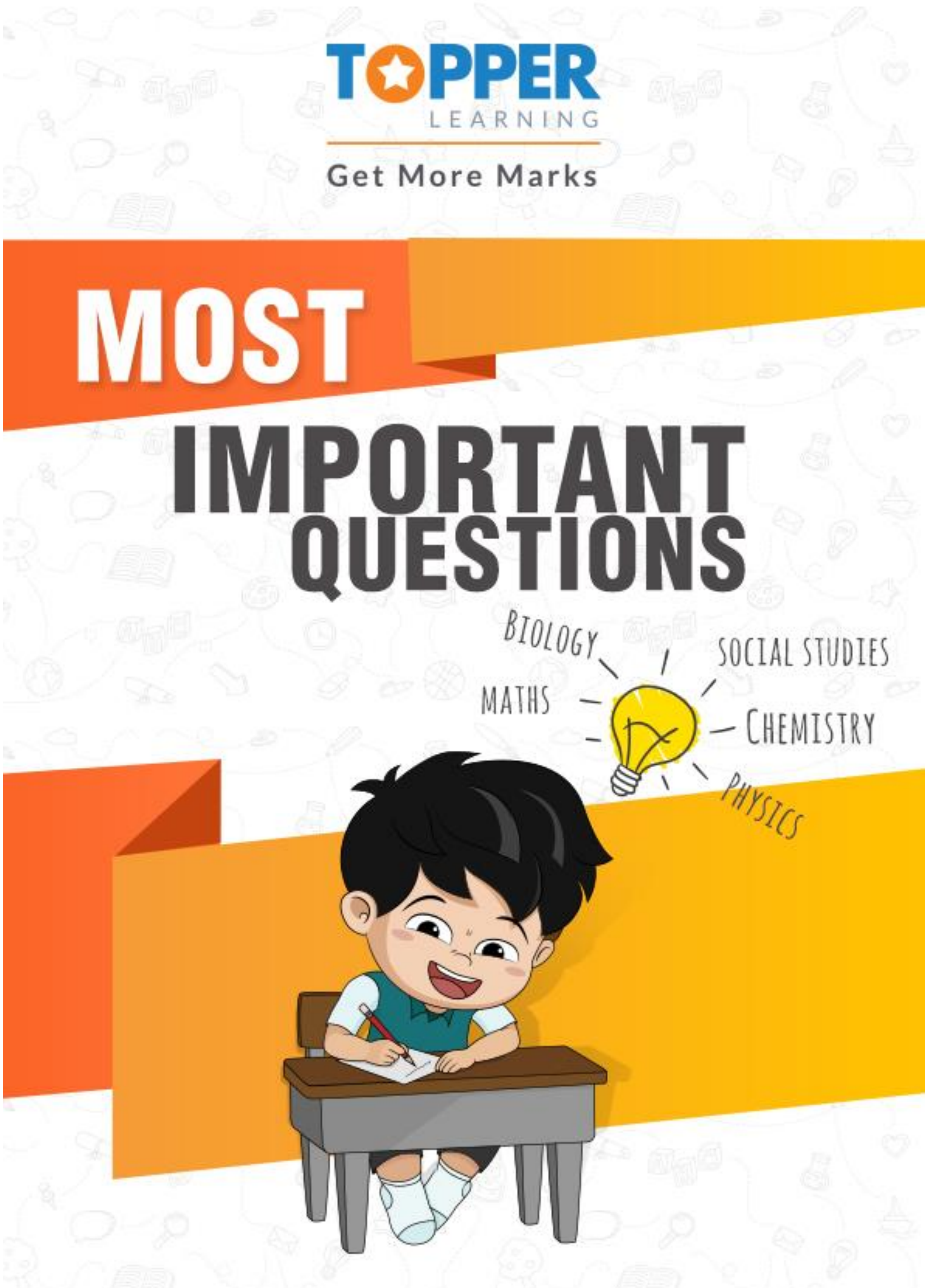
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Class X Maths

Most Important Questions (2021) - Solutions

Chapter 1: Real Numbers

1. Dividend = (divisor × quotient) + remainder

$$\Rightarrow 1365 = (\text{divisor} \times 31) + 32 \Rightarrow \frac{1365-32}{31} = \text{divisor} \Rightarrow \frac{1333}{31} = \text{divisor} \Rightarrow \text{Divisor} = 43$$

2. According to the question,
HCF (26, 169) = 13
HCF × LCM = Product of numbers

$$\Rightarrow \text{LCM} = \frac{169 \times 26}{13} \Rightarrow \text{LCM} = 338$$

3. Let us assume that $5 + 3\sqrt{2}$ is a rational number.

Hence, we can find co – prime p and q $(q \neq 0)$ such that

$$5+3\sqrt{2} = \frac{p}{q}$$

$$\therefore \frac{p}{q} - 5 = 3\sqrt{2}$$

$$\therefore \frac{p}{3q} - \frac{5}{3} = \sqrt{2}$$

$$\therefore \frac{p-5q}{3q} = \sqrt{2}$$

But $\sqrt{2}$ is irrational.
Thus, our assumption is wrong.
Hence, $5+3\sqrt{2}$ is an irrational.

4. (i) $\frac{13}{3125}$

$3125 = 5^5$

The denominator is of the form 5^m.

Hence, the decimal expansion of $\frac{13}{3125}$ is terminating.

(ii) $\frac{17}{8}$

$8 = 2^3$

The denominator is of the form 2^m.

Hence, the decimal expansion of $\frac{17}{8}$ is terminating.

5. Subtracting 6 from each number
378 – 6 = 372, 510 – 6 = 504

2	372
2	186
3	93
	31

2	504
2	252
2	126
7	63
3	9
	3

372 = 2 × 2 × 3 × 31 $= 2^2 \times 3 \times 31$, 504 = 2 × 2 × 2 × 3 × 3 × 7 $= 2^3 \times 3^2 \times 7$

H.C.F of 372 and 504 $= 2^2 \times 3 = 12$

Therefore the largest number which divides 378 and 510 leaving the remainder in each case is 12.

6.

2	336
2	168
2	84
2	42
3	21
	7

2	240
2	120
2	60
2	30
3	15
	5

2	96
2	48
2	24
2	12
2	6
	3

$336 = 2\times2\times2\times2\times3\times7 = 2^4\times3\times7$

$240 = 2\times2\times2\times2\times3\times5 = 2^4\times3\times5$

$96 = 2\times2\times2\times2\times2\times3 = 2^5\times3$

H.C.F $= 2^4\times3 = 16\times3 = 48$

Each stack will contain 48 books.

Number of stacks of the same height

$= \frac{240}{48} + \frac{336}{48} + \frac{96}{48} = 5+7+2 = 14$

Hence, 14 stacks will be there.

7. If possible, let $\sqrt{6}$ be rational and let its simplest form be $\frac{a}{b}$, then a and b are integers having no common factor other than 1, and $b \neq 0$.

Now,

$$\sqrt{6} = \frac{a}{b}$$

$$\Rightarrow 6 = \frac{a^2}{b^2} \text{[on squaring both sides]}$$

$$\Rightarrow 6b^2 = a^2 \ldots\ldots\ldots\ldots\text{(i)}$$

$$\Rightarrow 6 \text{ divides } a^2 \quad [\because 6 \text{ divides } 6b^2]$$

$$\Rightarrow 6 \text{ divides } a$$

Let a = 6c for some integer c.

Putting a = 6c in (i), we get

$$6b^2 = 36c^2 \Rightarrow b^2 = 6c^2$$

$$\Rightarrow 6 \text{ divides } b^2 \quad [\because 6 \text{ divides } 6c^2]$$

$$\Rightarrow 6 \text{ divides } b \quad [\because 6 \text{ divides } b^2 = 6 \text{ divides } b]$$

Thus, 6 is a common factor of a and b.

But, this contradicts the fact that a and b have no common factor other than 1.

The contradiction arises by assuming that $\sqrt{6}$ is rational.

Hence, $\sqrt{6}$ is irrational.

Chapter 2: Polynomials

1. The polynomial p(x) can be divided by the polynomial g(x) as follows:

$$\begin{array}{r|l} & \quad x-3 \\ x^2-2 & x^3-3x^2+5x-3 \\ & x^3 \qquad\ -2x \\ & \underline{- \qquad\quad +} \\ & \quad -3x^2+7x-3 \\ & \quad -3x^2 \qquad +6 \\ & \underline{\quad + \qquad\quad -} \\ & \qquad\quad 7x-9 \end{array}$$

Quotient = x – 3

Remainder = 7x – 9

2. Given polynomial is $p(x) = ax^2 + 7x + b$

$\frac{2}{3}$ and -3 are the zeroes of p(x)

$$\Rightarrow p\left(\frac{2}{3}\right) = 0 \text{ and } p(-3) = 0$$

$\Rightarrow a\left(\frac{2}{3}\right)^2 + 7\left(\frac{2}{3}\right) + b = 0$

$\Rightarrow \frac{4a}{9} + \frac{14}{3} + b = 0$

$\Rightarrow 4a + 9b + 42 = 0$ ...... (i)

Also, $a(-3)^2 + 7(-3) + b = 0$

$\Rightarrow 9a + b - 21 = 0$ ...... (ii)

Solving (i) and (ii) simultaneously, we get $a = 3$

Substituting the value of a in (ii), we get $b = -6$

3. Let $p(x) = 3x^4 + 6x^3 - 2x^2 - 10x - 5$

It is given that the two zeroes of p(x) are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

$\therefore \left(x - \sqrt{\frac{5}{3}}\right)\left(x + \sqrt{\frac{5}{3}}\right) = \left(x^2 - \frac{5}{3}\right)$ is a factor of p(x).

Therefore, we divide the given polynomial by $x^2 - \frac{5}{3}$.

$$
\begin{array}{r|l}
 & 3x^2 + 6x + 3 \\
x^2 + 0.x - \frac{5}{3} & 3x^4 + 6x^3 - 2x^2 - 10x - 5 \\
 & 3x^4 + 0x^3 - 5x^2 \\
 & - \quad - \quad + \\
\hline
 & 6x^3 + 3x^2 - 10x - 5 \\
 & 6x^3 + 0x^2 - 10x \\
 & - \quad - \quad + \\
\hline
 & 3x^2 + 0x - 5 \\
 & 3x^2 + 0x - 5 \\
 & - \quad - \quad + \\
\hline
 & 0 \\
\hline
\end{array}
$$

$3x^4 + 6x^3 - 2x^2 - 10x - 5 = \left(x^2 - \frac{5}{3}\right)\left(3x^2 + 6x + 3\right) = 3\left(x^2 - \frac{5}{3}\right)\left(x^2 + 2x + 1\right)$

Now, $x^2 + 2x + 1 = (x + 1)^2$

So, the two zeroes of $x^2 + 2x + 1$ are –1 and –1.

Hence, the zeroes of the given polynomial are $\sqrt{\frac{5}{3}}$, $-\sqrt{\frac{5}{3}}$, –1 and –1.

4. By division algorithm,

Dividend = Divisor × Quotient + Remainder

Divisor × Quotient = Dividend – Remainder

Divisor × Quotient = $x^4 - 6x^3 + 16x^2 - 25x + 10 - x - a$

$= x^4 - 6x^3 + 16x^2 - 26x + 10 - a$

It will be perfectly divisible by $x^2 - 2x + k$.

