CBSE Board
Class XII Mathematics
Sample Paper 1 – Solution

Section A

1. Correct option: B

Explanation:-

Coordinates of a point on the z-axis are (0, 0, c)

Therefore, we have

Distance of a point P(a, b, c) from the z-axis

$$= \sqrt{a^2 + b^2 + (c - c)^2}$$

$$= \sqrt{a^2 + b^2}$$

2. Correct option: D

Explanation:-

Probability of getting a one = $\frac{1}{6}$

Required probability = P(one on the first die) × P(one on the second die)

$= \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

3. Correct option: C

Explanation:-

$A' = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$

$A = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix}$

$B = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$

$(A + 2B) = \begin{bmatrix} -4 & 1 \\ 5 & 6 \end{bmatrix}$

$(A + 2B)' = \begin{bmatrix} -4 & 5 \\ 1 & 6 \end{bmatrix}$

4. Correct option: D
Explanation:-

Angle between the two diagonals of a cube is $\cos^{-1}\left(\frac{1}{3}\right)$.

5. Correct option: A
Explanation:-

Projection of a vector a on another vector b is $\vec{a}.\frac{\vec{b}}{|\vec{b}|}$

$$= \left(\hat{i} - 3\hat{k}\right).\frac{\left(3\hat{i} + \hat{j} - 4\hat{k}\right)}{\sqrt{9 + 1 + 16}} = \left|\frac{3 + 12}{\sqrt{26}}\right| = \frac{15}{\sqrt{26}}$$

6. Correct option: C
Explanation:-

Let $x = \tan^{-1}(-1)$

$\tan x = -1$

$\Rightarrow \tan x = -\tan\frac{\pi}{4}$

$\Rightarrow \tan x = \tan\left(\pi - \frac{\pi}{4}\right)$ ....$\left[\because \tan(\pi - \theta) = -\tan\theta\right]$

$\Rightarrow \tan x = \tan\frac{3\pi}{4}$

$\Rightarrow x = \frac{3\pi}{4}$

7. Correct option: B
Explanation:-

Two cards are drawn from 52 cards

Let E_1 and E_2 be the events of getting queen in the first and second draws respectively.

$\therefore P\left(E_1 \cap E_2\right) = \frac{4}{52} \times \frac{4}{52} = \frac{1}{13} \times \frac{1}{13}$

8. Correct option: A
Explanation:-

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 + \sin\theta & 1 \\ 1 + \cos\theta & 1 & 1 \end{vmatrix}$$

$R_1 \rightarrow R_1 - R_2$

$$\Delta = \begin{vmatrix} 0 & -\sin\theta & 0 \\ 1 & 1+\sin\theta & 1 \\ 1+\cos\theta & 1 & 1 \end{vmatrix}$$

$$\Delta = \sin\theta(1-1-\cos\theta) = -\sin\theta\cos\theta = -\frac{\sin 2\theta}{2}$$

We know that $-1 \le \sin x \le 1$

Therefore, $-\frac{1}{2} \le -\frac{\sin x}{2} \le \frac{1}{2}$

Hence, the minimum value of Δ is $-\frac{1}{2}$.

9. Correct option: D

Explanation:-

Given: $2x + y > 1$ and $2x - y \ge -3$

$\Rightarrow y > 1 - 2x$ and $2x + 3 \ge y$

$\Rightarrow 1 - 2x \le 2x + 3$

$\Rightarrow x \ge -\frac{1}{2}$

10. Correct option: C

Explanation:-

Let $I = \int \cos^{-1}(\sin x)dx$

We know that $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

$$\Rightarrow \cos^{-1}x = \frac{\pi}{2} - \sin^{-1}x$$

$$\therefore I = \int\left[\frac{\pi}{2} - \sin^{-1}(\sin x)\right]dx$$

$$\therefore I = \int \frac{\pi}{2}dx - \int x\,dx$$

$$\therefore I = \frac{\pi}{2}x - \frac{x^2}{2} = \frac{x}{2}(\pi - x)$$

11. Correct option: A

Explanation:-

$$f(x) = x^9 + 3x^7 + 64$$

$$\Rightarrow f'(x) = 9x^8 + 21x^6 = 3x^6(3x^2 + 7)$$

As $3x^6(3x^2+7) > 0 \ \forall x \in R$

Hence, f is increasing on R.

