Nagaland Board
Class IX
Mathematics
Sample Paper – 2 Solution

Time: 3 hrs **Total Marks: 80**

Section A

1.

(i) Correct answer: C

$$\left(6+\sqrt{27}\right)-\left(3+\sqrt{3}\right)+\left(1-2\sqrt{3}\right)$$
$$=6+3\sqrt{3}-3-\sqrt{3}+1-2\sqrt{3}$$
$$=4$$

It is positive and rational.

(ii) Correct answer: B

The value of the polynomial $x^2 - x - 1$ at $x = -1$ equals $(-1)^2 - (-1) - 1 = 1 + 1 - 1 = 1$

(iii) Correct answer: B

The number of line segments determined by 3 non-collinear points is three.

(iv) Correct answer: A

If we divide or multiply both sides of a linear equation with a non-zero number, then the solution of the linear equation remains the same as the graph of the equation remains same in both the cases.

(v) Correct answer: C

m∠PSR = m∠RQP = 125° (since PQRS is a parallelogram, opposite angles will be equal)
⇒m∠PQT = 180° (PQT is a straight line)
⇒m∠PQR + m∠RQT = 180°
⇒125° + m∠RQT = 180°
⇒m∠RQT = 55°

(vi) Correct answer: A

Class size is the difference between two successive class marks, i.e. 10 – 6 = 4

(vii) Correct answer: B

$$\frac{56}{1000}=0.056$$

(viii) Correct answer: C

The abscissa or x-coordinate of any point on Y-axis is zero

(ix) Correct answer: B

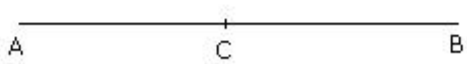
(x) Correct answer: A
Since AOB is a straight line,
$\angle$ AOB = 180°
$\Rightarrow$ x + 10° + x + x + 20° = 180°
$\Rightarrow$ 3x = 150°
$\Rightarrow$ x = 50°

Section B

2. Factorising $ky^2 - 6ky + 8k$, we have
$= k(y^2 - 6y + 8)$
$= k(y^2 - 4y - 2y + 8)$
$= k(y - 4)(y - 2)$
Thus, the dimensions of cuboid are given by the expressions k, (y-4) and (y 2).

3.

Given: AC = BC



AC +AC = BC + AC

(If equals are added to equal the wholes are equal)

$\Rightarrow$ 2AC = AB

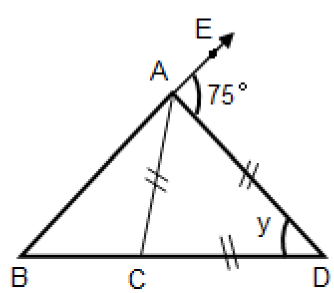
Hence, AC = $\frac{1}{2}$AB

4. Here $\angle$ADC = y = $\angle$ACD
Ext. $\angle$ACD = $\angle$ABC + $\angle$BAC
$\therefore$ 2$\angle$BAC = $\angle$ACD = y

$\Rightarrow \angle BAC = \frac{y}{2}$

$\therefore \frac{y}{2} + (180° - 2y) = 180° - 75°$

$\Rightarrow \frac{y}{2} + 180° - 2y = 180° - 75$



$\Rightarrow \frac{y}{2} - 2y = -75°$

$\Rightarrow -\frac{3y}{2} = -75°$

$\Rightarrow y = 50°$

5. Let 'l' be the length of the cube.
Now, T.S.A. of the cube = 294 cm^2 …(given)
$\therefore 6l^2 = 294$
$\therefore l^2 = \frac{294}{6} = 49$
$\therefore$ Side (l) = 7 cm.
Volume of cube = l × l × l = 7 × 7 × 7 = 343 cm^3

6. Number of students born in August = 6
Total number of students = 40
Required probability= $\frac{\text{Number of students born in August}}{\text{Total number of students}} = \frac{6}{40} = \frac{3}{20}$

Section C
(Questions 13 to 22 carry 3 marks each)

7. Given: a = 3 + b

a − b = 3

Applying the cubic identity on both the sides

$(a - b)^3 = 3^3$

$\Rightarrow a^3 - b^3 - 3(a)(b)(a - b) = 27$

$\Rightarrow a^3 - b^3 - 3ab(3) = 27$ $\quad$ ($\because$ a - b = 3)

$\Rightarrow a^3 - b^3 - 9ab = 27$

8.