So, now $x^4 - 6x^3 + 16x^2 - 26x + 10 - a$ can be divided by $x^2 - 2x + k$ as follows:

$$
\begin{array}{r|l}
 & x^2 - 4x + (8-k) \\
\hline
x^2 - 2x + k & x^4 - 6x^3 + 16x^2 - 26x + 10 - a \\
 & x^4 - 2x^3 + kx^2 \\
 & - \quad + \quad - \\
\hline
 & -4x^3 + (16-k)x^2 - 26x \\
 & -4x^3 + 8x^2 - 4kx \\
 & + \quad - \quad + \\
\hline
 & (8-k)x^2 - (26-4k)x + 10 - a \\
 & (8-k)x^2 - (16-2k)x + (8k - k^2) \\
 & - \quad + \quad - \\
\hline
 & (-10+2k)x + (10 - a - 8k + k^2)
\end{array}
$$

As $x^4 - 6x^3 + 16x^2 - 26x + 10 - a$ is divisible by $x^2 - 2x + k$, so, the remainder will be 0.

$\therefore (-10+2k)x + (10 - a - 8k + k^2) = 0$

$\Rightarrow (-10+2k) = 0$ and $(10 - a - 8k + k^2) = 0$

Now, $(-10+2k) = 0 \quad \Rightarrow 2k = 10 \Rightarrow k = 5$

$10 - a - 8k + k^2 = 0$

$\Rightarrow 10 - a - 8 \times 5 + 25 = 0$

$\Rightarrow 10 - a - 40 + 25 = 0$

$\Rightarrow -5 - a = 0$

$\Rightarrow a = -5$

Thus, k = 5 and a = –5.

Chapter 3: Pairs of Linear Equations in Two Variables

1. Let the fraction be $\frac{x}{y}$.

When 2 is added to both numerator and denominator, the fraction becomes

$\frac{x+2}{y+2} = \frac{1}{3}$ or $3x + 6 = y + 2$

$\Rightarrow 3x - y = -4$ ---(1)

When 3 is added both to numerator and denominator, the fractions becomes

$\frac{x+3}{y+3}=\frac{2}{5}$ or $5x+15=2y+6$

$\Rightarrow 5x - 2y = -9$ ----(2)

Multiplying (1) by 2, we get

$6x - 2y = -8$ ---(3)

Eq.(2) is $5x - 2y = -9$ ---(4)

Subtracting (4) from (3), we get x = 1

From (1), 3 − y = −4, y = 7

$\therefore$ Required fraction is $\frac{1}{7}$

2. On a graph paper, draw a horizontal line X'OX and a vertical line YOY' as the x-axis and the y-axis, respectively.

Given equations are 2x + 3y = 2 and x − 2y = 8

Graph of 2x + 3y = 2:

$y=\frac{2(1-x)}{3}$

Putting x = 1, we get y = 0

Putting x = −2, we get y = 2

Putting x = 4, we get y = −2

$\therefore$ Table for 2x + 3y = 2 is

x	1	−2	4
y	0	2	−2

Plot the points A(1, 0), B(−2, 2) and C(4, −2) on the graph paper. Join AB and AC to

get the graph line BC. Extend it both ways.

Thus, line BC is the graph of 2x + 3y = 2.

Graph of x − 2y = 8:

$y=\frac{x-8}{2}$

Putting x = 2, we get y = −3

Putting x = 4, we get y = −2

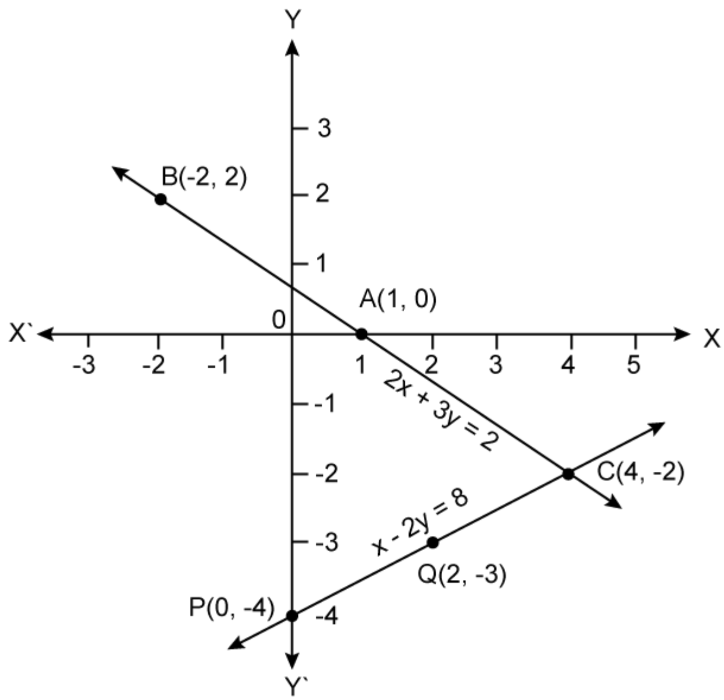
Putting x = 0, we get y = −4

Table for x − 2y = 8 is

x	2	4	0
y	−3	−2	−4

Now, on the same graph paper as above, plot the points P(0, −4) and Q(2, −3). Point C(4,−2) has already been plotted. Join QC and extend it. Thus, the line PC is the graph of x − 2y = 8.

Scale: 1 cm = 1 unit on both axes



The two graph lines interest at C(4, −2)

∴ x = 4, y = −2 is the solution of the given system of equations.

3. Let 1-day work of a man be $\frac{1}{x}$ and that of a boy be $\frac{1}{y}$.

Then, $\frac{2}{x}+\frac{5}{y}=\frac{1}{4}$

Let $u=\frac{1}{x}$ and $v=\frac{1}{y}$

$\Rightarrow 2u+5v=\frac{1}{4}\ ---(1)$

and $\frac{3}{x}+\frac{6}{y}=\frac{1}{3}$

$\Rightarrow 3u+6v=\frac{1}{3}\ ---(2)$

Multiplying (1) by 6 and (2) by 5, we get

$12u+30v=\frac{6}{4}\ ---(3)$

$15u+30v=\frac{5}{3}\ ---(4)$

Subtracting (3) from (4), we get

$3u=\frac{1}{6}\Rightarrow u=\frac{1}{18}$

Putting $u=\frac{1}{18}$ in (1), we get

$2\times\frac{1}{18}+5v=\frac{1}{4}\Rightarrow\frac{1}{9}+5v=\frac{1}{4}\Rightarrow 5v=\frac{1}{4}-\frac{1}{9}$

$\Rightarrow 5v=\frac{5}{36}\Rightarrow v=\frac{1}{36}$

Now $u=\frac{1}{18}\Rightarrow x=\frac{1}{u}=18$ and $v=\frac{1}{36}\Rightarrow y=\frac{1}{v}=36$

$\therefore$ x = 18, y = 36

A man will finish the work in 18 days and a boy will finish the work in 36 days when they work alone.

4. Let the speed of the boat in still water be x km/hr and the speed of the stream be y km/hr.
So,
Speed upstream = (x − y) km/hr
Speed downstream = (x + y) km/hr

Time taken to cover 12 km upstream = $\frac{12}{x-y}$hrs

Time taken to cover 40 km downstream = $\frac{40}{x+y}$hrs

Total time taken = 8 hrs

$\therefore\frac{12}{x-y}+\frac{40}{x+y}=8$

Again, time taken to cover 16 km upstream = $\frac{16}{(x-y)}$

Time taken to taken to cover 32 km downstream = $\frac{32}{(x+y)}$

Total time taken = 8 hrs

$\therefore\frac{16}{(x-y)}+\frac{32}{(x+y)}=8$

Putting $\frac{1}{(x-y)}=u$ and $\frac{1}{(x+y)}=v$, we get

12u + 40v = 8 $\Rightarrow$ 3u + 10v = 2 ---(1)
16u + 32v = 8 $\Rightarrow$ 2u + 4v = 1 ---(2)
Multiplying (1) by 4 and (2) by 10, we get
12u + 40v = 8 --- (3)
20u + 40v = 10 --- (4)
Subtracting (3) from (4), we get

$8u=2\Rightarrow u=\frac{1}{4}$

Putting $u=\frac{1}{4}$ in (3), we get

$3\times\frac{1}{4}+10v=2\Rightarrow 10v=\frac{5}{4}\Rightarrow v=\frac{1}{8}$

$u=\frac{1}{4}\Rightarrow\frac{1}{x-y}=\frac{1}{4}\Rightarrow x-y=4---(5)$

$v=\frac{1}{8}\Rightarrow\frac{1}{x+y}=\frac{1}{8}\Rightarrow x+y=8---(6)$

On adding (5) and (6), we get 2x = 12 ⇒ x = 6

Putting x = 6 in (6), we get

6 + y = 8 ⇒ y = 8 − 6 = 2

∴ x = 6, y = 2

Hence, the speed of the boat in still water = 6 km/hr and the speed of the stream = 2 km/hr.

Chapter 4: Quadratic Equations

1. The given equation is $3x^2-2x+8=0$

Comparing it with $ax^2+bx+c=0$, we get

a = 3, b = −2, c = 8

$$\begin{aligned}\therefore D&=\left(b^2-4ac\right)\\&=(-2)^2-(4\times3\times8)\\&=(4-96)\\&=-92\end{aligned}$$

2. Since x = 1 is a solution of $x^2+kx+3=0$, it must satisfy the equation $\therefore(1)^2+k(1)+3=0\Rightarrow k=-4$

Hence, the required value of k = −4.

3. The given quadratic equation is

$x^2-8x+18=0$

The above equation can be written as

$\therefore x^2-8x+16-16+18=0$

$\therefore x^2-8x+16=-2$

$\therefore(x-4)^2=-2$

$\therefore x-4=\sqrt{-2}$

Now square root of a negative number is not possible

So the given quadratic equation has no roots.

4. $x^2-(1+\sqrt{2})x+\sqrt{2}=0$

$\Rightarrow x^2-1.x-\sqrt{2}x+\sqrt{2}=0$

$\Rightarrow x(x-1)-\sqrt{2}(x-1)=0$

$\Rightarrow (x-1)(x-\sqrt{2})=0$

$\Rightarrow (x-1)=0$ or $x-\sqrt{2}=0$

$\Rightarrow x=1$ or $x=\sqrt{2}$

Hence, 1 and $\sqrt{2}$ are the roots of the given equation.

5. Comparing with $ax^2 + bx + c = 0$ we get a = 1, b = 3 and c = -7

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-3\pm\sqrt{3^2-4\times1\times-7}}{2\times1}=\frac{-3\pm\sqrt{37}}{2}$$

6. The given equation is $3x^2-32x+12=0$

Comparing it with $ax^2+bx+c=0$, we get

a = 3, b = −32, c = 12

$\therefore\ D=(b^2-4ac)=\left[(-32)^2-4\times3\times12\right]=(1024-144)=880>0$

So, the given equation has real and distinct roots given by

$$\alpha=\frac{-b+\sqrt{D}}{2a}=\frac{32+\sqrt{880}}{2\times3}=\frac{32+4\sqrt{55}}{6}=\frac{2(16+2\sqrt{55})}{6}=\frac{16+2\sqrt{55}}{3}$$

$$\beta=\frac{-b-\sqrt{D}}{2a}=\frac{32-\sqrt{880}}{2\times3}=\frac{32-4\sqrt{55}}{6}=\frac{2(16-2\sqrt{55})}{6}=\frac{16-2\sqrt{55}}{3}$$

Hence, $\frac{16+2\sqrt{55}}{3}$ and $\frac{16-2\sqrt{55}}{3}$ are the roots of the given equation.