12. Correct option: C
Explanation:-

Given: $A=\begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix} = i\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$A^4 = i^4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^{4n} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

13. Correct option: A
Explanation:-

Given: $\vec{a} = 2\hat{i} - 3\hat{j} - k$ and $\vec{b} = \hat{i} + 4\hat{j} - 2\vec{k}$,

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 2 & -3 & -1 \\ 1 & 4 & -2 \end{vmatrix}$$

$$\vec{a} \times \vec{b} = 10\hat{i} + 3\hat{j} + 11k$$

14. Correct option: C
Explanation:-

Given: $a*b = a^2 + b^2$

So, $4*5 = 4^2 + 5^2$

$$(4*5)*3 = (4*5)^2 + 3^2$$

$$= (4^2 + 5^2)^2 + 3^2$$

$$= 41^2 + 3^2 = 1690$$

15. Correct option: C
Explanation:-

Function f is continuous at every point of its domain, so f is continuous at 1.

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x)$$

$$\Rightarrow \lim_{x\to 1} 5x - 4 = \lim_{x\to 1} 4x^2 + 3bx$$

$$\Rightarrow 5 - 4 = 4 + 3b$$

$$\Rightarrow 1 = 4 + 3b$$

$$\Rightarrow b = -1$$

16. Correct option: A

Explanation:-

Let $I = \int \tan^{-1}\left(\frac{\sin 2x}{1+\cos 2x}\right)dx$

$$I = \int \tan^{-1}\left(\frac{2\sin x\cos x}{1+2\cos^2 x-1}\right)dx$$

$$= \int \tan^{-1}\left(\frac{\sin x}{\cos x}\right)dx$$

$$= \int \tan^{-1}(\tan x)dx$$

$$= \int x\,dx$$

$$= \frac{x^2}{2}+c$$

17. Correct option: D

Explanation:-

The required area is given by

$$A = \int_0^{2\pi} \cos x\,dx$$

$$= \int_0^{\frac{\pi}{2}} \cos x\,dx + \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \cos x\,dx + \int_{\frac{3\pi}{2}}^{2\pi} \cos x\,dx$$

$$= [\sin x]_0^{\frac{\pi}{2}} + [\sin x]_{\frac{\pi}{2}}^{\frac{3\pi}{2}} + [\sin x]_{\frac{3\pi}{2}}^{2\pi}$$

$$= 1+2+1 \ldots (\text{Since area can't be} -\text{ve})$$

$$= 4 \text{ sq. units}$$

18. Correct option: B

Explanation:-

Given:

$$\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$$

$$\Rightarrow 2x^2 - 40 = 18 + 14$$

$$\Rightarrow 2x^2 = 32 + 40$$

$$\Rightarrow 2x^2 = 72$$

$$\Rightarrow x^2 = 36$$

$$\Rightarrow x = \pm 6$$

19. Correct option: B

Explanation:-

Given:

$$\int_0^{\alpha} \frac{1}{1+4x^2}dx = \frac{\pi}{8}$$

$$\Rightarrow \frac{1}{2}\left[\tan^{-1}(2x)\right]_0^{\alpha} = \frac{\pi}{8}$$

$$\Rightarrow \tan^{-1}(2\alpha) = \frac{\pi}{4}$$

$$\Rightarrow 2\alpha = \tan\frac{\pi}{4} \Rightarrow \alpha = \frac{1}{2}$$

20. Correct option: C

Explanation:-

Given differential equation is:

$$(x\log x)\frac{dy}{dx} + y = 2\log x$$

$$\Rightarrow \frac{dy}{dx} + \frac{1}{x\log x}y = \frac{2\log x}{x\log x}$$

$$\Rightarrow \frac{dy}{dx} + \frac{1}{x\log x}y = \frac{2}{x}$$

$$\text{Comparing with } \frac{dy}{dx} + Py = Q$$

$$\Rightarrow P = \frac{1}{x\log x},\ Q = \frac{2}{x}$$

$$\text{I.F.} = e^{\int P\,dx} = e^{\int \frac{1}{x\log x}dx} = e^{\log(\log x)} = \log x$$

Section B

21. Given two vectors are $\hat{i} - 2\hat{j} + 3k$ and $3\hat{i} - 2\hat{j} + k$.

$$\sin\theta = \frac{\left|\vec{a} \times \vec{b}\right|}{\left|\vec{a}\right|\left|\vec{b}\right|} \text{.........(i)}$$