Since AB ||DC

$\angle x = 30^0$ [Alternate angles]

In $\triangle$ABD

$80^0 + 30^0 + \angle y = 180^0$

$\angle y = 180^0 - 110^0 = 70^0$

In $\triangle$BDC

$30^0 + (70^0 - 30^0) + \angle z = 180^0$

$\angle z = 110^0$

Or

Let the angles of a quadrilateral be 2x, 5x, 8x and 9x respectively.

By the angle sum property of a quadrilateral, we have

2x + 5x + 8x + 9x = 360°

$\therefore$ 24x = 360°

$\therefore$ x = 15°

Now,

First angle = 2x = 2 × 15 = 30°,

Second angle = 5x = 5 × 15 = 75°,

Third angle = 8x = 8 × 15 = 120° and

Fourth angle = 9x = 9 × 15 = 135°.

Thus, the angles of a quadrilateral are 30°, 75°, 120° and 135°.

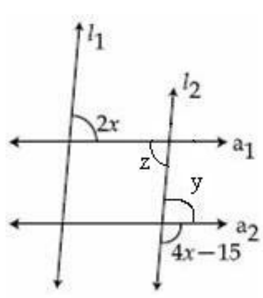
9. $27p^3 + 8q^3 + 54p^2q + 36pq^2$
$=(3p)^3 + (2q)^3 + 18pq(3p+2q)$
$=(3p)^3 + (2q)^3 + 3 \times 3p \times 2q\ (3p + 2q)$
$=(3p + 2q)^3$ [$(a + b)^3 = a^3 + b^3 + 3ab\ (a + b)$ where $a = 3p$ and $b = 2q$]
$= (3p + 2q)\ (3p + 2q)\ (3p + 2q)$

10. $b^2 + c^2 + 2(ab + bc + ca)$
$= a^2 + b^2 + c^2 + 2ab + 2bc + 2ca - a^2$ [Adding and suntracting a^2]
$=[a^2 + b^2 + c^2 + 2ab + 2bc + 2ca] - a^2$
$=(a + b + c)^2 - (a)^2$ [Using $x^2 + y^2 + 2xy + 2yz + 2zx = (x + y + z)^2$]
$=(a + b + c + a)(a + b + c - a)$ [Because $a^2 - b^2 = (a+ b)\ (a - b)$]
$=(2a + b +c)(b + c)$

Or

Let $p(z) = az^3 + 4z^2 + 3z - 4$ and $q(z) = z^3 - 4z + a$
When p(z) is divided by z-3, the remainder is given by:
$p(3) = a \times 3^3 + 4 \times 3^2 + 3 \times 3 - 4$
$= 27a + 36 + 9 - 4$
$= 27a + 41$ ………………..(i)
When q(z) is divided by z-3 the remainder is given by:
$q(3)= 3^3 - 4 \times 3 + a$
$= 27 - 12 + a$
$= 15 + a$ …………….(ii)
Given that p(3) = q(3). So, from (i) and (ii), we have:
$27a + 41 = 15 + a$
$27a - a = -41 + 15$
$26a = -26$
$a = -1$

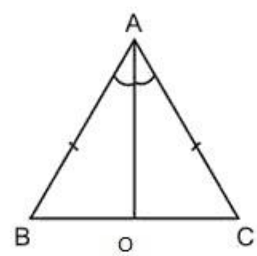
11. $2x = z$ (Alternate angles, as $l_1 \parallel l_2$)
$y = z$ (Alternate angles, as $a_1 \parallel a_2$)
So, $2x = y$
Now, $y + 4x - 15 = 180°$(linear pair)
$2x + 4x - 15 = 180°$
$6x = 195°$
$x = 32.5$



Or

Let ABC be an isosceles triangle with AB = AC.

Construction: Draw the bisector AO of ∠A.
In ΔABO and ΔACO, we have:
AB=AC (Given)



AO=OA (Common)
$\angle$BAO=$\angle$CAO (By Construction)
$\triangle$ABO $\cong$ $\triangle$ACO (By SAS congruence criteria)

12. Number of white balls = x
Total number of balls = 12

$$\therefore \text{P(white ball)} = \frac{x}{12}$$

If 6 white balls are added, we have
Total number of balls = 18
Number of white balls = x + 6

$$\text{Now, P(getting a white ball)} = \frac{x+6}{18}$$

According to the given information,

$$\frac{x+6}{18} = 2\left(\frac{x}{12}\right)$$

$$\therefore \frac{x+6}{18} = \frac{x}{6}$$

$$\therefore 6x + 36 = 18x$$

$$\therefore 12x = 36$$

$$\therefore x = 3$$

13. Length (l_1) of the storehouse = 40 m

Breadth (b_1) of the storehouse = 25 m

Height (h_1) of the storehouse = 10 m

Volume of storehouse = $l_1 \times b_1 \times h_1 = (40 \times 25 \times 10)\ m^3 = 10000\ m^3$

Length (l_2) of a wooden crate = 1.5 m

Breadth (b_2) of a wooden crate = 1.25 m

Height (h_2) of a wooden crate = 0.5 m

Volume of a wooden crate = $l_2 \times b_2 \times h_2 = (1.5 \times 1.25 \times 0.5)\ m^3 = 0.9375\ m^3$

Let the number of wooden crates stored in the storehouse be ‘n’.

