Tripura
Class XII Mathematics
Sample Paper

Time allowed: 3 hours **Maximum Marks: 100**

Section-A

Question number 1 to 18 carry 1 mark each.

1. Find the value of $\sin^{-1}(-\frac{\sqrt{3}}{2})$ in the interval $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

2. For what values of x is the following matrix singular $A=\begin{bmatrix}3-2x & x+1\\ 2 & 4\end{bmatrix}$

3. Evaluate: $\int\frac{1+\cot x}{x+\log\sin x}dx$

4. Show that the function $f(x)=\begin{cases}\frac{\sin x}{x}+\cos x, & x\neq 0\\ 2, & x=0\end{cases}$ is continuous at x = 0.

5. Find the value of 'α' for which $\alpha(\hat{i}+\hat{j}+\hat{k})$ is a unit vector.

6. In the given mapping, is f is a bijection? Give reason for your answer. f: N→N defined by f(x) = 2x ∀x ∈ N

7. If f, g: R → R be functions defined as f(x) = 2x – 6, $g(x) = x^2 + 3$. Find gof (4).

8. Without expanding, find the value of the following determinant

$$\begin{vmatrix}0 & q-r & r-s\\ r-q & 0 & p-q\\ s-r & q-r & 0\end{vmatrix}$$

9. Cartesian equation of a line is $\frac{x-5}{2}=\frac{y-(-2)}{1}=\frac{z-4}{3}$. Find its vector equation.

10. If the sum of mean and variance of a binomial distribution for 5 trials is 1.8, find the distribution.

11. If $\underset{a}{\rightarrow}$ is a unit vector and $(\vec{x}-\vec{a}).(\vec{x}+\vec{a})=15$ then find $|x|$.

12. Evaluate : $\int_0^p\frac{\sqrt{x}}{\sqrt{x}+\sqrt{p-x}}dx$

13. Give an example of two non-zero matrices A and B such that AB = 0 but BA ≠ 0.

14. Evaluate: $\int e^{\log \sin x}.\cos x.dx$

15. For what value of λ the vectors $\vec{a}=2\hat{i}+\lambda\hat{j}+\hat{k}$ and $\vec{b}=-\hat{i}+2\hat{j}-3\hat{k}$ are orthogonal to each other.

16. Show that the points A(3, −5, 1) , B (−1, 0 , 8) and C (7 , −10 , −6) are collinear.

17. Write the intercept cut off by the plane 2x + y – z = 5 on x-axis.

18. Form a 2 x 2 matrix A = $[a_{ij}]$ where a_{ij} is given by

$$a_{ij}=\frac{|2i-j|}{3j}$$

Section-B

Questions number 19 to 28 carry 4 marks each.

19. A random variable X has the following probability distribution :

X	0	1	2	3	4	5	6	7
P(X)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

Determine: (i) k (ii) P(X < 3) (iii) P(X > 6) (iv) P(1 ≤ X < 3)

20. If $A=\begin{bmatrix}1 & 1 & 1\\ 1 & 2 & -3\\ 2 & -1 & 3\end{bmatrix}$, find A^{-1}

Using A^{-1}, solve the following system of equation

x + y + 2z = 0

x + 2y – z = 9

x – 3y + 3z = -14

21. The vector equations of two lines are:

$\vec{r}=\hat{i}+2\hat{j}+3\hat{k}+\lambda\left(\hat{i}-3\hat{j}+2\hat{k}\right)$ and $\vec{r}=4\hat{i}+5\hat{j}+6\hat{k}+\mu\left(2\hat{i}-3\hat{j}+\hat{k}\right)$

Find the shortest distance between the above lines.

OR

If $u=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ and $v=\tan^{-1}\left(\frac{2x}{1-x^2}\right)$, where -1 < x < 1, then write the value of $\frac{du}{dv}$.

22. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ and the straight line $\frac{x}{3}+\frac{y}{2}=1$.

23. Find $\int \frac{x^4 dx}{(x-1)(x^2+1)}$

24. Two dice are rolled twice. Find the probability distribution of the random variable X, which denotes the number of doublets. Find its mean and variance.

25. The probability that a student entering a university will graduate is 0.4. Find the probability that out of 3 students of the university:

i. none will graduate
ii. only one will graduate
iii. all will graduate

26. Find the angle between the line

$\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$ and the plane $10x + 2y - 11z = 3$

27. Two bags A and B contain 3 red and 4 black balls, and 4 red and 5 black balls respectively. From bag A, one ball is transferred to bag B and then a ball is drawn from bag B. The ball is found to be red in colour. Find the probability that

(a)the transferred ball is black ?

(b) the transferred ball is red ?

28. A doctor is to visit a patient. Form past experience, it is known that the probabilities that he will come by train, bus, scooter or by other means of transport are respectively $\frac{3}{10}, \frac{1}{5}, \frac{1}{10}$ and $\frac{2}{5}$. The probabilities that he will be late are $\frac{1}{4}, \frac{1}{3}$ and $\frac{1}{12}$ if he comes by train, bus and scooter respectively. But if he comes by other means of transport, then he will not be late. When he arrives, he is late. What is the probability that the doctor came by train?

SECTION-C

Questions number 29 to 35 carry 6 marks each.

29. A point on the hypotenuse of a right angled triangle is at distance of a units and b units from the sides. Show that the minimum length of hypotenuse is $\left(a^{\frac{2}{3}} + b^{\frac{2}{3}}\right)^{\frac{3}{2}}$

30. Find the volume of the largest cylinder which can be inscribed in a sphere of radius r.

31. Obtain the differential equation of all the circles touching the x-axis at the origin.

32. Evaluate :

$$\int \frac{(3\sin\alpha - 2)\cos\alpha}{5 - \cos^2\alpha - 4\sin\alpha} d\alpha$$

33. Find A^{-1}, by using elementary row transformations. Given $A = \begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$.

34. $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$; Find $A^2 - 5A + 7I$. Hence, find A^4

35. Prove that the curves $y = x^2$ and $x = y^2$ divide the square bounded by $x = 0$, $y = 0$, $x = 1$ and $y = 1$ into three parts which are equal in area.