To find $\vec{a} \times \vec{b}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 1 & -2 & 3 \\ 3 & -2 & 1 \end{vmatrix}$$

$$\Rightarrow \vec{a} \times \vec{b} = \left[-2-(-6)\right]\hat{i} - (1-9)\hat{j} + \left[-2-(-6)\right]k$$

$$\Rightarrow \vec{a} \times \vec{b} = 4\hat{i} + 8\hat{j} + 4k$$

$$\Rightarrow \left|\vec{a} \times \vec{b}\right| = \sqrt{4^2 + 8^2 + 4^2} = \sqrt{96} = 4\sqrt{6} \text{............(i)}$$

$$\left|\vec{a}\right| = \sqrt{1^2 + (-2)^2 + 3^2} = \sqrt{14} \text{........(ii) and}$$

$$\left|\vec{b}\right| = \sqrt{3^2 + (-2)^2 + 1^2} = \sqrt{14} \text{...........(iii)}$$

$$\Rightarrow \sin\theta = \frac{4\sqrt{6}}{\sqrt{14}\sqrt{14}} \text{......From (i), (ii) and (iii)}$$

$$\Rightarrow \sin\theta = \frac{4\sqrt{6}}{14} = \frac{2\sqrt{6}}{7}$$

22. The function f(x) is continuous at x = a if $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$

We need to check the continuity of f(x) at x = 1

So consider,

$$\text{L.H.L.} = \lim_{x \to 1^-} f(x)$$

$$= \lim_{x \to 1}\left(\frac{3}{2} - x\right)$$

$$= \frac{3}{2} - 1 = \frac{1}{2}$$

And,

$$\text{R.H.L.} = \lim_{x \to 1^+} f(x)$$

$$= \lim_{x \to 1}\left(\frac{3}{2} + x\right)$$

$$= \frac{3}{2} + 1 = \frac{5}{2}$$

$\Rightarrow$ L.H.L.≠R.H.L.

Therefore, function f(x) is not continuous at x = 1.

23. Consider $(x^2 - y^2)dx + 2xy\,dy = 0$ .... (i)

Given equation is homogeneous, then take y = vx

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

Substituting the values in (i), we get

$$\left(x^2 - v^2x^2\right) + 2x^2v.\frac{dy}{dx} = 0$$

$$\Rightarrow x^2\left(1 - v^2\right) + 2x^2v\left(v + x\frac{dv}{dx}\right) = 0$$

$$\Rightarrow x^2\left(1 - v^2\right) + 2x^2v^2 + 2x^3v.\frac{dv}{dx} = 0$$

$$\Rightarrow x^2\left(1 - v^2 + 2v^2\right) + 2x^3v\frac{dv}{dx} = 0$$

$$\Rightarrow x^2\left(1 + v^2\right) + 2x^3v\frac{dv}{dx} = 0$$

$$\Rightarrow \int \frac{v}{1 + v^2}dv + \int \frac{x^2}{2x^3}dx = 0$$

$$\Rightarrow \frac{1}{2}\log\left(1 + v^2\right) + \frac{1}{2}\log x = \log C$$

$$\Rightarrow \left(1 + \frac{y^2}{x^2}\right)x = C_1$$

$$\Rightarrow \left(x^2 + y^2\right) = C_1x$$

Now y = 1 when x = 1

So, $2 = C_1$

∴ Solution is $x^2 + y^2 = 2x$

24. Let $I = \int_{-1}^{2}(7x - 5)dx$

Take a = − 1, b = 2; $h = \frac{2 + 1}{n} \Rightarrow nh = 3, f(x) = 7x - 5$

$$\therefore I = \lim_{h \to 0} h\left[f(-1) + f(-1 + h) + f(-1 + 2h) + ... + f\left(-1 + \overline{n - 1}h\right)\right] ...(i)$$

$$f(-1) = -7 - 5 = -12; f(-1 + h) = 7(-1 + h) - 5 = 7h - 12$$

$$f\left(-1 + \overline{n - 1}\,h\right) = 7\left(-1 + \overline{n - 1}\,h\right) - 5 = 7(n - 1)h - 12$$

Substituting in (i)

$$\int_{-1}^{2}(7x - 5)dx = \lim_{h \to 0} h\left[(-12) + (7h - 12) + (14h - 12) + ... + \{7(n - 1)h - 12\}\right]$$

$$= \lim_{h\to 0} h\left[7h\left(1+2+....+\overline{n-1}\right)-12n\right] = \lim_{h\to 0} h\left[7h\frac{(n-1)n}{2}-12n\right]$$

$$= \lim_{h\to 0}\left[\frac{7}{2}(nh)(nh-h)-12nh\right] = \lim_{h\to 0}\left[\frac{7}{2}(3)(3-h)-36\right]$$

$$= \frac{7}{2}\times 9-36 = \frac{63}{2}-36 = -\frac{9}{2}$$

OR

Let $I = \int_2^3 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{5-x}}dx \quad ......(i)$