7. $\frac{(x-3)}{(x+3)}-\frac{(x+3)}{(x-3)}=6\frac{6}{7}$

$$\Rightarrow\frac{(x-3)^2-(x+3)^2}{(x+3)(x-3)}=\frac{48}{7}$$

$$\Rightarrow\frac{(x^2+9-6x)-(x^2+9+6x)}{(x+3)(x-3)}=\frac{48}{7}$$

$$\Rightarrow\frac{-12x}{x^2-9}=\frac{48}{7}$$

$\Rightarrow -84x=48x^2-432$

$\Rightarrow 48x^2 + 84x - 432 = 0$

$\Rightarrow 4x^2 + 7x - 36 = 0$

$\Rightarrow 4x^2 + 16x - 9x - 36 = 0$

$\Rightarrow 4x(x+4) - 9(x+4) = 0$

$\Rightarrow (4x-9)(x+4) = 0$

$\Rightarrow 4x - 9 = 0$ or $x + 4 = 0$

$\Rightarrow x = \frac{9}{4}$ or $x = -4$

Hence, −4 and $\frac{9}{4}$ are the roots of the given equation.

8. Let the speed of the boat in still water be x kmph, then
Speed of the boat downstream = (x + 2) kmph
And the speed of the boat upstream = (x – 2) kmph

Time taken to cover 8 km downstream = $\frac{8}{(x+2)}$hrs

Time taken to cover 8 km upstream = $\frac{8}{(x-2)}$hrs

Total time taken = $\frac{5}{3}$ hrs

$\frac{8}{(x+2)} + \frac{8}{(x-2)} = \frac{5}{3}$

$\Rightarrow \frac{1}{x+2} + \frac{1}{x-2} = \frac{5}{24}$

$\Rightarrow \frac{x-2+x+2}{(x+2)(x-2)} = \frac{5}{24}$

$\Rightarrow \frac{2x}{x^2-4} = \frac{5}{24}$

$\Rightarrow 5x^2 - 20 - 48x = 0$

$\Rightarrow 5x^2 - 48x - 20 = 0$

$\Rightarrow 5x^2 - 50x + 2x - 20 = 0$

$\Rightarrow 5x(x-10) + 2(x-10) = 0$

$\Rightarrow (x-10)(5x+2) = 0$

$\Rightarrow x = 10$ or $x = \frac{-2}{5}$

$\Rightarrow x = 10$ (speed cannot be negative)

Then the speed of the boat in still water is 10 kmph.

Chapter 5: Arithmetic Progressions

1. The given AP is 5, 7, 9, ... 201.
The last term l = 201, common difference d = 7 − 5 = 2
6^{th} term from the end = l − (n − 1)d= 201 − (6 − 1) × 2 = 201 − 10 = 19

2. Let the three consecutive numbers be a − d, a, a + d.
According to the question,
a − d + a + a + d = 3
∴3a = 3
∴a = 1
(a − d)a(a + d) = −35
(1 − d)(1 + d) = −35
$1 - d^2 = -35$
$d^2 = 36 \Rightarrow d = \pm 6$

3. In the given AP, let the first term = a common difference = d
Then, $a_n = a + (n-1)d$
$\Rightarrow a_7 = a + (7-1)d$, and $a_{13} = a + (13-1)d$
$\Rightarrow a_7 = a + 6d$, $a_{13} = a + 12d$
$\Rightarrow a_7 = -4 \Rightarrow a + 6d = -4 ---(1)$
$\Rightarrow a_{13} = -16 \Rightarrow a + 12d = -16 ---(2)$
Subtracting (1) from (2), we get
$6d = -12 \Rightarrow d = -2$
Putting $d = -2$ in (1), we get
$a + 6 \times (-2) = -4 \Rightarrow a - 12 = -4 \Rightarrow a = 8$
Thus, $a = 8$, $d = -2$
So, the required AP is 8, 6, 4, 2, 0....

4. Let a be the first term and d be the common difference.
p^{th} term = a +(p − 1)d = q (given) -----(1)
q^{th} term = a +(q − 1) d = p (given) -----(2)
Subtracting (2) from (1),
(p − q)d = q − p
(p − q)d= −(p − q)
∴ d = −1
Putting d = −1 in (1),
$a - (p-1) = q \Rightarrow a = p + q - 1$
$\therefore (p+q)^{th}$ term $= a + (p+q-1)d = (p+q-1) - (p+q-1) = 0$

5. Let the first term of the given AP = a and common difference = d then, $a_n = a + (n-1)d$

$\Rightarrow a_4 = a + (4-1)d,\ a_{25} = a + (25-1)d,$ and $a_{11} = a + (11-1)d$

$\Rightarrow a_4 = a + 3d,\ a_{25} = a + 24d,$ and $a_{11} = a + 10d$

Now, $a_4 = 0 \Rightarrow a + 3d = 0 \Rightarrow a = -3d$

$\therefore a_{25} = a + 24d = (-3d + 24d) = 21d$

And $a_{11} = a + 10d = -3d + 10d = 7d$

$\therefore a_{25} = 21d = 3 \times (7d) = 3 \times a_{11}$

Hence, it is proved that 25^{th} term of the given AP is triple its 11^{th} term.

6. 24, 21, 18,...

a = 24, d = 21 – 24 = -3, S_n = 78

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow \frac{n}{2}[2a + (n-1)d] = 78$

$\Rightarrow \frac{n}{2}[48 - 3(n-1)] = 78$

$\Rightarrow \frac{n}{2}[51 - 3n] = 78$

$\Rightarrow 3n^2 - 51n + 156 = 0$

$\Rightarrow n^2 - 17n + 52 = 0$

$\Rightarrow (n-4)(n-13) = 0$

$\Rightarrow n = 4$ or $n = 13$

The number of terms either 4 or 13.

7. Let the three terms of an A.P. be a – d, a, a + d.

The sum of three terms of an A.P. is 21.

⇒ a – d + a + a + d = 21

⇒ 3a = 21

⇒ a = 7

The product of the first and the third terms exceeds the second term by 6.

⇒ (a – d)(a + d) = a + 6

⇒ $a^2 - d^2 = a + 6$

⇒ $49 - d^2 = 13$

⇒ $d^2 = 36$

⇒ d = ± 6

When a = 7 and d = 6 then the three terms are 1, 7, 13.

When a = 7 and d = -6 then the three terms are 13, 7, 1.

8. Let a_1 and a_2 be the first terms and d_1 and d_2 the common differences of the two given A.P.'s

We have $S_n = \frac{n}{2}[2a_1 + (n-1)d_1]$ and $S_n' = \frac{n}{2}[2a_2 + (n-1)d_2]$

$$\Rightarrow \frac{S_n}{S_n'} = \frac{\frac{n}{2}\left[2a_1 + (n-1)d_1\right]}{\frac{n}{2}\left[2a_2 + (n-1)d_2\right]}$$

It is given that $\frac{S_n}{S_n'} = \frac{3n+5}{5n+7}$

$$\Rightarrow \frac{2a_1 + (n-1)d_1}{2a_2 + (n-1)d_2} = \frac{3n+5}{5n+7} \Rightarrow \frac{a_1 + \left(\frac{n-1}{2}\right)d_1}{a_2 + \left(\frac{n-1}{2}\right)d_2} = \frac{3n+5}{5n+7} \quad ....(i)$$

Now, ratio of their Nth terms...... (Considering 'N' instead of 'n' to avoid confusion)

$$\Rightarrow \frac{a_1 + (N-1)d_1}{a_2 + (N-1)d_2} \quad .... (ii)$$

In (i) if we replace

$\frac{n-1}{2}$ by N - 1 we will get required ratio

$$\Rightarrow \frac{n-1}{2} = N-1$$

$$\Rightarrow n-1 = 2N-2$$

$$\Rightarrow n = 2N-1 \text{ ...substitute in (ii)}$$

$$\Rightarrow \frac{S_n}{S_n'} = \frac{a_1 + (N-1)d_1}{a_2 + (N-1)d_2} = \frac{3(2N-1)+5}{5(2N-1)+27} = \frac{3N+1}{5N+11}$$

Hence, the ratio of their N^{th} terms is 3N + 1: 5N + 11.

Chapter 6: Triangles

1. The ratio of the areas of two similar triangles are equal to the ratio of the squares of any two corresponding sides.

$$\therefore \frac{ar\Delta ABC}{ar\Delta PQR} = \frac{AB^2}{PQ^2} = \frac{16}{81}$$

2. Given: $\Delta OAB \sim \Delta OCD$,

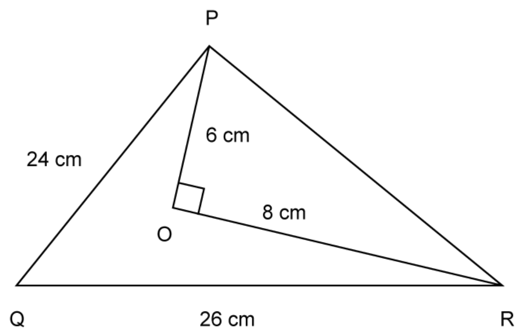
AB = 8 cm, BO = 6.4 cm, CD = 5 cm, OC = 3.5 cm

$$\Rightarrow \frac{OA}{OC} = \frac{AB}{CD} = \frac{BO}{DO} \Rightarrow \frac{OA}{3.5} = \frac{8}{5} = \frac{6.4}{DO} \Rightarrow \frac{OA}{3.5} = \frac{8}{5} \text{ and } \frac{6.4}{DO} = \frac{8}{5}$$

$$\Rightarrow OA = \frac{3.5 \times 8}{5} = 5.6 \text{ and } DO = \frac{6.4 \times 5}{8} = 4$$

$$\therefore OA = 5.6 \text{cm and } DO = 4 \text{ cm}$$

3. Given: In ΔPQR, ∠POR = 90°, OP = 6 cm, PQ = 24 cm and QR = 26 cm



In ΔPOR, m∠POR = 90° .... given

∴ By Pythagoras theorem, we get

$PR^2 = PO^2 + OR^2$

$PR^2 = \left(6^2 + 8^2\right) = (36 + 64) = 100$

$\Rightarrow PR = \sqrt{100} = 10\,cm$

In ΔPQR,

$QR^2 = (26)^2 = 676\,cm^2$ and $QP^2 + PR^2 = \left(24^2 + 10^2\right) = (576 + 100) = 676$

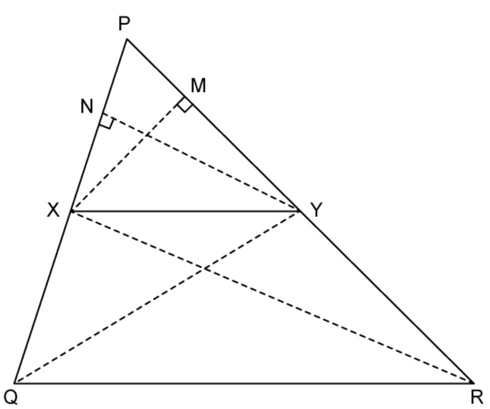
Hence, $QR^2 = QP^2 + PR^2$

(This means that the sum of the square of two sides is equal to the square of the greatest side.)

By the converse of the Pythagoras' theorem,

ΔPQR is a right-angled triangle which is right angled at P.

4.



Given: ΔPQR in which XY || QR, XY intersects PQ and PR at X and Y respectively.

To prove: $\frac{PX}{XQ}=\frac{PY}{YR}$

Construction: Join RX and QY and draw YN perpendicular to PQ and XM perpendicular to PR.

