 5non} - \text{green marbles)}$$

$$= 1 - \frac{117}{4484}$$

$$= \frac{4484 - 117}{4484} = \frac{4367}{4484}$$

(ii)On the dice two faces are with number '1', three faces are with number '2' and one face is with number '3'

$$\therefore P(1) = \frac{2}{6} = \frac{1}{3}; P(2) = \frac{3}{6} = \frac{1}{2}; P(3) = \frac{1}{6}$$

$$\therefore \text{(i) } P(2) = \frac{1}{2}$$

(ii) $P(1 \text{ or } 3) = P(1) + P(3)$[The events are mutually exclusive]

$$= \frac{1}{3} + \frac{1}{6} = \frac{1}{2}$$

$$\text{(iii) } P(\text{not}3) = 1 - P(3) = 1 - \frac{1}{6} = \frac{5}{6}$$