Using $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$, we get

$$I = \int_2^3 \frac{\sqrt{2+3-x}}{\sqrt{2+3-x}+\sqrt{2+3-5+x}}dx$$

$$\Rightarrow I = \int_2^3 \frac{\sqrt{5-x}}{\sqrt{5-x}+\sqrt{x}}dx \; ......(ii)$$

Adding (i) and (ii), we get

$$2I = \int_2^3 1.\,dx$$

$$= x\Big]_2^3 = 1$$

$$\Rightarrow I = \frac{1}{2}$$

25. Let E: Standard quality

$$P(E/I) = \frac{30}{100};\; P(E/II) = \frac{90}{100}$$

$$P(II/E) = \frac{P(II)\cdot P(E/II)}{P(I)\cdot P(E/I)+P(II)\cdot P(E/II)}$$

$$= \frac{\frac{30}{100}\times\frac{90}{100}}{\frac{70}{100}\times\frac{30}{100}+\frac{30}{100}\times\frac{90}{100}}$$

$$= \frac{9}{16}$$

26. $f(n) = n - (-1)^n$

$f(n) = n - 1$, n is even

$= n + 1$, n is odd

Injectivity:

Let n, m be any two even/odd natural numbers.

Then, f(n) = f(m)

⇒ n – 1 = m – 1

$\Rightarrow n = m$

Thus, in both cases $f(n) = f(m) \Rightarrow n = m$

If n is even and m is odd, then $n \neq m$. Also f(n) is odd and f(m) is even.

So, $f(n) \neq f(m)$.

Thus $n \neq m \Rightarrow f(n) \neq f(m)$

So, f is an injective map.

Surjectivity:

Let n be an arbitrary natural number.

If n is an odd natural number, then there exists an even natural number n + 1 such that

$f(n + 1) = n + 1 - 1 = n$

If n is an even natural number, then there exists an odd natural number n + 1 such that

$f(n + 1) = n + 1 + 1 = n + 2$

Thus, every $n \in N$ has its pre-image in N.

So, $f: N \to N$ is a surjection.

Hence, $f: N \to N$ is a bijection.

OR

$(a, b) R (c, d) = a + d = b + c$

For reflexive: $(a, b) R (a, b) \Rightarrow a + b = b + a$, true in N

For symmetric: $(a, b) R (c, d) \Rightarrow a + d = b + c \Rightarrow c + b = d + a$

$\Rightarrow (c, d) R (a, b)$, for $(a, b), (c, d) \in N \times N$

For transitive: Let $(a, b), (c, d), (e, f), \in N \times N$

$(a, b) R (c, d)$ and $(c, d) R (e, f)$

$(a, b) R (c, d) \Rightarrow a + d = b + c$ ....(i)

$(c, d) R (e, f) \Rightarrow c + f = d + e$ ....(ii)

Adding (i) and (ii), we get

$a + d + c + f = b + c + d + e \Rightarrow a + f = b + e \Rightarrow (a, b) R (e, f)$

Hence, R is transitive

Hence, the relation R is an equivalence relation.

27. The given planes are

$\vec{r} \cdot (\hat{i} + \hat{j} + k) = 1 \Rightarrow \vec{r} \cdot (\hat{i} + \hat{j} + k) - 1 = 0 \ldots (i)$

And, $\vec{r} \cdot (2\hat{i} + 3\hat{j} - k) + 4 = 0 \ldots (ii)$

The equation of plane passing through the line of intersection of these planes is

$$\left[\vec{r} \cdot (\hat{i} + \hat{j} + k) - 1\right] + \lambda \left[\vec{r} \cdot (2\hat{i} + 3\hat{j} - k) + 4\right] = 0$$

$$\Rightarrow \vec{r} \cdot \left[(2\lambda + 1)\hat{i} + (3\lambda + 1)\hat{j} + (1 - \lambda)k\right] + (4\lambda - 1) = 0 \ldots (iii)$$

Its direction ratios are $(2\lambda + 1), (3\lambda + 1)$ and $(1 - \lambda)$.

The required plane is parallel to x-axis. So, its normal will be perpendicular to x-axis.

We know that, the direction ratios of x-axis are 1, 0 and 0

$\Rightarrow 1\cdot(2\lambda+1)+0(3\lambda+1)+0(1-\lambda)=0$

$\Rightarrow 2\lambda+1=0$

$\Rightarrow \lambda=-\frac{1}{2}$

Substituting $\lambda=-\frac{1}{2}$ in equation (iii), we get

$\Rightarrow \vec{r}\cdot\left[-\frac{1}{2}\hat{j}+\frac{3}{2}\hat{k}\right]+(-3)=0$

$\Rightarrow \vec{r}\cdot\left(\hat{j}-3\hat{k}\right)+6=0$

Therefore, its Cartesian equation is y – 3z + 6 = 0
This is the equation of the required plane.