Proof:

Since, ar(ΔPXY) = $\frac{1}{2}\times PX\times YN$ .....(i)

ar(ΔPXY) = $\frac{1}{2}\times PY\times XM$ ....(ii)

Similarly, ar(ΔQXY) = $\frac{1}{2}\times QX\times NY$ .....(iii)

ar(ΔRXY) = $\frac{1}{2}\times YR\times XM$ .....(iv)

Dividing (i) by (iii) we get,

$$\therefore \frac{ar(PXY)}{ar(QXY)}=\frac{\frac{1}{2}\times PX\times YN}{\frac{1}{2}\times QX\times YN}=\frac{PX}{QX} \quad ....(v)$$

Again dividing (ii) by (iv)

$$\therefore \frac{ar(PXY)}{ar(RXY)}=\frac{\frac{1}{2}\times PY\times XM}{\frac{1}{2}\times YR\times XM}=\frac{PY}{YR} \quad ....(vi)$$

Since the area of triangles with same base and between same parallel lines are equal, so

∴ ar(ΔQXY) = ar(ΔRXY) .....(vii)

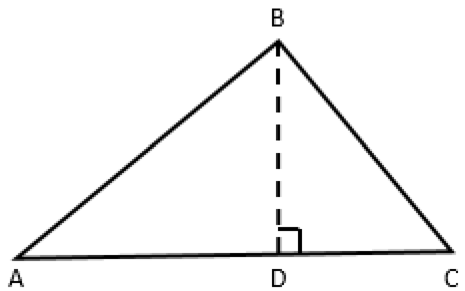
As ΔQXY and ΔRXY are on same base XY and between same parallel lines XY and QR.

Therefore, from (v), (vi) and (vii) we get

$\therefore \frac{PX}{XQ}=\frac{PY}{YR}$

Hence proved.

5.



Given: A right angle triangle ABC at B.

To prove : $AC^2 = AB^2 + BC^2$

Construction : Draw BD perpendicular to AC.

Proof :
If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse, then triangles on both sides of the perpendicular are similar to the whole triangle and to each other.

$\therefore \Delta ADB \sim \Delta ABC$

$\therefore \frac{AD}{AB} = \frac{AB}{AC}$ $\because$ c. p. c. t.

$\therefore AD \times AC = AB^2$ .....(i)

$\Delta BDC \sim \Delta ABC$

$\therefore \frac{CD}{BC} = \frac{BC}{AC}$ $\because$ c. p. c. t.

$\therefore CD \times AC = BC^2$ .....(ii)

Adding (i) and (ii)

$\therefore AD \times AC + CD \times AC = AB^2 + BC^2$

$\therefore AC(AD + CD) = AB^2 + BC^2$

$\therefore AC \times AC = AB^2 + BC^2$

$\therefore AB^2 + BC^2 = AC^2$

Hence proved.

6. i.

In ΔAMD, $AM^2 + MD^2 = AD^2$ (1)

In ΔAMC. $AM^2 + MC^2 = AC^2$ (2)

$\Longrightarrow AM^2 + (MD + DC)^2 = AC^2$

$\Longrightarrow (AM^2 + MD^2) + DC^2 + 2MD.DC = AC^2$

Using equation (1) we may get

$AD^2 + DC^2 + 2MD.DC = AC^2$

Now using the result $DC = \frac{BC}{2}$

$$AD^2 + \left(\frac{BC}{2}\right)^2 + 2MD.\left(\frac{BC}{2}\right) = AC^2$$

$$AD^2 + \left(\frac{BC}{2}\right)^2 + MD \times BC = AC^2$$

ii. In ΔABM applying Pythagoras theorem, $AB^2 = AM^2 + MB^2$

$= (AD^2 - DM^2) + MB^2$..............from(1)

$= (AD^2 - DM^2) + (BD - MD)^2$

$= AD^2 - DM^2 + BD^2 + MD^2 - 2BD \times MD$

$= AD^2 + BD^2 - 2BD \times MD$

$$= AD^2 + \left(\frac{BC}{2}\right)^2 - 2\left(\frac{BC}{2}\right) \times MD$$

$$= AD^2 + \left(\frac{BC}{2}\right)^2 - BC \times MD$$

iii. In ΔAMB, $AM^2 + MB^2 = AB^2$ (1)

In ΔAMC, $AM^2 + MC^2 = AC^2$ (2)

Adding equation (1) and (2)

$2AM^2 + MB^2 + MC^2 = AB^2 + AC^2$

$2AM^2 + (BD - DM)^2 + (MD + DC)^2 = AB^2 + AC^2$

$2AM^2 + BD^2 + DM^2 - 2BD.DM + MD^2 + DC^2 + 2MD.DC = AB^2 + AC^2$

$2AM^2 + 2MD^2 + BD^2 + DC^2 + 2MD(-BD + DC) = AB^2 + AC^2$

$$2\left(AM^2 + MD^2\right) + \left(\frac{BC}{2}\right)^2 + \left(\frac{BC}{2}\right)^2 + 2MD\left(-\frac{BC}{2} + \frac{BC}{2}\right) = AB^2 + AC^2$$

$$2AD^2 + \frac{BC^2}{2} = AB^2 + AC^2$$

Chapter 7: Coordinate Geometry

1. The line segment joining the points A(3, -4) and B(1, 2) is trisected at the points P and Q.

We have P(a, -2) and $Q\left(\frac{5}{3}, b\right)$.

The point P is the point of trisection of the line segment AB.
So, P divides AB in the ratio 1:2.

$(a, -2) = \left(\frac{2\times 3+1\times 1}{1+2}, \frac{2\times -4+1\times 2}{1+2}\right) = \left(\frac{7}{3}, -2\right) \Rightarrow a = \frac{7}{3}$

Similarly,
The point Q is the point of trisection of the line segment AB.
Hence, Q divides AB in the ratio 2:1.

$Q\left(\frac{5}{3}, b\right) = \left(\frac{2\times 1+1\times 3}{1+2}, \frac{2\times 2+1\times -4}{1+2}\right) = \left(\frac{5}{2}, 0\right) \Rightarrow b = 0$

2. Let A(6, -1), B(1,3) and C(k,8) be the given points.

$AB=\sqrt{(1-6)^2+(3+1)^2}=\sqrt{(-5)^2+(4)^2}=\sqrt{25+16}$

$=\sqrt{41}$ units

$BC=\sqrt{(k-1)^2+(8-3)^2}=\sqrt{k^2+1-2k+25}=\sqrt{k^2-2k+26}$

we know that, $AB=BC$

$\Rightarrow \sqrt{41}=\sqrt{k^2-2k+26}$

[on squaring both sides, we get]

$k^2-2k+26=41$

$\Rightarrow k^2-2k+26-41=0$

$\Rightarrow k^2-2k-15=0$

$\Rightarrow k^2-5k+3k-15=0$

$\Rightarrow k(k-5)+3(k-5)=0$

$\Rightarrow (k-5)(k+3)=0$

$\therefore k=5$ or $k=-3$

3. From the diagram given we have Anita, Atharva and Atish are seated at C(2, 4), B(3, 5) and A(5, 7) respectively.
Using distance formula, we have

AB = $\sqrt{(5-3)^2+(7-5)^2}=\sqrt{2^2+2^2}=\sqrt{8}=2\sqrt{2}$ units

BC = $\sqrt{(3-2)^2+(5-4)^2}=\sqrt{1+1}=\sqrt{2}$ units

AC = $\sqrt{(5-2)^2+(7-4)^2}=\sqrt{3^2+3^2}=\sqrt{18}=3\sqrt{2}$ units

Since, AB + BC = $2\sqrt{2}+\sqrt{2}=3\sqrt{2}=AC$

Hence, A, B and C are collinear.

Thus, Anita, Atharva and Atish are seated in a line.

4. Let A(2, 1), B(5, 2), C(6, 4) and D(3, 3) be the angular points of a parallelogram ABCD. Then

$AB=\sqrt{(5-2)^2+(2-1)^2}=\sqrt{(3)^2+(1)^2}=\sqrt{10}=\sqrt{10}$ units

$BC=\sqrt{(6-5)^2+(4-2)^2}=\sqrt{(1)^2+(2)^2}=\sqrt{1+4}=\sqrt{5}$ units

$DC=\sqrt{(6-3)^2+(4-3)^2}=\sqrt{(3)^2+(1)^2}=\sqrt{9+1}=\sqrt{10}$ units

$AD=\sqrt{(3-2)^2+(3-1)^2}=\sqrt{(1)^2+(2)^2}=\sqrt{1+4}=\sqrt{5}$ units

Thus, $AB=DC$ and $AD=BC$

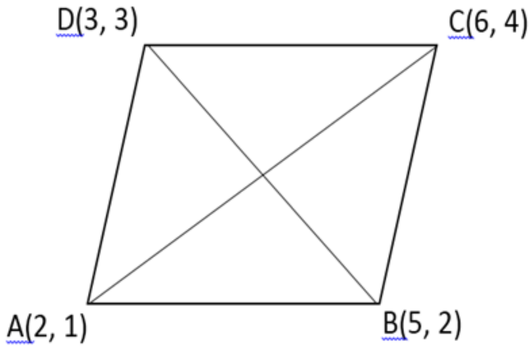
$\therefore$ ABCD is a parallelogram because its opposite sides are equal.

Diagonal $AC=\sqrt{(6-2)^2+(4-1)^2}=\sqrt{(4)^2+(3)^2}=\sqrt{16+9}=\sqrt{25}=5$ units

Diagonal $BD=\sqrt{(3-5)^2+(3-2)^2}=\sqrt{(2)^2+(1)^2}=\sqrt{5}$ unit

Hence, diagonal AC ≠ diagonal BD

Thus, ABCD is not a rectangle, because its diagonals are not equal.



5. Let P(m, 6) divides the line segment joining A(-4, 3) and B(2, 8) in the ratio k:1.
Hence, the point P is

$$\left(\frac{k\times 2+1\times(-4)}{k+1}, \frac{k\times 8+1\times 3}{k+1}\right)=\left(\frac{2k-4}{k+1}, \frac{8k+3}{k+1}\right)$$

$$\Rightarrow \frac{2k-4}{k+1}=m \quad -----(1)$$

and $\frac{8k+3}{k+1}=6 \Rightarrow 8k+3=6k+6 \Rightarrow 2k=3 \Rightarrow k=\frac{3}{2}$

Putting value of k in (1)

$$\frac{2\times\frac{3}{2}-4}{\frac{3}{2}+1}=m \Rightarrow m=\frac{3-4}{\frac{5}{2}} \Rightarrow m=-\frac{2}{5}$$

Hence, $m=-\frac{2}{5}, k=\frac{3}{2}$

Chapter 8: Introduction to Trigonometry

1.

$$\begin{aligned} \text{L.H.S.} &= (\sin 72^\circ+\cos 18^\circ)(\sin 72^\circ-\cos 18^\circ) \\ &= \sin^2 72^\circ-\cos^2 18^\circ \\ &= \sin^2 72^\circ-\cos^2(90^\circ-72^\circ) \\ &= \sin^2 72^\circ-\sin^2 72^\circ \\ &= 0 \\ &= \text{R.H.S.} \end{aligned}$$

2. ΔABC is right angled at C.

