Tripura Board
Class X
Mathematics
Sample Paper 2 – Solution

Group A

1. We know that,

3Median = Mode + 2 Mean

3 (21.2) - 21.4 = 2 Mean

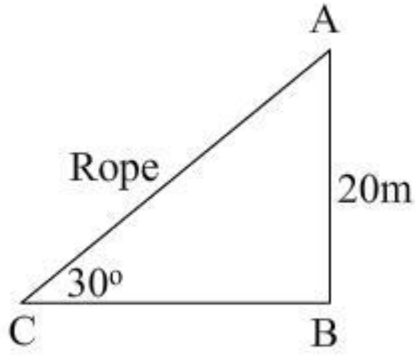
63.6 - 21.4 = 2 Mean

42.2 = 2 Mean

Mean = 21.1

2. Tangent: A tangent to a circle is a line that intersects the circle in exactly one point.
Secant: A line that intersects a circle in two distinct points is called a secant of the circle

3.



AB is the pole = 20 m, $\angle$ACB = 30°

$\sin 30^\circ = \frac{20}{AC} \Rightarrow \frac{1}{2} = \frac{20}{AC}$

$\Rightarrow$AC = 40 m.

Hence the length of the rope = 40 m.

Group B

4. Since a – b, a and a + b are the zeros of f(x).

$\therefore (a-b)+a+(a+b) = -\dfrac{\text{Coeff. of } x^2}{\text{Coeff. of } x^3}$

$\Rightarrow 3a = -\dfrac{-3}{1}$

$\Rightarrow 3a = 3$

$\Rightarrow a = 1$

And, $(a-b)a(a+b) = -\dfrac{\text{Coeff. term}}{\text{Coeff. of } x^3}$

$\Rightarrow a(a^2-b^2) = -\dfrac{1}{1}$

$\Rightarrow 1(1-b^2) = -1$

$\Rightarrow b^2 = 2$

$\Rightarrow b = \pm\sqrt{2}$

5. 455 = 84 $\times$ 5 + 35

$\Rightarrow$ 84 = 35 $\times$ 2 + 14

$\Rightarrow$ 35 = 14 $\times$ 2 + 7

$\Rightarrow$ 14 = 7 $\times$ 2 + 0

Therefore, HCF = 7

6. Since the lengths of tangents from an exterior point to a circle are equal.

Therefore, XP = XQ (from X) ….(i)

AP = AR (from A) ….(ii)

BQ = BR (from B) ….(iii)

Now, XP = XQ

$\Rightarrow$ XA + AP = XB + BQ

$\Rightarrow$ XA + AR = XB + BR (Using (ii) and (iii))

7. Let r be the radius of the wheel.

Distance covered in 1 revolution = $2\pi r$

Distance covered in 5000 revolutions = 5000 $\times$ $2\pi r$ = 11 km

$5000\times\dfrac{22}{7}(2r) = 11\times1000$ metres

2r = $\dfrac{7}{10}$ metres = 70 cm

Thus, the diameter of the wheel is 70 cm.

8. In right triangle ABC, we have

$\sin\ 45^\circ = \frac{BC}{AC}$

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{BC}{150}$

$\Rightarrow BC = \frac{150}{\sqrt{2}}$

$\Rightarrow BC = 75\sqrt{2}$ m

Thus, the width of the river is $75\sqrt{2}$ metres.

Group C

9.

$$\text{LHS} = \frac{\sec A + \tan A}{\sec A - \tan A}$$

$$= \frac{\sec A + \tan A}{\sec A - \tan A} \times \frac{\sec A + \tan A}{\sec A + \tan A}$$

$$= \frac{(\sec A + \tan A)^2}{\sec^2 A - \tan^2 A}$$

$$= (\sec A + \tan A) \qquad \left(\because \sec^2\theta = 1 + \tan^2\theta \therefore \sec^2\theta - \tan^2\theta = 1\right)$$

$$= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)^2$$

$$= \left(\frac{1 + \sin A}{\cos A}\right)^2$$

Hence L.H.S. = R.H.S

10. Area of quadrilateral ABCD = Area of ΔABC + Area of ΔACD

Area of triangle $= \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$

Area of ΔABC $= \frac{1}{2}$ [1(−3 − 2) + 7(2 − 1) + 12(1 + 3)]

$= \frac{1}{2}$ [−5 + 7 + 48]

= 25 sq. units

Area of ΔACD $= \frac{1}{2}$ [1(2 − 21) + 12(21 − 1) + 7(1 − 2)]

$= \frac{1}{2}$ [−19 + 240 − 7]

= 107 sq. units

Therefore, area of quadrilateral ABCD = 25 + 107 = 132 sq. units.