28. The given equation is $\tan^{-1}\frac{x+1}{x-1}+\tan^{-1}\frac{x-1}{x}=\tan^{-1}(-7)$

$\Rightarrow \tan^{-1}\left(\frac{\frac{x+1}{x-1}+\frac{x-1}{x}}{1-\frac{x+1}{x-1}\times\frac{x-1}{x}}\right)=\tan^{-1}(-7)$

$\Rightarrow \tan^{-1}\left(\frac{x^2+x+x^2-2x+1}{x^2-x-x^2+1}\right)=\tan^{-1}(-7)$

$\Rightarrow \tan^{-1}\left(\frac{2x^2-x+1}{-x+1}\right)=\tan^{-1}(-7)$

$\Rightarrow \frac{2x^2-x+1}{-x+1}=-7$

$\Rightarrow 2x^2-8x+8=0$

$\Rightarrow x^2-4x+4=0$

$\Rightarrow (x-2)^2=0$

$\Rightarrow x=2$

But x = 2 does not satisfy the equation $\tan^{-1}\frac{x+1}{x-1}+\tan^{-1}\frac{x-1}{x}=\tan^{-1}(-7)$

Hence, no solution exist.

29. LHS = $\begin{vmatrix} 3p & -p+q & -p+r \\ -q+p & 3q & -q+r \\ -r+p & -r+q & 3r \end{vmatrix}$

Applying $C_1 \rightarrow C_1 + C_2 + C_3$, we get

$$\text{LHS} = \begin{vmatrix} p+q+r & -p+q & -p+r \\ p+q+r & 3q & -q+r \\ p+q+r & -r+q & 3r \end{vmatrix} = (p+q+r)\begin{vmatrix} 1 & -p+q & -p+r \\ 1 & 3q & -q+r \\ 1 & -r+q & 3r \end{vmatrix}$$

Applying $R_1 \to R_1 - R_2$

$$\text{LHS} = (p+q+r)\begin{vmatrix} 0 & -p-2q & -p+q \\ 1 & 3q & -q+r \\ 1 & -r+q & 3r \end{vmatrix}$$

Applying $R_2 \to R_2 - R_3$

$$\text{LHS} = (p+q+r)\begin{vmatrix} 0 & -p-2q & -p+q \\ 0 & r+2q & -q-2r \\ 1 & -r+q & 3r \end{vmatrix}$$

Expanding this determinant along C_1, we get

LHS = $(p + q + r)[(-p - 2q)(-q - 2r) - (-p + q)(r + 2q)]$

= $(p + q + r)(pq + 2pr + 2q^2 + 4qr + pr + 2pq - qr - 2q^2)$

= $(p + q + r)(3pq + 3qr + 3pr)$

= $3(p + q + r)(pq + qr + rp)$ = RHS

Hence, $\begin{vmatrix} 3p & -p+q & -p+r \\ -q+p & 3q & -q+r \\ -r+p & -r+q & 3r \end{vmatrix} = 3(p+q+r)(pq+qr+rp)$.

30. Taking $(\log x)^2$ as the first function and x as the second function for applying integrating by parts, we have

$$I = (\log x)^2 \int x\,dx - \int \left\{ \left[\frac{d}{dx}(\log x)^2 \right] \int x\,dx \right\} dx$$

$$= (\log x)^2 \left(\frac{x^2}{2} \right) - \int \left[\left(\frac{2\log x}{x} \right) \left(\frac{x^2}{2} \right) \right] dx$$

$$= (\log x)^2 \left(\frac{x^2}{2} \right) - \int (x \log x)\,dx$$

Again applying By parts, we get

$$I = (\log x)^2\left(\frac{x^2}{2}\right) - \left\{\log x\int x\,dx - \int\left[\left(\frac{d}{dx}\log x\right)\int x\,dx\right]dx\right\}$$

$$= (\log x)^2\left(\frac{x^2}{2}\right) - \left\{\log x\left(\frac{x^2}{2}\right) - \int\left[\left(\frac{1}{x}\right)\frac{x^2}{2}\right]dx\right\}$$

$$= (\log x)^2\left(\frac{x^2}{2}\right) - (\log x)\left(\frac{x^2}{2}\right) + \int\frac{x}{2}dx$$

$$= (\log x)\left(\frac{x^2}{2}\right)(\log x - 1) + \frac{x^2}{4} + C$$

Hence, $\int x(\log x)^2\,dx = (\log x)\left(\frac{x^2}{2}\right)(\log x - 1) + \frac{x^2}{4} + C.$