Now, $\angle A+\angle B+\angle C=180^\circ$

$\Rightarrow A+B+C=180^\circ \Rightarrow A+B+90^\circ=180^\circ \Rightarrow A+B=90^\circ$

$\therefore \cos(A+B)=\cos 90^\circ=0$

3. We know that

$\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$$\Rightarrow (3x)^2 - \left(\frac{3}{x}\right)^2 = 1 \Rightarrow 9x^2 - \frac{9}{x^2} = 1 \Rightarrow 9\left(x^2 - \frac{1}{x^2}\right) = 1 \Rightarrow 3\times 3\left(x^2 - \frac{1}{x^2}\right) = 1 \Rightarrow 3\left(x^2 - \frac{1}{x^2}\right) = \frac{1}{3}$$

4. $\text{L.H.S.} = \dfrac{1+\sec\theta-\tan\theta}{1+\sec\theta+\tan\theta}$

$$= \frac{1+(\sec\theta-\tan\theta)}{1+\sec\theta+\tan\theta}$$

$$= \frac{(\sec^2\theta-\tan^2\theta)+(\sec\theta-\tan\theta)}{1+\sec\theta+\tan\theta}$$

$$= \frac{(\sec\theta-\tan\theta)(\sec\theta+\tan\theta)+(\sec\theta-\tan\theta)}{1+\sec\theta+\tan\theta}$$

$$= \frac{(\sec\theta-\tan\theta)[\sec\theta+\tan\theta+1]}{1+\sec\theta+\tan\theta}$$

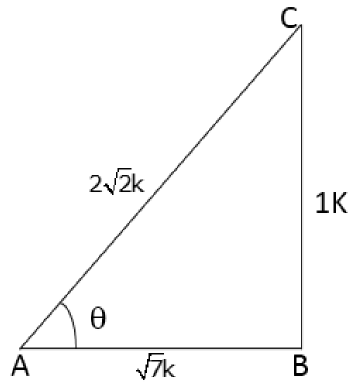
$$= \sec\theta - \tan\theta$$

$$= \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$$

$$= \frac{1-\sin\theta}{\cos\theta}$$

$$= \text{R.H.S.}$$

5. Let us draw a ΔABC in which $\angle B = 90°$ and $\angle A = \theta$.



Given: $\tan\theta = \dfrac{BC}{AB} = \dfrac{1}{\sqrt{7}}$

Let $BC = 1k$ and $AB = \sqrt{7}k$

where k is positive

By pythagoras theorem, we have

$$AC^2 = \left(AB^2 + BC^2\right)$$

$$\Rightarrow AC^2 = \left[\left(\sqrt{7}k\right)^2 + (1k)^2\right] = 7k^2 + 1k^2 = 8k^2 \Rightarrow AC = 2\sqrt{2}k$$

$$\text{cosec}\,\theta=\frac{AC}{BC}=\frac{2\sqrt{2}k}{1k}=2\sqrt{2},\ \ \sec\theta=\frac{AC}{AB}=\frac{2\sqrt{2}k}{\sqrt{7}k}=\frac{2\sqrt{2}}{\sqrt{7}}$$

$$\Rightarrow\frac{\left(\text{cosec}^2\theta-\sec^2\theta\right)}{\left(\text{cosec}^2\theta+\sec^2\theta\right)}=\left[\frac{\left(2\sqrt{2}\right)^2-\left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2}{\left(2\sqrt{2}\right)^2+\left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2}\right]=\frac{\left(8-\frac{8}{7}\right)}{\left(8+\frac{8}{7}\right)}=\frac{\left(\frac{48}{7}\right)}{\left(\frac{64}{7}\right)}=\frac{48}{64}=\frac{3}{4}$$

$$\text{Hence, }\left(\frac{\text{cosec}^2\theta-\sec^2\theta}{\text{cosec}^2\theta+\sec^2\theta}\right)=\frac{3}{4}$$

6. $x=\cot A+\cos A$ and $y=\cot A-\cos A$

Thus, we have

$$x+y=(\cot A+\cos A)+(\cot A-\cos A)=2\cot A$$
$$x-y=(\cot A+\cos A)-(\cot A-\cos A)=2\cos A$$

$$\text{L.H.S.}=\left(\frac{x-y}{x+y}\right)^2+\left(\frac{x-y}{2}\right)^2=\left(\frac{2\cos A}{2\cot A}\right)^2+\left(\frac{2\cos A}{2}\right)^2$$

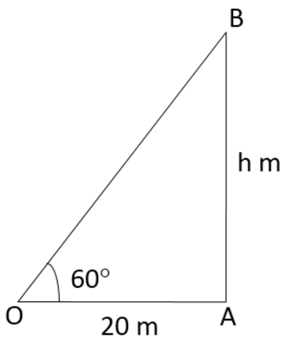
Thus, we have

$$x+y=(\cot A+\cos A)+(\cot A-\cos A)=2\cot A$$
$$x-y=(\cot A+\cos A)-(\cot A-\cos A)=2\cos A$$

$$\text{L.H.S.}=\left(\frac{x-y}{x+y}\right)^2+\left(\frac{x-y}{2}\right)^2$$
$$=\left(\frac{2\cos A}{2\cot A}\right)^2+\left(\frac{2\cos A}{2}\right)^2$$
$$=\left(\frac{\cos A}{\cot A}\right)^2+(\cos A)^2$$
$$=\left(\frac{\cos A}{\cos A/\sin A}\right)^2+(\cos A)^2$$
$$=(\sin A)^2+(\cos A)^2$$
$$=\sin^2 A+\cos^2 A$$
$$=1$$
$$=\text{R.H.S.}$$

Chapter 9: Some Applications of Trigonometry

1. Let AB be the tower standing on a level ground and O be the position of the observer. Then OA = 20 m and ∠OAB = 90° and ∠AOB = 60°.



Let AB = h metres.

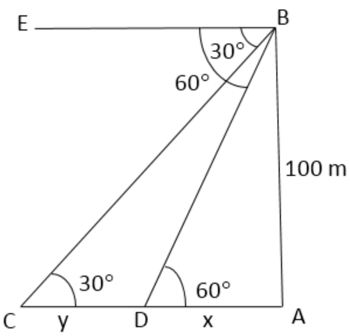
From the right-angled ΔOAB, we have

$$\frac{AB}{OA}=\tan 60^\circ=\sqrt{3}\Rightarrow\frac{h}{20}=\sqrt{3}\Rightarrow h=\left(20\times\sqrt{3}\right)\Rightarrow h=20\times 1.732\Rightarrow h=34.64m$$

Hence, the height of the tower is $20\sqrt{3}m=34.64\,m$

2. Let AB be the lighthouse, and let C and D be the positions of the ship.

Let AD = x, CD = y



In BDA, $\frac{x}{100}=\cot 60^\circ\Rightarrow x=\frac{100}{\sqrt{3}}m$

Similarly in ΔBCA, $\frac{x+y}{100}=\cot 30^\circ$

$\Rightarrow(x+y)=100\sqrt{3}m$

$$y=(x+y)-x=\left(100\sqrt{3}-\frac{100}{\sqrt{3}}\right)m=\left(\frac{200}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}\right)m=115.46m$$

The distance travelled by the ship during the period of observation = 115.46 m

3. Let CD be a tower of height = 50 m
AB be another tower of height = 40 m
The observe is at M and angle of elevation ∠ CMD = ∠ AMB = 45°
Distance between BD = BM + MD
In right angle triangle CDM,

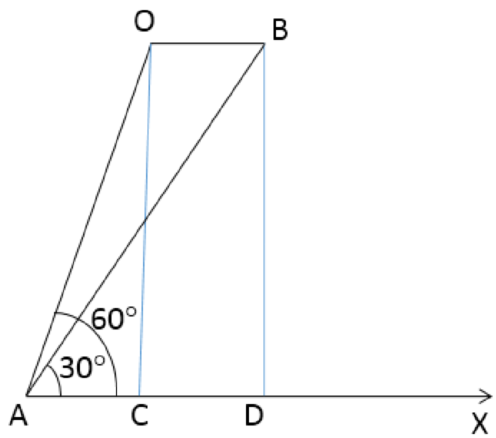
$$\tan\theta = \frac{CD}{DM} \Rightarrow \tan 45° = \frac{50}{DM} \Rightarrow 1 = \frac{50}{DM} \Rightarrow DM = 50 \text{ m}$$

In right angle triangle AMB,

$$\tan\theta = \frac{AB}{MB} \Rightarrow \tan 45° = \frac{40}{BM} \Rightarrow 1 = \frac{40}{BM} \Rightarrow BM = 40 \text{ m}$$

Hence, BD = BM + MD = 40 + 50 = 90 m
The distance between the windmills on either side of field is 90 m.

4. Let O and B be the two positions of the jet plane, and let A be the point of observation.
Let AX be the horizontal ground.
Draw OC ⊥AX and BD ⊥ AX.
Then
$\angle CAO = 60°$, $\angle DAB = 30°$ and OC = BD = $1500\sqrt{3}$ m



From right ΔOCA, we have $\frac{AC}{OC} = \cot 60° = \frac{1}{\sqrt{3}} \Rightarrow \frac{AC}{1500\sqrt{3}} = \frac{1}{\sqrt{3}} \Rightarrow AC = 1500\text{m}$ .....(1)

From right ΔADB, we have $\frac{AD}{BD} = \cot 30° = \sqrt{3} \Rightarrow \frac{AD}{1500\sqrt{3}} = \sqrt{3} \Rightarrow AD = \left(1500\sqrt{3} \times \sqrt{3}\right) = 4500\text{m}$

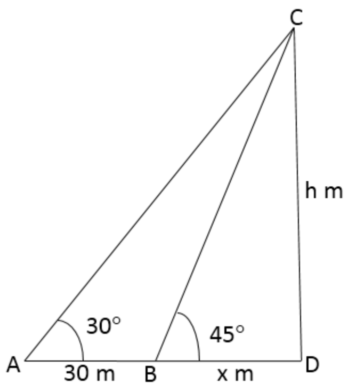
$\therefore CD = (AD - AC) = (4500 - 1500)\text{m} = 3000 \text{ m}$
$\therefore OB = CD = 3000\text{m}$

Thus, the jet plane covers 3000 m in 15 seconds.

Hence, the speed of the jet plane is

$$=\left(\frac{3000}{15}\times\frac{60\times60}{1000}\right)\text{kmph}=720\,\text{kmph}$$

5. Let CD be the height of the building.
Then ∠CAB = 30°, ∠CBD = 45°, ∠ADC = 90° and AB = 30 m
CD = h metres and BD = x metres



From right-angled ΔCAD, we have

$$\frac{CD}{DA}=\tan 30^\circ=\frac{1}{\sqrt{3}}\Rightarrow\frac{h}{30+x}=\frac{1}{\sqrt{3}}\Rightarrow 30+x=h\sqrt{3}\Rightarrow x=\left(h\sqrt{3}-30\right)\qquad .....(1)$$

From right-angled ΔBCD, we have

$$\frac{CD}{BD}=\tan 45^\circ=1\Rightarrow\frac{h}{x}=1\Rightarrow h=x\qquad ....(2)$$

from $(1)\,\&\,(2)$, we get

$$h\sqrt{3}-30=h\Rightarrow h\sqrt{3}-h=30$$

$$\Rightarrow h=\frac{30}{\left(\sqrt{3}-1\right)}\times\frac{\left(\sqrt{3}+1\right)}{\left(\sqrt{3}+1\right)}=\frac{30\sqrt{3}+30}{3-1}=\frac{30\left(\sqrt{3}+1\right)}{2}$$

$$\Rightarrow h=15\left(1.732+1\right)=15\times2.732=40.98$$

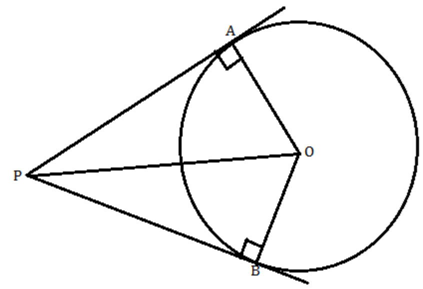
Putting h = 40.98 in (2), we get x = 40.98 m