11.

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{x}=\frac{1}{a+b+x}$$

$$\Rightarrow \frac{1}{a}+\frac{1}{b}=\frac{1}{a+b+x}-\frac{1}{x}$$

$$\Rightarrow \frac{a+b}{ab}=\frac{x-a-b-x}{(a+b+x)x}$$

$$\Rightarrow \frac{a+b}{ab}=\frac{-(a+b)}{(a+b+x)x}$$

$$\Rightarrow ax + bx + x^2 = -ab$$

$$\Rightarrow x^2 + ax + bx + ab = 0$$

$$\Rightarrow x(x + a) + b(x + a) = 0$$

$$\Rightarrow (x + a)(x + b) = 0$$

$$\Rightarrow x + a = 0 \text{ or } x + b = 0$$

$$\Rightarrow x = -a, -b$$

12.

$$\frac{x}{a}+\frac{y}{b} = 2$$

$\Rightarrow$ bx + ay = 2ab... (1)
ax – by = $a^2 - b^2$... (2)

Multiplying (1) with a and (2) with b, we get

$$\begin{aligned} \cancel{abx} + a^2y &= 2a^2b \\ \cancel{abx} - b^2y &= a^2b - b^3 \\ - \quad + \quad & \quad - \quad + \\ \hline y(a^2+b^2) &= a^2b + b^3 \\ \Rightarrow y(a^2+b^2) &= b(a^2+b^2) \\ \Rightarrow y &= b \end{aligned}$$

From (1), bx + ab = 2ab
$\Rightarrow$ bx = ab
$\Rightarrow$ x = a
Hence, x = a and y = b.

13. Let $\frac{3}{2\sqrt{5}}$ be a rational number. So, $\frac{3}{2\sqrt{5}}=\frac{a}{b}$ Where a and b are co-prime integers and b $\neq 0$.

$\sqrt{5}=\frac{3b}{2a}$

Now, a, b, 2 and 3 are integers Therefore, $\frac{3b}{2a}$ is a rational number.

$\Rightarrow \sqrt{5}$ is a rational number.

This is a contradiction as we know that $\sqrt{5}$ is irrational.

Therefore, our assumption is wrong.

Hence, $\frac{3}{2\sqrt{5}}$ is an irrational number.

14. Let us assume that Prema invests Rs. x @10% and Rs. y @8% in the first year.

We know that

Interest = $\frac{P\times R\times T}{100}$

According to given conditions,

$\frac{x\times 10\times 1}{100}+\frac{y\times 8\times 1}{100}=1640$

$\Rightarrow$ 10x + 8y = 164000 ...(i)

After interchanging the rates, we have

$\frac{y\times 10\times 1}{100}+\frac{x\times 8\times 1}{100}=1600$

$\Rightarrow$ 10y + 8x=160000

Or, 8x + 10y=160000 ...(ii)

Adding (i) and (ii), we get

18x + 18y=324000

$\Rightarrow$ x + y = 18000 ...(iii)

Subtracting (ii) from (i), we get

2x - 2y=4000

$\Rightarrow$ x - y = 2000 ...(iv)

Adding (iii) and (iv), we get

2x=20000

$\Rightarrow$ x = 10000

Substituting this value of x in (iii), we get

y = 8000

Thus, the sums invested in the first year at the rate 10% and 8% are Rs. 10000 and Rs. 8000, respectively.