OR

Let $I = \int\frac{\cos x}{(1-\sin x)(2-\sin x)}dx$

Take sin x = t ⇒ cos x dx = dt, we get

$$I = \int\frac{dt}{(1-t)(2-t)}$$

Let $\frac{1}{(1-t)(2-t)} = \frac{A}{1-t} + \frac{B}{2-t}$

$$\Rightarrow 1 = A(2-t) + B(1-t)$$

Take t = 2 and then t = 1, we get
B = –1 and A = 1

$$\Rightarrow \frac{1}{(1-t)(2-t)} = \frac{1}{1-t} - \frac{1}{2-t}$$

Therefore, we have

$$I = \int\left[\frac{1}{(1-t)} - \frac{1}{(2-t)}\right]dt$$

$$= -\log|1-t| + \log|2-t| + C$$

$$= \log\left|\frac{2-t}{1-t}\right| + C$$

$$= \log\left|\frac{2-\sin x}{1-\sin x}\right| + C$$

31. Given: $x = a\left(\cos t + \log\tan\frac{t}{2}\right)$ and y = a sin t

Differentiating x w.r.t. t, we get

$$\frac{dx}{dt} = a\left[-\sin t + \cot\frac{t}{2} \times \frac{d}{dt}\left(\tan\frac{t}{2}\right)\right]$$

$$\Rightarrow \frac{dx}{dt} = a\left[-\sin t + \cot\frac{t}{2} \times \sec^2\frac{t}{2} \times \frac{d}{dt}\left(\frac{t}{2}\right)\right]$$

$$\Rightarrow \frac{dx}{dt} = a\left[-\sin t + \cot\frac{t}{2} \times \sec^2\frac{t}{2} \times \frac{1}{2}\right]$$

$$\Rightarrow \frac{dx}{dt} = a\left[-\sin t + \frac{1}{\sin t}\right] = a\left[\frac{1-\sin^2 t}{\sin t}\right]$$

$$\Rightarrow \frac{dx}{dt} = a\left(\frac{\cos^2 t}{\sin t}\right) \ldots (i)$$

Differentiating y w.r.t. t, we get

$$\frac{dy}{dt} = a\cos t \ldots (ii)$$

From (i) and (ii), we have

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{a\cos t}{a\left(\frac{\cos^2 t}{\sin t}\right)} = \frac{\sin t}{\cos t} = \tan t$$

$$\Rightarrow \frac{dy}{dx} = \tan t \ldots (iii)$$

Now, $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$

From (iii), we get

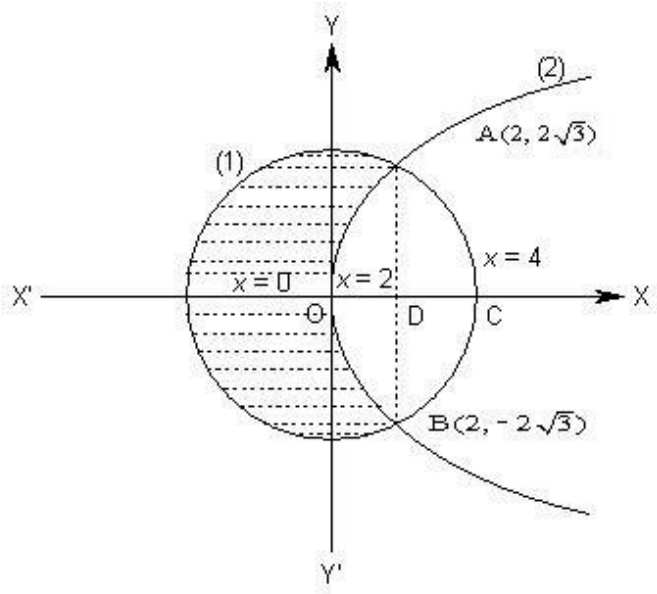
$$\frac{d^2y}{dt^2} = \frac{d}{dx}(\tan t) = \sec^2 t \cdot \frac{dt}{dx} = \sec^2 t \cdot \frac{\sin t}{a\cos^2 t} = \frac{\sin t}{a\cos^4 t}$$

$$\Rightarrow \frac{d^2y}{dt^2} = \frac{\sec^4 t \cdot \sin t}{a}$$

Section C

32. $x^2 + y^2 = 16$ .... (i)

$y^2 = 6x$ .... (ii)



Points of intersection of curve (i) and (ii) are obtained as follows:

$x^2 + 6x - 16 = 0$

$\Rightarrow (x+8)(x-2) = 0$

$\Rightarrow x = 2 \qquad (\because x \neq -8)$

$y^2 = 12$

$y = \pm 2\sqrt{3}$

Therefore, points of intersection are $A(2, 2\sqrt{3})$ and $B(2, -2\sqrt{3})$

Also, C(0, 4)

Area(OBCAO) = 2 (Area ODA + Area DCA)

$$= 2\left[\int_0^2 \sqrt{6x}\,dx + \int_2^4 \sqrt{16-x^2}\,dx\right]$$

$$= 2\left[\sqrt{6}.\left\{\frac{2}{3}x^{3/2}\right\}_0^2 + \left\{\frac{x\sqrt{16-x^2}}{2} + \frac{16}{2}\sin^{-1}\frac{x}{4}\right\}_2^4\right]$$

$$= 2\left[\frac{2\sqrt{6}}{3}.2\sqrt{2} + \left\{0 + 8\sin^{-1}1\right\} - \left\{\frac{2.2\sqrt{3}}{2} + 8\sin^{-1}\frac{1}{2}\right\}\right]$$

$$= \frac{16\sqrt{3}}{3} + 16.\frac{\pi}{2} - \left(4\sqrt{3} + 16.\frac{\pi}{6}\right)$$

$$=\left(\frac{4\sqrt{3}}{3}+\frac{16}{3}\pi\right)\text{sq. units.}$$

Required Area = Area of a circle $-\left(\frac{4\sqrt{3}}{3}+\frac{16}{3}\pi\right)$

$$=16\pi-\frac{4\sqrt{3}}{3}-\frac{16}{3}\pi$$

$$=\frac{32}{3}\pi-\frac{4\sqrt{3}}{3}=\frac{4}{3}\left(8\pi-\sqrt{3}\right)\text{sq. units}$$

33. $3l+m+5n=0$ …. (1)

$6mn-2nl+5lm=0$ ... (2)

From (1), $m=-(3l+5n)$ ... (3)

Substituting the value of m in (2)

$\Rightarrow -6(3l+5n)n-2nl-5l(3l+5n)=0$

$\Rightarrow -18ln-30n^2-2nl-15l^2-25nl=0$

$\Rightarrow -30n^2-45nl-15l^2=0$

$\Rightarrow 2n^2+3ln+l^2=0$ [Divide by (-15)]

$\Rightarrow (2n+l)(n+l)=0$

Either $2n+l=0$ or $n+l=0$

(I) when $2n+l=0 \Rightarrow l=-2n$

From (3), $m=-(-6n+5n)=n$

(II) when $n+l=0 \Rightarrow l=-n$

From (3), $m=-(-3n+5n)=-2n$

Direction ratios of two lines are

$-2n, n, n$ and $-n, -2n, n$

$-2, 1, 1$ and $1, 2, -1$

Angle between two lines is

$$\cos\theta=\left|\frac{(-2)1+(1)(2)+(1)(-1)}{\sqrt{(-2)^2+1^2+1^2}\sqrt{1^2+2^2+(-1)^2}}\right|=\frac{1}{6}$$

Hence, $\theta=\cos^{-1}\left(\frac{1}{6}\right)$.

OR

Equation of line is $\vec{r} = 2\hat{i} - \hat{j} + 2k + \lambda\left(3\hat{i} + 4\hat{j} + 2k\right)$

$\vec{r} = (2+3\lambda)\hat{i} + (-1+4\lambda)\hat{j} + (2+2\lambda)k$

This point lies on the plane $\vec{r}\cdot\left(\hat{i} - \hat{j} + k\right) = 5$

$\Rightarrow \left[(2+3\lambda)\hat{i} + (-1+4\lambda)\hat{j} + (2+2\lambda)k\right]\cdot\left(\hat{i} - \hat{j} + k\right) = 5$

$\Rightarrow (2+3\lambda) + (-1+4\lambda)(-1) + (2+2\lambda) = 5$

$\Rightarrow \lambda = 0$

Coordinates are

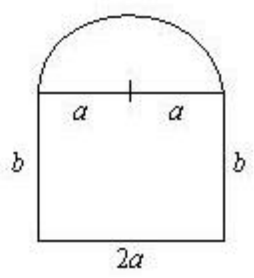
$(2+3\lambda, -1+4\lambda, 2+2\lambda) = (2,-1,2)$