Hence, the height of the building = 40.98 m

And the distance of its base from point A = AD = (30 + x) m = (30 + 40.98) m = 70.98 m

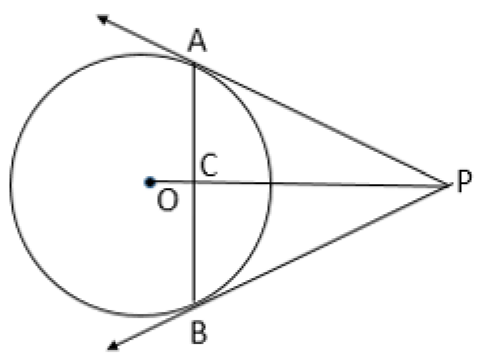
Chapter 10: Circles

1. Consider the following diagram.



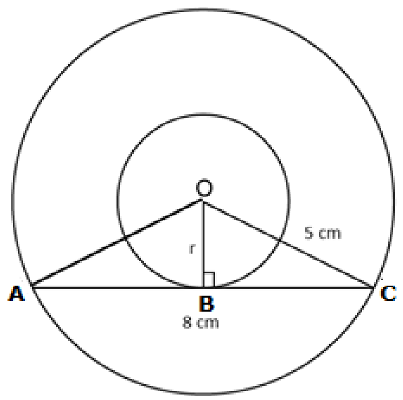
Let P be an external point and PA and PB be tangents to the circle.
We need to prove that PA = PB
Now consider the triangles ΔOAP and ΔOBP
m ∠A = m∠B = 90° .... PA and PB are tangents
OP = OP [common]
OA = OB (radii of the circle)
Thus, by Right Angle-Hypotenuse-Side criterion of congruence we have,
ΔOAP ≅ ΔOBP
The corresponding parts of the congruent triangles are congruent.
Thus,
PA = PB
The lengths of the tangents drawn from an external point to a circle are equal.

2.



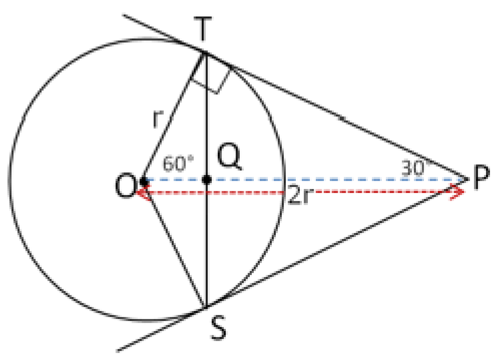
Let AB be a chord of circle with centre O.
Let AP and BP be two tangents at A and B respectively.
Suppose the tangents meet at point P. Join OP.
Suppose OP meets AB at C.
Now, in ΔPCA and ΔPCB,
PA = PB ....(tangents from an external point are equal)
∠APC = ∠BPC ....(PC is the angle bisector of ∠ APB)
PC = PC ....(common)
Hence, ΔPAC ≅ ΔPBC ....(by SAS congruence criterion)
⇒ ∠PAC = ∠PBC ... C. A. C. T.

3.



Let O be the centre of concentric circles.
Since AC is a tangent to the inner circle.
$\angle OBC = 90°$ ....(tangent is perpendicular to the radius of a circle)
AC is a chord of the outer circle.
We know that, the perpendicular drawn from the
centre to a chord of a circle, bisects the chord.
So, $AC = 2BC \Rightarrow BC = 4$ cm
In ΔOBC,
By Pythagoras theorem,
$OC^2 = OB^2 + BC^2 \Rightarrow 5^2 = r^2 + 4^2 \Rightarrow r^2 = 5^2 - 4^2 \Rightarrow r^2 = 25 - 16 \Rightarrow r^2 = 9 \text{ cm} \Rightarrow r = 3 \text{ cm}$
Hence, the radius of the smaller circle is 3 cm.

4. In the given figure,



OP = 2r ... (Given)
$\angle OTP = 90°$ ... (radius drawn at the point of contact is perpendicular to the tangent)
In ΔOTP,

$\sin\angle OPT = \frac{OT}{OP} = \frac{1}{2} = \sin 30° \Rightarrow \angle OPT = 30°$

$\therefore \angle TOP = 60°$
$\therefore \Delta OTP$ is a 30°-60°-90°, right triangle.
In ΔOTS,

OT = OS ... (Radii of the same circle)
∴ ΔOTS is an isosceles triangle.
∴ ∠OTS = ∠OST ... (Angles opposite to equal sides of an isosceles triangle are equal)
In ΔOTQ and ΔOSQ
OS = OT ... (Radii of the same circle)
OQ = OQ ...(side common to both triangles)
∠OTQ = ∠OSQ ... (angles opposite to equal sides of an isosceles triangle are equal)
∴ ΔOTQ ≅ ΔOSQ ... (By S.A.S)

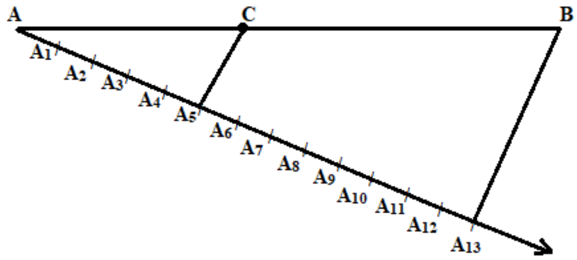
∴ ∠TOQ = ∠SOQ = 60° ... (C.A.C.T)

∴ ∠TOS = 120° ... (∠TOS = ∠TOQ + ∠SOQ = 60° + 60° = 120°)
∴ ∠OTS + ∠OST = 180° – 120° = 60°
∴ ∠OTS = ∠OST = 60° ÷ 2 = 30° ∵ OT = OS

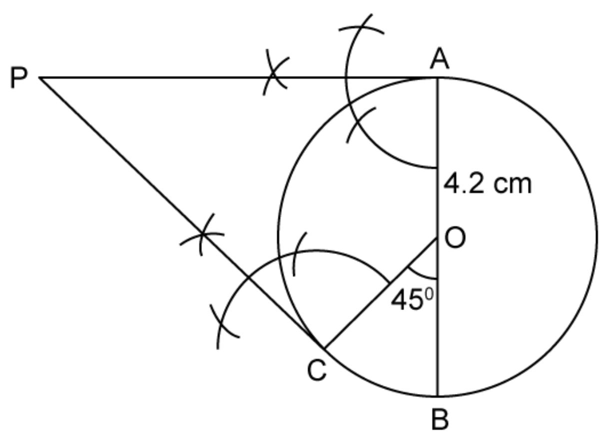
Chapter 11: Constructions

1.

1. Draw a line segment of AB length 7.6 cm.
2. Draw a ray AX making acute angle with AB and take 13 points A_1, A_2, ... A_{13} on it at equal distances
3. Join A_{13} and B. Draw line parallel to $A_{13}B$ from A_5 meeting AB at C.
4. AC: CB = 5:8. Measuring the lengths, we get AC = 2.9 cm and CB = 4.7 cm.



2.



Steps of construction:
(i) A circle of radius 4.2 cm at centre O is drawn.
(ii) A diameter AB is drawn.
(iii) With OB as the base, an angle BOC of 45° is drawn.
(iv) At A, a line perpendicular to OA is drawn.
(v) At C, a line perpendicular to OC is drawn.
(vi) These lines intersect each other at P.
PA and PC are the required tangents.

Chapter 12: Areas Related to Circles

1. Area of the first circle = $\pi r^2 = 962.5 \text{ cm}^2$

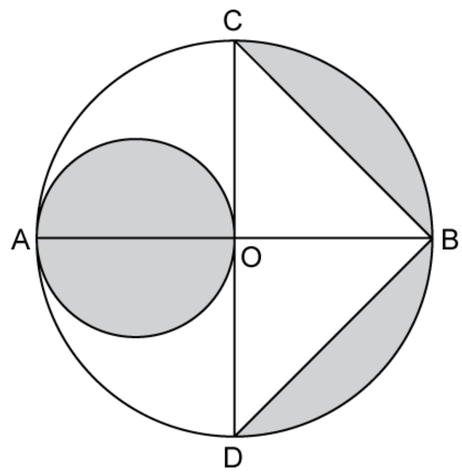
$$r^2 = \left(962.5 \times \frac{7}{22}\right) \text{cm} \Rightarrow r^2 = 306.25 \Rightarrow r = 17.5 \text{ cm}$$

Area of the second circle = $\pi R^2 = 1386 \text{cm}^2$

$$R^2 = \left(1386 \times \frac{7}{22}\right) \text{cm} \Rightarrow R^2 = 441 \Rightarrow R = 21 \text{cm}$$

Width of ring R – r = (21 - 17.5) cm = 3.5 cm

2.



Area of the shaded region

= (area of circle with OA as the diameter)+ (area of semi-circle DBC) - (area of ΔBCD)

Area of the circle with OA as the diameter = $\pi r^2 = \left(\frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\right) \text{cm}^2 = 38.5 \text{ m}^2$

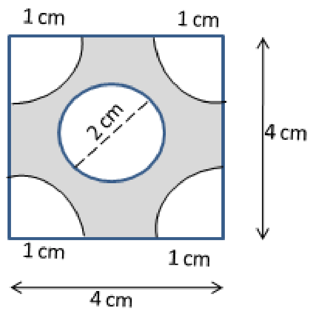
OB = 7 cm, CD = AB = 14 cm … diameter of the same circle

Area of semi-circle DBC = $\frac{1}{2}\pi r^2 = \left(\frac{1}{2} \times \frac{22}{7} \times 7 \times 7\right) \text{cm}^2 = 77 \text{ cm}^2$

$$\text{Area of } \Delta BCD = \frac{1}{2} \times DC \times OB = \frac{1}{2} \times 14 \times 7 = 49\,cm^2$$

$$\text{Area of shaded region} = (38.5 + 77 - 49) = 66.5\,cm^2$$

3.



Area of the square = (4 × 4) cm^2 = 16 cm^2

Area of four quadrant corners $= 4\left[\frac{1}{4}\pi r^2\right] = \pi r^2 = (\pi \times 1 \times 1)cm^2 = 3.14\ cm^2$

Radius of the inner circle = $\frac{2}{2} = 1$ cm

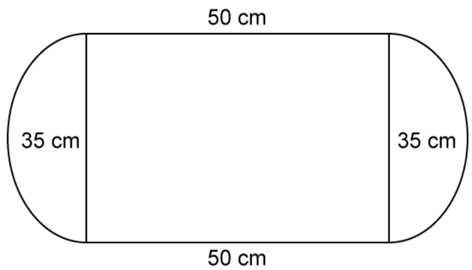
Area of the circle at the centre = $\pi r^2 = (3.14 \times 1 \times 1)cm^2$ = 3.14 cm^2

Area of the shaded region

= [area of the square - area of four corner quadrants - area of the circle at the centre] cm^2

= [16 - 3.14 - 3.14] cm^2 = 9.72 cm^2

4.