Hence, volume of ‘n’ wooden crates = Volume of storehouse

0.9375 × n = 10000

$$\therefore n = \frac{10000}{0.9375} = 10666.66$$

Thus, 10666 wooden crates can be stored in the storehouse.

14. Inner radius of hemispherical bowl = 5 cm

Thickness of the bowl = 0.25 cm

∴ Outer radius (r) of hemispherical bowl = (5 + 0.25) cm = 5.25 cm

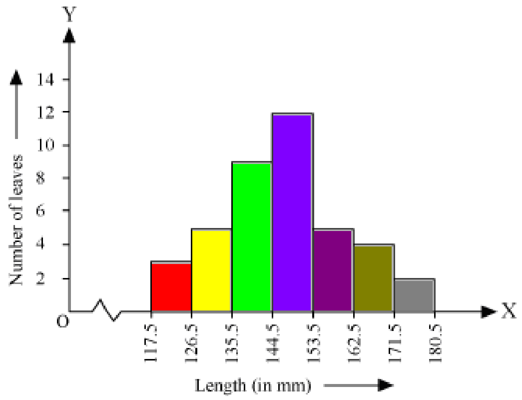
Outer C.S.A. of hemispherical bowl = $2\pi r^2 = 2 \times \frac{22}{7} \times (5.25)^2 = 173.25 \text{ cm}^2$

Thus, the outer curved surface area of the bowl is 173.25 cm^2.

15. Lengths of the leaves are represented in discontinuous class intervals. Hence we have to add 0.5 mm to each upper class limit and also have to subtract 0.5 mm from the lower class limits so as to make our class intervals continuous.

Length (in mm)	Number of leaves
117.5 – 126.5	3
126.5 – 135.5	5
135.5 – 144.5	9
144.5 – 153.5	12
153.5 – 162.5	5
162.5 – 171.5	4
171.5 – 180.5	2

Now taking the length of leaves on the x-axis and number of leaves on the y-axis we can draw the histogram of this information as below:

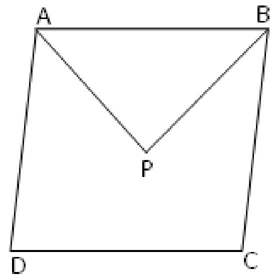


Here 1 unit on the y-axis represents 2 leaves.

i. Other suitable graphical representation of this data could be a frequency polygon.

ii. No, as maximum numbers of leaves (i.e. 12) have their length in between of 144.5 mm and 153.5 mm. It is not necessary that all have a length of 153 mm.

16. Given: ABCD is a parallelogram such that angle bisector of adjacent angles A and B intersect at point P.



To prove that m∠APB = 90°.

AD ∥ BC

∴ m∠A + m∠B = 180° [Consecutive interior angles]

$$\therefore \frac{1}{2}m\angle A + \frac{1}{2}m\angle B = 90^\circ$$

But,

$$\frac{1}{2}m\angle A + \frac{1}{2}m\angle B + m\angle APB = 180^\circ \text{ ...(Angle sum property of a triangle)}$$

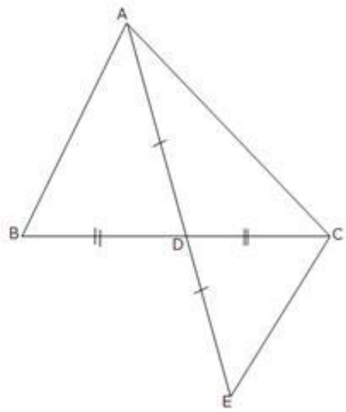
$$\therefore 90^\circ + m\angle APB = 180^\circ$$

$$\therefore m\angle APB = 90^\circ$$

Thus, the angle bisectors of two adjacent angles intersect at right angles.