15. Let P(x, y), Q(a + b, b – a) and R(a – b, a + b) be the given points.
It is given that PQ = PR ⇒ $PQ^2 = PR^2$

$\{x - (a + b)\}^2 + \{y - (b - a)\}^2 = \{x - (a - b)\}^2 + \{y - (a + b)\}^2$

$\Rightarrow x^2 - 2x(a + b) + (a + b)^2 + y^2 - 2y(b - a) + (b - a)^2$

$= x^2 + (a - b)^2 - 2x(a - b) + y^2 - 2y(a + b) + (a + b)^2$

$\Rightarrow -2x(a + b) - 2y(b - a) = -2x(a - b) - 2y(a + b)$

$\Rightarrow -ax - bx - by + ay = -ax + bx - ay - by$

$\Rightarrow 2bx = 2ay$

$\Rightarrow bx = ay$

16. Consider the following table:

C.I	f_i	x_i	d_i	f_id_i
100 - 150	4	125	-2	-8
150 - 200	5	175	-1	-5
200 - 250	12	225	0	0
250 - 300	2	275	1	2
300 - 350	2	325	2	4
Total	25			-7

Let A = 225

$$d_i = \frac{x_i - 225}{50}$$

$$\bar{x} = A + \frac{\sum f_i d_i}{\sum f_i} \times h = 225 - \frac{7}{\cancel{25}} \times \cancel{50}^{2} = 225 - 14 = 211$$

17. Given : In ΔXYZ and ΔDEF

$\frac{XY}{DE} = \frac{YZ}{EF} = \frac{XA}{DB}$ ...(1)

To prove: ΔXYZ ~ ΔDEF

Proof: Since XA and DB are medians

2YA = YZ

2EB = EF ...(2)

From (1) and (2)

$\frac{XY}{DE} = \frac{2YA}{2EB} = \frac{XA}{DB}$

⇒ΔXYA ~ ΔDEB (BY SAS rule)

⇒∠Y = ∠E ...(3)

Now, in ΔXYZ and ΔDEF,

$\frac{XY}{DE} = \frac{YZ}{EF}$ From (1)

∠Y = ∠E from (3)

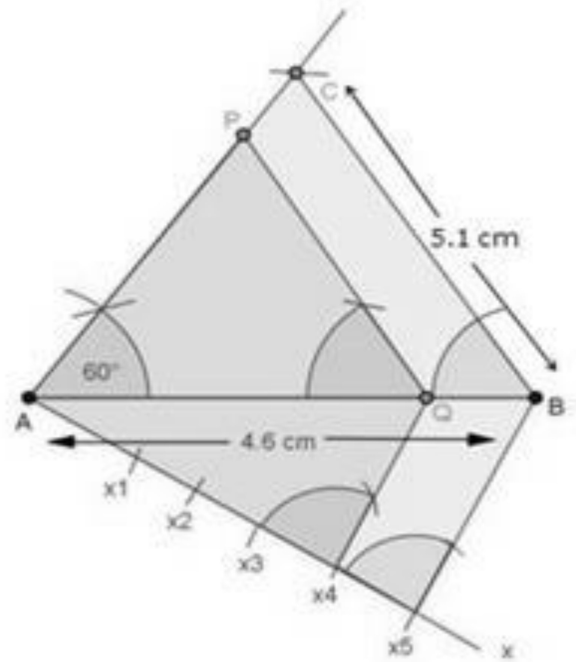
⇒ΔXYZ ~ ΔDEF (BY SAS rule)

Group D

18. Steps of construction:-

(1) Draw a line segment AB of 4.6 cm.
(2) At A draw an angle of 60°.
(3) With centre B and radius 5.1 cm draw an arc which intersects line AC of angle at C.
(4) Join BC.
(5)At A draw an acute angle BAX of any measure.
(6) Starting from A, cut 5 equal parts on Ax_5.
(7) Join X_5B
(8) Through X_4, Draw X_4Q || X_5B
(9) Through Q, Draw QP || BC

$\therefore \Delta PAQ \sim \Delta CAB$



19. There are three sections of each class and it is given that the number of trees planted by any class is equal to class number.

The number of trees planted by class I = number of sections × 1 = 3 × 1 = 3
The number of trees planted by class II = number of sections × 2 = 3 × 2 = 6
The number of trees planted by class III = number of sections × 3 = 3 × 3 = 9
Therefore, we have the sequence: 3, 6, 9, ..., (12 terms)

To find total number of trees planted by all the students, we need to find sum of the 12 terms of the sequence.