Area of the rectangular lawn in the middle = (50 × 35) m^2 = 1750 m^2

Radius of semi-circles $= \frac{35}{2} = 17.5$ m

Area of two semicircles

$$= 2(\text{area of semi circle}) = \left[2\left(\frac{1}{2}\pi r^2\right)\right]m^2 = \left(2 \times \frac{1}{2} \times \frac{22}{7} \times 17.5 \times 17.5\right)m^2 = 962.5\ m^2$$

Area of the lawn = (area of the rectangle + area of the semi-circle)

$= (1750 + 962.5)\ m^2 = 2712.5\ m^2$

Chapter 13: Surface Areas and Volumes

1. R = 20 cm, r = 8 cm and h = 16 cm

$$\therefore\ l=\sqrt{h^2+(R-r)^2}=\sqrt{(16)^2+(20-8)^2}$$
$$=\sqrt{256+144}\text{cm}=20\text{cm}$$

Total surface area of the container = $\pi l(R+r)+\pi r^2$

$$=\left[3.14\times20\times(20+8)+3.14\times8\times8\right]\text{cm}^2$$
$$=(3.14\times20\times28+3.14\times8\times8)\text{cm}^2$$
$$=(1758.4+200.96)\text{cm}^2$$
$$=1959.36\ \text{cm}^2$$

Cost of the metal sheet used = $\text{Rs}\left(1959.36\times\frac{15}{100}\right)=\text{Rs}.293.90$

2. Radius of the cone = r = 5 cm and height h = 12 cm

Volume of the cone = $\frac{1}{3}\pi r^2 h=\frac{1}{3}\pi 5^2\times12$

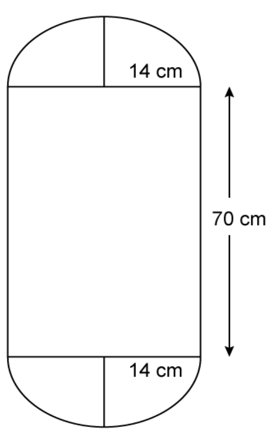
Diameter of the ball = 4 cm, radius = 2 cm

Volume of the sphere = $\frac{4}{3}\pi r^3=\frac{4}{3}\pi 2^3$

According to the question,

The number of balls = $\frac{\frac{1}{3}\pi 5^2\times12}{\frac{4}{3}\pi 2^3}=9.375\approx9$

3.



Radius of each hemispherical end = $\frac{28}{2}=14\text{cm}$

Height of each hemispherical part = Its radius

Height of the cylindrical part = (98 – 2 × 14) = 70 cm

Area of surface to be polished
= 2(curved surface area of the hemisphere) + (curved surface area of the cylinder)

$=\left[2\left(2\pi r^2\right)+2\pi rh\right]\text{sq.unit}$

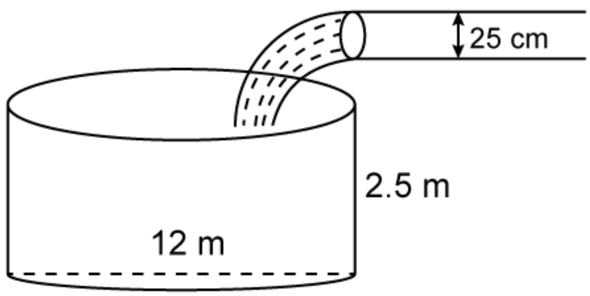
$=2\pi r(2r+h)\text{cm}^2$

$=2\times\frac{22}{7}\times14\times[2\times14+70]\text{cm}^2$

$=(88\times98)=8624\ \text{cm}^2$

Cost of polishing the surface of the solid = Rs. (0.15 × 8624) = Rs. 1293. 60

4.



Height of the cylindrical tank = 2.5 m

Its diameter = 12 m, radius = 6 m

Volume of the tank = $\pi r^2 h=\frac{22}{7}\times6\times6\times2.5\text{m}^3=\frac{1980}{7}\text{m}^3$

Water is flowing at the rate of 3.6 km/ hr = 3600 m/hr

Diameter of the pipe = 25 cm, radius = 0.125 m

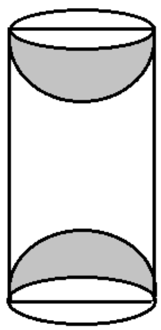
Volume of water flowing per hour $=\frac{22}{7}\times0.125\times0.125\times3600\ \text{m}^3=\frac{22\times3600}{7\times8\times8}\text{m}^3=\frac{2475}{14}\text{m}^3$

Time taken to fill the tank$=\frac{1980}{7}\div\frac{2475}{14}\ \text{hr}=\frac{1980}{7}\times\frac{14}{2475}\text{hr}=\frac{792}{495}\text{hr}$

$=1.6\ \text{hr}=1\ \text{hr}\,36\ \text{mins}$

$\text{Water charges}=\text{Rs.}\frac{1980}{7}\times0.07=\text{Rs.}19.80$

5.



Radius of a wooden cylinder = 4.2 cm
Height of the wooden cylinder = 12 cm

Lateral surface area $= 2\pi rh \text{ sq.cm} = 2\times\pi\times4.2\times12 \text{cm}^2 = 100.8\pi \text{ cm}^2$

Radius of the hemisphere = 4.2 cm

Surface area of two hemispheres $= 2\times2\pi r^2 \text{ sq.unit} = 4\pi\times4.2\times4.2 \text{ cm}^2 = 70.56\pi \text{ cm}^2$

Total surface area = (100.8 + 70.56) π = 171.36 π = 171.36 × 22/7 = 538.56 cm^2

Further, the volume of the cylinder = $\pi r^2 h = 4.2\times4.2\times12\pi \text{ cm}^2$ = 211. 68 π cm^2

Volume of two hemispheres = $2\times\frac{2}{3}\pi r^3 = \frac{4}{3}\pi\times\times4.2\times4.2 = 98.784\pi \text{ cm}^3$

Volume of wood left = (211.68 – 98.784) π =112.896 π = = $112.896\times\frac{22}{7}$ = 354.816 cm^3

Chapter 14: Statistics

1. We have

Class	Frequency f_i	Class Mark x_i	$f_i x_i$
10–20	11	15	165
20–30	15	25	375
30–40	20	35	700
40–50	30	45	1350
50–60	14	55	770
60–70	10	65	650
	$\sum f_i = 100$		$\sum f_i x_i = 4010$

$$\therefore \text{ Mean } \bar{x} = \frac{\sum(f_i \times x_i)}{\sum f_i} = \frac{4010}{100} = 40.10$$

2. We have, $17+f_1+32+f_2+19=120 \Rightarrow f_2=52-f_1$

Class	Frequency f_i	Mid Value x_i	f_ix_i
0–20	17	10	170
20–40	f_1	30	$30f_1$
40–60	32	50	1600
60–80	$52 - f_1$	70	$3640 - 70f_1$
80–100	19	90	1710
	$\sum f_i = 120$		$\sum f_ix_i = 7120 - 40f_1$

$\therefore$ Mean, $\bar{x}=\dfrac{\sum(f_i \times x_i)}{\sum f_i}=\dfrac{7120-40f_1}{120}=50$

$\Rightarrow 7120-40f_1=6000 \Rightarrow 40f_1=1120 \Rightarrow f_1=28$

Thus, $f_1=28$ and $f_2=(52-28)=24$

3. We prepare the frequency table as given below:

Class	Frequency f_i	C.F
5–10	5	5
10–15	6	11
15–20	15	26
20–25	10	36
25–30	5	41
30–35	4	45
35–40	2	47
40–45	2	49
	$\sum f_i = 49$	

Now, N = 49 $\Rightarrow \dfrac{N}{2}=\dfrac{49}{2}=24.5$

The cumulative frequency just greater than and nearest to 24.5 is 26 and the corresponding class is 15–20. Thus, the median class is 15–20.

$\therefore$ l = 15, h = 5, f = 15, c = CF preceding median class = 11 and $\left(\dfrac{N}{2}\right)=24.5$

$$\text{Median } m_e = l + \left[h \times \frac{\left(\frac{N}{2}-c\right)}{f}\right] = 15 + \left(5 \times \frac{(24.5-11)}{15}\right) = 15 + \left(5 \times \frac{13.5}{15}\right) = 15 + 4.5 = 19.5$$

The median of the frequency distribution is 19.5.

Chapter 15: Probability

1. Total number of tickets sold = 250, Number of prizes = 5
Let E be the event of getting a prize.

Number of favourable outcomes = 5

$\therefore$ P(getting a prize) = $P(E) = \dfrac{5}{250} = \dfrac{1}{50}$

2. A one rupee coin is tossed 3 times then the total number of possible outcomes = 2^3 = 8
S = {HHH, HTH, THH, TTH, HHT, HTT, THT, TTT} $\Rightarrow$ n(S) = 8
Let A be the event that Harsh will lose the game.
A = {HTH, THH, TTH, HHT, HTT, THT} $\Rightarrow$ n(A) = 6
Required probability = $\dfrac{n(A)}{n(S)} = \dfrac{6}{8} = \dfrac{3}{4}$

3. Face cards in a pack of cards are jacks, queens and kings.
$\therefore$ The number of face cards = 4 × 3 = 12
Total number of cards = 52

(i) Probability of getting a face card = $\dfrac{12}{52} = \dfrac{3}{13}$

(ii) Number of king cards which are not red = 2
Number of red cards = 26

$\therefore$ Probability of getting a red card or a black king card $= \dfrac{28}{52} = \dfrac{7}{13}$

Probability of getting neither a red card nor a king card $= 1 - \dfrac{7}{13} = \dfrac{6}{13}$

4. Two dice are thrown simultaneously.
Total number of outcomes = 6 × 6 = 36
(i) Favourable cases are
(1, 1), (1, 2), (1, 3), (1, 4), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 6), (3, 1), (3, 2),
(3, 3), (3, 4), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 6)
Number of favourable cases = 25

$\therefore$ Probability that 5 will not come upon either die $= \dfrac{25}{36}$

(ii) Favorable cases are

(1, 5), (2, 5), (3, 5), (4, 5), (5, 5), (6, 5), (5, 1), (5, 2), (5, 3), (5, 4), (5, 6) = 11

Probability that 5 will come at least once $= \dfrac{11}{36}$

(iii) 5 will come up on both dice in 1 case = (5, 5)

$\therefore$ Probability that 5 will come on both dice = $\frac{1}{36}$

5. Total number of tickets = 100

(i) Even numbers are 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 44, 46, 48, 50, 52, 54, 56, 58, 60, 62, 64, 66, 68, 70, 72, 74, 76, 78, 80, 82, 84, 86, 88, 90, 92, 94, 96, 98, 100.
Total number of even numbers = 50
P(getting an even number) = $\frac{50}{100} = \frac{1}{2}$

(ii) Numbers less than 16 are 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15.
Total number of numbers less than 16 is 14.
P(getting a number less than 16) = $\frac{14}{100} = \frac{7}{50}$

(iii) Numbers which are a perfect square are 4, 9, 16, 25, 36, 49, 64, 81, 100.
Total number of perfect squares = 9
P(getting a perfect square) = $\frac{9}{100}$

(iv) Prime numbers less than 40 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37.
Total number of prime numbers = 12
P(getting a prime number less 40) = $\frac{12}{100} = \frac{3}{25}$

Case Study

1.

(a) In the Garden 1 (Larger garden) one angle is 65°.
Larger garden is similar to the smaller garden (Garden 2),
$\Rightarrow$ The second angle of the larger garden = 53°
In Garden 1,
53° + 65° + third angle = 180° ..... Angle sum property of a triangle
$\Rightarrow$ Third angle = 180° – 118° = 62°

(b) In the Garden 2 (Smaller garden) one angle is 53°.
Smaller garden is similar to the larger garden (Garden 1),
$\Rightarrow$ The second angle of the smaller garden = 65°
In Garden 2,
53° + 65° + third angle = 180° ..... Angle sum property of a triangle
$\Rightarrow$ Third angle = 180° –118° = 62°

(c) Larger garden is similar to the smaller garden.

$$\Rightarrow \frac{\text{Area of Larger Garden}}{\text{Area of Larger Garden}} = \frac{(32)^2}{(24)^2} = \left(\frac{4}{3}\right)^2$$

$$\Rightarrow \frac{\text{Area of Larger Garden}}{\text{Area of Larger Garden}} = \frac{16}{9}$$

Therefore, ratio of the area of larger garden to that of the area of smaller garden is 16: 9.