Section D
(Questions 23 to 30 carry 4 marks each)

17. Given: AD is median of triangle ABC
To Prove: AB+AC>2AD
Proof: Produce AD so that AD = DE



Now, in triangles ADB and EDC,
AD = DE
BD = DC
∠ADB = ∠EDC
Thus, triangles ADB and EDC are congruent (By SAS congruence criterion)
Hence, AB = EC (CPCT)
Now, in triangle AEC,
AC + CE > AE
AC + CE > 2AD
AC + AB > 2AD (since, AB = EC, proved above)

18.

$$\frac{16\times 2^{n+1}-4\times 2^{n}}{16\times 2^{n+2}-2\times 2^{n+2}}$$

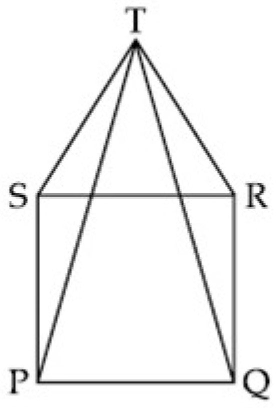
$$=\frac{2^{4}\times 2^{n+1}-2^{2}\times 2^{n}}{2^{4}\times 2^{n+2}-2\times 2^{n+2}}$$

$$=\frac{2^{n+5}-2^{n+2}}{2^{n+6}-2^{n+3}}$$

$$=\frac{2^{n+5}-2^{n+2}}{2.2^{n+5}-2.2^{n+2}}$$

$$=\frac{\left(2^{n+5}-2^{n+2}\right)}{2\left(2^{n+5}-2^{n+2}\right)}=\frac{1}{2}$$

19.



□PQRS is a square

$\therefore$ PQ = QR = RS = SP ... (i)

Also $\angle$RSP = $\angle$SRQ = $\angle$RQP = $\angle$SPQ = 90° ... (ii)

Also ΔTSR is equilateral

TS = TR = SR ... (iii)

Also $\angle$STR = $\angle$TSR = $\angle$TRS = 60°

From (i) and (iii)

TR = QR

Also $\angle$TSP = $\angle$RSP + $\angle$TSR = 90° + 60° = 150°

Similarly $\angle$TRQ = 150°

In ΔTSP and ΔTRQ,

PS = QR [by (i)]

$\angle$TSP= $\angle$TRQ [both 150°]

TS = TR [by (iii)]

$\therefore$ ΔTSP $\cong$ ΔTRQ [by SAS criterion]

20.

a)

$$\left\{5\left(2^{3\times\frac{1}{3}}+3^{3\times\frac{1}{3}}\right)^{3}\right\}^{\frac{1}{4}}$$

$$=\left[5(2+3)^{3}\right]^{\frac{1}{4}}$$

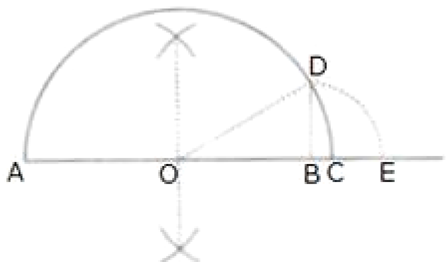
$$=\left(5\times5^{3}\right)^{\frac{1}{4}}$$

$$=5^{4\times\frac{1}{4}}$$

$$=5$$

b) In order to represent $\sqrt{7}$ on number line, we follow the steps given below:

Step 1: Draw a line and mark a point A on it.
Step 2: Mark a point B on the line drawn in step 1 such that AB = 7 cm.
Step 3: Mark a point C on AB produced such that BC = 1 unit.
Step 4: Find mid-point of AC. Let the mid-point be O.
Step 5: Taking O as the centre and OC = OA as radius draw a semicircle. Then, draw a line passing through B perpendicular to OB. Let the perpendicular cut the semicircle at D.
Step6: Taking B as the centre and radius BD draw an arc cutting OC produced at E. Point E so obtained represents $\sqrt{7}$.



21. Arranging the given data in ascending order, we have

41, 43, 57, 58, 61, 71, 92, 99, 127

Here, n = 9 (odd)

$$\therefore \text{Median} = \left(\frac{n+1}{2}\right)^{th} \text{ value}$$

$$= \left(\frac{9+1}{2}\right)^{th} \text{ value}$$

$$= 5^{th} \text{ value}$$

$$= 61$$

If 58 is replaced by 85, we get the following data:

41, 43, 57, 61, 71, 85, 92, 99, 127

$$\therefore \text{New median} = \left(\frac{n+1}{2}\right)^{th} \text{ value}$$

$$= \left(\frac{9+1}{2}\right)^{th} \text{ value}$$

$$= 5^{th} \text{ value}$$

$$= 71$$

Or

Diameter = 24 m ⇒ radius = 12 m

Radius of the conical part = Radius of the cylindrical part (r) = 12 m

Height of cylindrical part (h) = 11 m, height of the cone (h) = 5 m

For the conical part of the circus tent,

$l^2 = r^2 + h^2$

$\therefore l = \sqrt{r^2 + h^2}$

$\therefore l = \sqrt{12^2 + 5^2} = \sqrt{144 + 25} = \sqrt{169} = 13$ m

Surface area of the tent = Curved surface area of the conical part + Curved surface area of the cylindrical part

$$\begin{aligned}\therefore \text{Surface area of the tent} &= \pi rl + 2\pi rh \\ &= \pi r(l + 2h) \\ &= \frac{22}{7} \times 12(13 + 22) \\ &= \frac{22}{7} \times 12 \times 35 \\ &= 1320 \text{ m}^2\end{aligned}$$

Breadth of the canvas (B) = 5 m

Let the length of the canvas = L

Now, area of canvas required = surface area of the tent

$\therefore$ L × B = 1320

$\therefore$ L × 5 = 1320

$\therefore$ L = 264 m

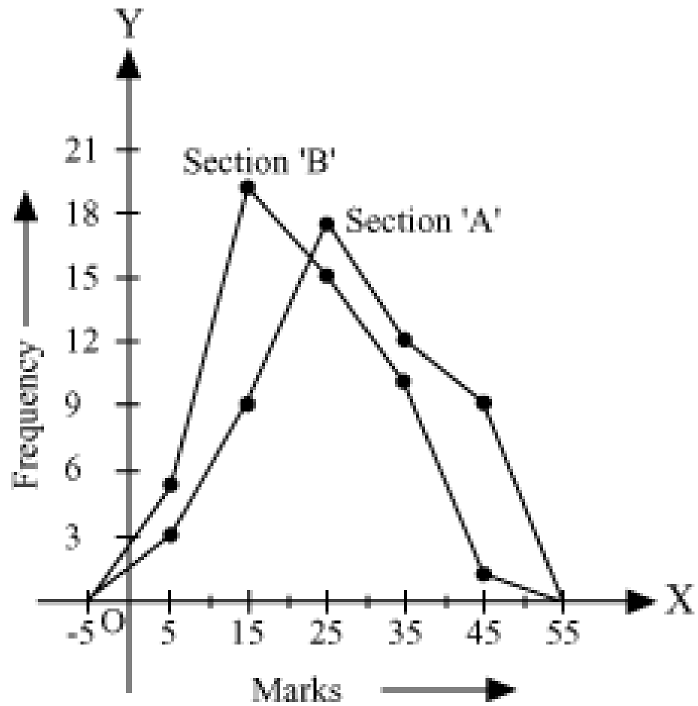
Thus, 264 m long canvas is required to make the tent.

22. We can find class marks of the given class intervals by using the formula –

$$\text{Class mark} = \frac{\text{upper class limit} + \text{lower class limit}}{2}$$

Section A			Section B		
Marks	Class marks	Frequency	Marks	Class marks	Frequency
0 – 10	5	3	0 – 10	5	5
10 – 20	15	9	10 – 20	15	19
20 – 30	25	17	20 – 30	25	15
30 – 40	35	12	30 – 40	35	10
40 – 50	45	9	40 – 50	45	1

Now taking the class marks on the x-axis and frequency on the y-axis and choosing an appropriate scale (1 cm = 3 units on the y-axis) we can draw a frequency polygon as below:



From the graph we can see that the performance of students of section 'A' is better than the students of section 'B'.

Or

Let us assume that Laxmi purchased x bananas and y oranges.
Since each banana costs Rs. 2, x bananas cost Rs. 2 × x = Rs. 2x
Similarly, each orange costs Rs. 3.
Thus, y oranges cost Rs. 3 × y = Rs. 3y
Thus, the total amount paid by Laxmi is Rs. (2x + 3y), which equals Rs. 30
Thus, we can express the given information in the form of a linear equation as $2x + 3y = 30$
Now, we know that Laxmi purchased 6 oranges, i.e., the value of y is 6.
Substitute this value of y in the equation $2x + 3y = 30$, thereby reducing it to a linear equation in one variable.
We can then solve the equation to obtain the value of x.
$2x + 3 \times 6 = 30$
$\Rightarrow 2x + 18 = 30$
This is a linear equation in one variable.
$\Rightarrow 2x = 30 - 18$
$\Rightarrow 2x = 12$
$\Rightarrow x = 6$
Thus, we see that the value of x is 6, i.e., Laxmi purchased 6 bananas.