First term = a = 3

Common difference = d = 6 - 3 = 3

n = 12

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$\Rightarrow S_{12} = \frac{12}{2}[2\times3+(12-1)3]$$
$$= 6[6+33]$$
$$= 6\times39$$
$$= 234$$

Thus, in total 234 trees will be planted by the students.

Values inferred are environmental friendly and social.

20. In right ΔPQR,

$PR^2 = PQ^2 + QR^2 = 25 + 144 = 169$

$\therefore PR = 13$ cm

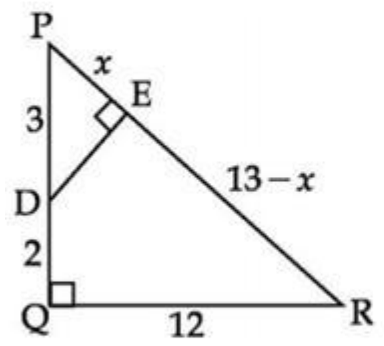
Let PE = x, then ER = 13 - x

In ΔPQR and ΔPED,

$\angle PQR = \angle PED$

$\angle QPR = \angle EPD$

$\therefore \Delta PQR \sim \Delta PED$ [AA similarity]



$$\therefore \frac{PQ}{PE} = \frac{QR}{ED} = \frac{PR}{PD}$$

$$\frac{5}{x} = \frac{12}{ED} = \frac{13}{3}$$

$$\therefore PE = x = \frac{5 \times 3}{13} = \frac{15}{13} = 1\frac{2}{13}$$

$$ED = \frac{12 \times 3}{13} = \frac{36}{13} = 2\frac{10}{13} \text{ cm}$$

21. Let the number of girls and boys in the class be x and y, respectively.

According to the given conditions, we have:

x + y = 10

x - y = 4

x + y = 10 ⇒ x = 10 - y

Three solutions of this equation can be written in a table as follows:

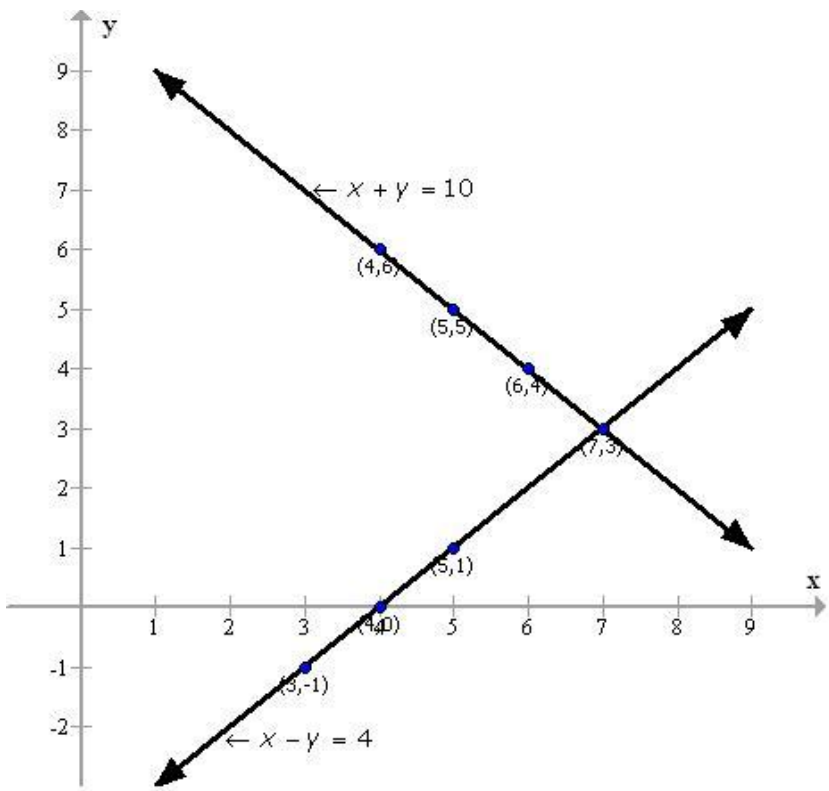
x	5	4	6
y	5	6	4

x - y = 4 ⇒ x = 4 + y

Three solutions of this equation can be written in a table as follows:

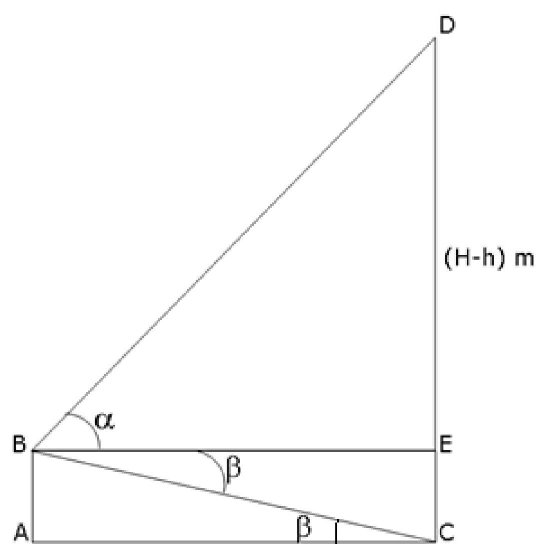
x	5	4	3
y	1	0	-

The graphs of the two equations can be drawn as follows:



From the graph, it can be observed that the two lines intersect each other at the point (7, 3).
So, x = 7 and y = 3 is the required solution of the given pair of equations.

22. Let B be the window of a house AB and let CD be the other house.
Then, AB = EC = h metres.
Let CD = H metres. Then, ED = (H − h) m



In ΔBED,

$\cot \alpha = \frac{BE}{ED}$

BE = (H − h) cot α ... (a)

In ΔACB,

$\frac{AC}{AB} = \cot \beta$

AC = h.cot β .... (b)

But BE = AC

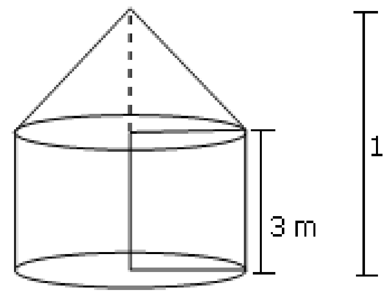
∴(H − h) cot α = hcot β .... [From (a) and (b)]

$H = h\frac{(\cot\alpha + \cot\beta)}{\cot\alpha}$

H = h(1 + tanα cotβ)

Thus, the height of the opposite house is h(1 + tanα.cotβ) metres.

23. Radius of conical portion = Radius of cylindrical portion = 14 m
Height of cylindrical portion = 3 m
Height of conical portion = 13.5 m − 3 m = 10.5 m



C.S.A. of tent = C.S.A. of cylinder + C.S.A. of cone

$= 2\pi rh + \pi rl$

$= 2\pi\ (14)\ (3) + \pi(14)$

$\sqrt{14^2 + 10.5^2}$

$= 264 + 14\pi\ \sqrt{306.25}$

$= 264 + 14\ \pi\ (17.5)$

$= 264 + 770$

$= 1034\ m^2$

Cost of painting the inside of tent,
i.e. 1034 m^2 at the rate of Rs. 2 per sq. m = Rs. 1034 x 2 = Rs. 2068.

24. Given, AC = BD = 7 cm and AB = CD = 1.75 cm = $\frac{7}{4}$ cm

Area of shaded region = 2 (area of semi-circle of radius $\frac{7}{2}$ cm) − 2(area of semi-circle of radius $\frac{7}{8}$ cm)

$$=2\left[\frac{1}{2}\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\right]-2\left[\frac{1}{2}\times\frac{22}{7}\times\frac{7}{8}\times\frac{7}{8}\right]$$

$$=\left(\frac{77}{2}-\frac{77}{32}\right)$$

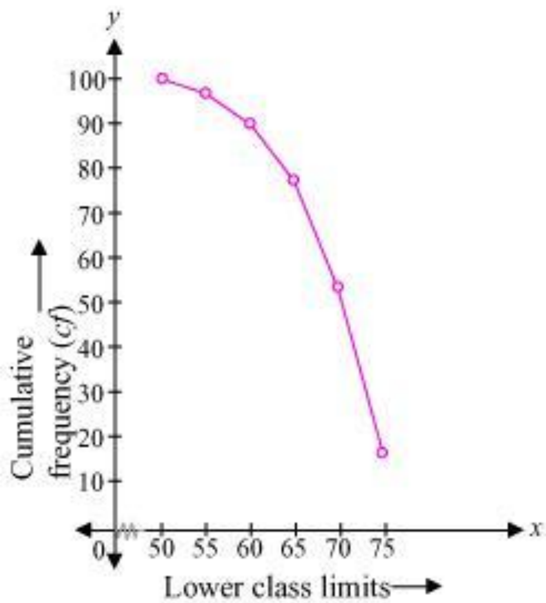
$$=\frac{77}{2}\left[1-\frac{1}{16}\right]=\frac{77}{2}\times\frac{15}{16}\text{ cm}^2=\frac{1155}{32}\text{ cm}^2=36.1\text{ cm}^2$$