(d) Let the unknown side of garden 1 be x.
Larger garden (garden 1) is similar to the smaller garden (garden 2).

$$\Rightarrow \frac{32}{24} = \frac{x}{18}$$

$$\Rightarrow \frac{4}{3} = \frac{x}{18} \Rightarrow x = 24\text{m}$$

(e) Three sides of the smaller garden are 18, 22 and 24.

$$\Rightarrow s = \frac{18+21+24}{2} = \frac{63}{2}$$

By Heron's Formula:
Area of a smaller garden

$$= \sqrt{\frac{63}{2}\left(\frac{63}{2}-18\right)\left(\frac{63}{2}-21\right)\left(\frac{63}{2}-24\right)} = \frac{1}{4}\sqrt{63\times27\times21\times15} = \frac{189}{4}\sqrt{15}\ \text{m}^2$$

2.

(a) In the given graph, the wire is intersecting x – axis at three places.
Therefore, the number of zeroes possible for the polynomial made by the wire is 3.

(b) In the given graph, the wire is intersecting x – axis at −4, −1 and 2.
Therefore, the zeroes of the polynomial are −4, −1 and 2.

(c) The graph of polynomial made by the wire is a Cubic polynomial.
Therefore, the degree of the polynomial is 3.

(d) The polynomial made by the wire is a Cubic polynomial.
Here, $\alpha = -4, \beta = -1$ and $\lambda = 2$ (from the graph) are the zeroes of the polynomial.
Equation of a Cubic polynomial = $ax^3 + bx^2 + cx + d$

$$\alpha + \beta + \lambda = -4 - 1 + 2 = -3 = \frac{-b}{a} \ldots (i)$$

$$\alpha\beta + \beta\lambda + \lambda\alpha = 4 - 2 - 8 = -6 = \frac{c}{a} \ldots (ii)$$

$$\alpha\beta\lambda = 8 = \frac{-d}{a} \ldots (iii)$$

From (i), (ii) and (iii), we get

Equation of the cubic polynomial = $a(x^3 + 3x^2 - 6x - 8)$, where a is some constant.

(e) Value of the polynomial at the point where x will be −2 is 2 (from the graph y = 2).

3.

(a)

Time(in sec)	No. of groups(f)	Cumulative Frequency
120 – 140	2	2
140 – 160	5	2 + 5 = 7
160 – 180	4	7 + 4 = 11
180 – 200	3	11 + 3 = 14
200 – 220	1	14 + 1 = 15
	N = 15	

Here, N = 15 $\Rightarrow \frac{N}{2} = 7.5$

Cumulative frequency just after 7.5 is 11.

$\therefore$ Median class is 160 – 180.

$\therefore$ l = 160, h = 20, N = 15, c = 7, f = 4

$$\therefore \text{Median } M_e = l + h\left(\frac{\frac{N}{2} - c}{f}\right) = 160 + 20\left(\frac{7.5 - 7}{4}\right) = 160 + 2.5 = 167.5 \text{ seconds}$$

(b) Modal class is 140 – 160 as it has the highest frequency i.e. 5.
$\Longrightarrow$ Upper limit of the modal class is 160.

(c) Mean, Median and Mode are the measures of Central Tendency.

(d) Median class is 160 – 180 and the Modal class is 140 – 160
$\Longrightarrow$ Sum of lower limits of Median class and Modal class = 160 + 140 = 300

(e) 2 + 5 + 4 = 11
$\Longrightarrow$ 11 groups finished the race in 3 minutes.

4.

(a) The shape of the small dome is hemisphere.

$$\text{Curved Surface Area of Hemisphere} = 2\pi r^2 = 2 \times \frac{22}{7} \times (3.5)^2 = 77\,m^2$$

$\Longrightarrow$ Fencing material will be required to cover the 2 small domes each with radius 3.5 m = 2 × 77 = 154 m^2

(b) Total surface area of a Cylinder

$$= 2\pi r(h + r) = 2 \times \frac{22}{7} \times 5(14 + 5) = 597.14\,m^2$$

So, the total surface area of the two cylindrical pillars(at the front) with height 14 m and radius 5 m each = 2 × 597.14 = 1194.28 m^2

(c) Lateral Surface area of a Cylinder $= 2\pi rh = 2 \times \frac{22}{7} \times 5 \times 14 = 440\,m^2$

So, the lateral surface area of the two cylindrical pillars with height 14 m and radius 5 m each = 2 × 440 = 880 m^2

(d) The shape of the small dome is hemisphere.

Volume of a Hemisphere = $\frac{2}{3}\pi r^3 = \frac{2}{3}\times\frac{22}{7}\times(4.2)^3 = 155.232\ m^3$

Volume of one dome when the radius is 4.2 m = 155.232 m^3

(e) Volume of a cylinder $=\pi r^2 h = \frac{22}{7}\times(7)^2\times 16 = 2464\ m^3$

The volume of the 2 pillars (at the front) if each has height 16 m and radius 7m = 2 × 2464 = 4928 m^3

5.

(a) Mid-point of J and I = $\left(\frac{6+9}{2},\frac{17+16}{2}\right)=\left(\frac{15}{2},\frac{33}{2}\right)$

(b) The distance of the point P from the y – axis is 4m.

(c) The distance between A and S is 16m.

(d) Coordinates of A are (1, 8) and that of B are (5, 10)
Coordinates of a point dividing AB in the ratio 1:3 is

$\left(\frac{1\times5+3\times1}{1+3},\frac{1\times10+3\times8}{1+3}\right)=\left(2,\frac{17}{2}\right)=(2.0,8.5)$

(e) (x, y) is equidistant from Q(9, 8) and S(17, 8).
$\Rightarrow (9 - x)^2 + (8 - y)^2 = (17 - x)^2 + (8 - y)^2$
$\Rightarrow 81 - 18x = 289 - 34x$
$\Rightarrow 16x = 208$
$\Rightarrow x = 13$
$\Rightarrow x - 13 = 0$

6.

(a) Width of the scale model = $\frac{1}{4}\times$ width of the boat $=\frac{1}{4}\times 60 = 15cm$

(b) If any two polygons are not the mirror image of one another then there similarity will effect.

(c) We know that, Scale factor = $\frac{\text{length in image}}{\text{corresponding length in object}}$

If two similar triangles have a scale factor of a: b then their altitudes have a ratio a: b

(d) This is an example of similarity.

$\Rightarrow \frac{5}{2}=\frac{12.5}{\text{Shadow of a tree}}$

$\Rightarrow$ Shadow of a tree $=\frac{25}{5}=5m$

(e) Here, ΔTEF and ΔTAB are similar triangles as they form the equal angles

Therefore, the ratio of their corresponding sides is same.
As E and F are the midpoints TA and TB, so TE = 6m and TF = 6m

$$\frac{EF}{AB} = \frac{TE}{TA} = \frac{1}{2} \Rightarrow EF = 6m$$

7.

(a) $x^2 - 2x - 8 = x^2 - 4x + 2x - 8 = x(x-4) + 2(x-4) = 0$
$\Rightarrow (x-4)(x+2) = 0$
$\Rightarrow x = 4$ or $x = -2$

(b) Zeroes of a polynomial can be expressed graphically. Number of zeroes of polynomial is equal to number of points where the graph of polynomial **Intersects x – axis.**

(c) Graph of a quadratic polynomial is a Parabola.

(d) A highway underpass is parabolic in shape and a parabola is the graph that results from $p(x)=ax^2+bx+c$ which has two zeroes. (As it is a quadratic polynomial)
Sum of zeroes = 0 and one of the zero = 6 $\Rightarrow$ other zero = –6

x^2 – (sum of zeroes)x + product of zeroes = $x^2 - 36$

(e) $f(x) = (x-2)^2 + 4 = x^2 - 4x + 8$ is a Quadratic Polynomial.
The number of zeroes that f(x) can have is 2.

8.

(a)

Time (in sec)	No. od students(f)	x	Fx
0 – 20	8	10	80
20 – 40	10	30	300
40 – 60	13	50	650
60 – 80	6	70	420
80 – 100	3	90	270
	$\Sigma f = 40$		$\Sigma fx = 1720$

Mean time taken by a student to finish the race = 1720/40 = 43 seconds

(b) The modal class is 40 – 60 as it has the highest frequency i.e 13.
Upper limit of the modal class = 60

(c) The construction of cumulative frequency table is useful in determining the Median.

(d)

Time (in sec)	No. od students(f)	cf
0 – 20	8	8
20 – 40	10	18
40 – 60	13	31
60 – 80	6	37
80 – 100	3	40
	N= Σf = 40	

Here N/2 = 40/2 = 20, Median Class = 40 – 60, Modal Class = 40 – 60
Sum of lower limits of median class and modal class = 40 + 40 = 80

(e) Number of students who finished the race within 1 minute
= 8 + 10 + 13 = 31

9.

(a) Angles ∠LKM and ∠JKL are called as Linear Pair of angles.

(b) m∠LKM + m∠JKL =180°…… Linear Pair
⟹ 2x – 15 + m∠LKM=180°
⟹ m∠LKM= 195° –2x

(c) In △LKM,
m∠LKM + m∠LMK + m∠KLM=180° …angle sum property of a triangle
⟹ 195° – 2x + 50 + x = 180°
⟹ x = 65° = m∠KLM

(d) m∠LKM = 195° –2x = 195 – 2(65) = 195 – 130 = 65°
In △LKM, m∠LKM = m∠KLM = 65°
⟹△LKM is an isosceles triangle.

(e) m∠LKJ = 2x – 15 = 2(65) – 15 = 130 – 15 = 115°

10.

(a) The coordinates of CAVE of DEATH is (5, 3).

(b) The coordinates of THREE PALMS is (6, 4).

(c) The coordinates FOUR CROSS CLIFF and CAVE of DEATH are (2, 3) and (5, 3) respectively.

Distance between them = $\sqrt{(5-2)^2+(3-3)^2}=\sqrt{9}=3$ units

(d) The distance of SKULL ROCK from x – axis is 5 units.

(e) The mid – point of CAVE of DEATH and THREE PALMS = $\left(\frac{5+6}{2}, \frac{3+4}{2}\right) = (5.5, 3.5)$

11.

(a) In the first round 1, 3, 5, 7 and 9 numbered triangles are removed.
This means, Rahul is the alternate removing triangles.
In the second round 4 and 8 numbered triangles are removed.
In the third round 6 numbered triangle is removed.
So, 2 numbered triangle will be left in the last.

(b) Removed triangles numbered in sequence are
3, 6, 9, 4, 8, 5, 2 and 7.
So, 1 numbered triangle will be left in the end.

(c) Removed triangles numbered in sequence are
8, 3, 9, 2, 6, 10, 11, 7, 4, 1
So, 5 numbered triangle will be left in the end.

(d) The perimeters of the triangle will follow the below pattern
3, 6, 9, 12, 15, 18, 21, 24 and 27
$\Longrightarrow$ They are multiples of 3.

(e) We know that, area of an equilateral triangle = $\frac{\sqrt{3}}{4}(\text{side})^2$

The ratio of the areas of first two triangles whose sides are 3 and 4 is 9: 16
The ratio of the areas of two triangles whose sides are 6 and 8 is 36: 64 = 9: 16.
Hence, they are in proportion as their ratio is same and that is 9: 16.