25. We can obtain cumulative frequency distribution of more than type as following:

Production yield (lower class limits)	Cumulative frequency
More than or equal to 50	100
More than or equal to 55	100 - 2 = 98
More than or equal to 60	98 - 8 = 90

More than or equal to 65	90 - 12 = 78
More than or equal to 70	78 - 24 = 54
More than or equal to 75	54 - 38 = 16

Now, taking lower class limits on x-axis and their respective cumulative frequencies on y-axis, we can obtain the ogive as follows:



26. Total number of outcomes = 36

Favorable outcomes with the sum as 11 are (5, 6) and (6, 5).

Favourable outcome with the sum as 12 is (6, 6).

P(sum 11) = $\frac{2}{36}$

P(sum 12) = $\frac{1}{36}$

P(sum $\leq$ 10) = 1 - [P(sum 11) + P(sum 12)]

$$= 1 - \left[\frac{2}{36} + \frac{1}{36}\right]$$

$$= 1 - \frac{3}{36}$$

$$= 1 - \frac{1}{12}$$

$$= \frac{11}{12}$$

Or

When two dice are thrown, sample space is given by
S
={(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)}

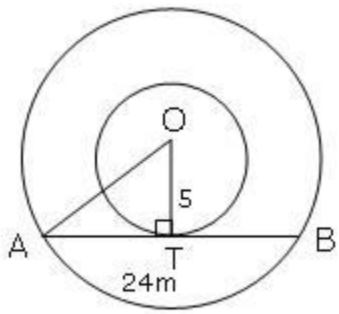
Thus, total number of outcomes when two dice are thrown = 36

Let A be the event that 5 will not come up on either die.

Thus, no. of favorable outcomes =
{(1,1),(1,2),(1,3),(1,4),(1,6),(2,1),(2,2),(2,3),(2,4),(2,6),(3,1),(3,2),(3,3),(3,4),(3,6),(4,1),(4,2),(4,3),(4,4),(4,6),(6,1),(6,2),(6,3),(6,4),(6,6)} = 25

Hence, P (A) = $\frac{25}{36}$

27.



Let O be the centre of circle and AB be the chord OT is radius of smaller circle

So OT $\perp$ AB since tangent is $\perp$ to radius at its point of contact.

AT = TB = 12 cm

(since perpendicular from centre to the chord bisects it)

So, In triangle OAT,

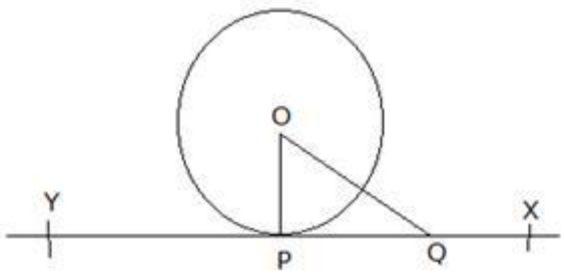
$OA^2 = OT^2 + AT^2$

$OA^2 = 5^2 + 12^2$

So, OA = 13 cm

Thus, the radius of the larger circle is 13 cm.

Or



Given: A circle with centre O and a tangent XY to the circle at a point P

To Prove: OP is perpendicular to XY.

Construction: Take a point Q on XY other than P and join OQ.

Proof: Here the point Q must lie outside the circle as if it lies inside the tangent XY will become secant to the circle.

Therefore, OQ is longer than the radius OP of the circle, That is, OQ > OP.

This happens for every point on the line XY except the point P.

So OP is the shortest of all the distances of the point O to the points on XY.

And hence OP is perpendicular to XY.

Hence, proved.