Distance between $(2,-1,2)$ and $(-1,-5,-10)$

$= \sqrt{(-1-2)^2 + (-5+1)^2 + (-10-2)^2}$

$= 13$ units

34. Let 2a cm be the length and b cm be the breadth of the rectangle. Then a cm is the radius of the semi-circle.



By hypothesis,

perimeter $= 2a + b + b + \pi a$

$\Rightarrow p = 2a + b + b + \pi a$

$\Rightarrow 2b = p - (\pi + 2)a$ ..... (1)

Also A = Area of the window

$= \frac{1}{2}\pi a^2 + 2a \times b$

$= \frac{1}{2}\pi a^2 + a \times \left[p - (\pi+2)a\right]$...[By (1)]

$= pa - \frac{1}{2}(\pi + 4)a^2$

$\Rightarrow \frac{dA}{da} = p - (\pi + 4)a$

For max or min $\frac{dA}{da} = 0$

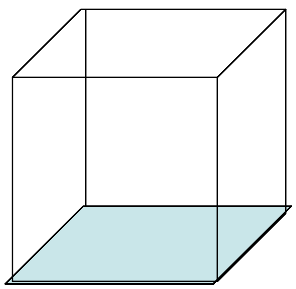
$\Rightarrow p-(\pi+4)a=0$

$\Rightarrow a=\dfrac{p}{\pi+4}.$

Also, $\dfrac{d^2A}{da^2}=-(\pi+4)<0$

$\therefore$ The light will be maximum when the radius of the semi-circle is, $a=\dfrac{p}{\pi+4}.$

OR



Let x be side of the square base and y be the height of the cuboid
Volume (V) = x. x. y = x^2 y … (i)

Surface area (S) = 2 (x.x + x.y + x.y) = $2x^2+4xy=2x^2+4x\dfrac{V}{x^2}$

$S=2x^2+\dfrac{4V}{x}\Rightarrow\dfrac{dS}{dx}=4x-\dfrac{4V}{x^2}$

For minimum S, $\dfrac{dS}{dx}=0\Rightarrow 4x-\dfrac{4V}{x^2}=0\Rightarrow x^3=V\Rightarrow x=\sqrt[3]{V}$

$\dfrac{d^2S}{dx^2}=4+\dfrac{8V}{x^3}\Rightarrow\left.\dfrac{d^2S}{dx^2}\right]_{x=\sqrt[3]{V}}=4+\dfrac{8V}{V}>0$

$\therefore$ For $x=\sqrt[3]{V}$, surface area is minimum

$\Rightarrow x^3=V\Rightarrow x^3=x^2y\,[\text{From (i)}]\Rightarrow x=y\Rightarrow$ cuboid is a cube

35. Consider, $A=\begin{bmatrix}1&3&-2\\-3&0&-5\\2&5&0\end{bmatrix}$

We write A = IA

$\Rightarrow\begin{bmatrix}1&3&-2\\-3&0&-5\\2&5&0\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}A$

$\Rightarrow\begin{bmatrix}1&3&-2\\0&9&-11\\2&5&0\end{bmatrix}=\begin{bmatrix}1&0&0\\3&1&0\\0&0&1\end{bmatrix}A$ [By performing $R_2\to R_2+3R_1$]

$$\Rightarrow \begin{bmatrix} 1 & 3 & -2 \\ 0 & 9 & -11 \\ 0 & -1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} A \qquad \text{[By performing } R_3 \rightarrow R_3 - 2R_1\text{]}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 10 \\ 0 & 9 & -11 \\ 0 & -1 & 4 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 3 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} A \qquad \text{[By performing } R_1 \rightarrow R_1 + 3R_3\text{]}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & -\frac{11}{9} \\ 0 & -1 & 4 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 3 \\ \frac{1}{3} & \frac{1}{9} & 0 \\ -2 & 0 & 1 \end{bmatrix} A \qquad \text{[By performing } R_2 \rightarrow \frac{1}{9} R_2\text{]}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & -\frac{11}{9} \\ 0 & 0 & \frac{25}{9} \end{bmatrix} = \begin{bmatrix} -5 & 0 & 3 \\ \frac{1}{3} & \frac{1}{9} & 0 \\ -\frac{5}{3} & \frac{1}{9} & 1 \end{bmatrix} A \qquad \text{[By performing } R_3 \rightarrow R_3 + R_2\text{]}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & -\frac{11}{9} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 3 \\ \frac{1}{3} & \frac{1}{9} & 0 \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix} A \qquad \text{[By performing } R_3 \rightarrow \frac{9}{25} R_3\text{]}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 3 \\ -\frac{2}{5} & \frac{4}{25} & \frac{11}{25} \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix} A \qquad \text{[By performing } R_2 \rightarrow R_2 + \frac{11}{9} R_3\text{]}$$

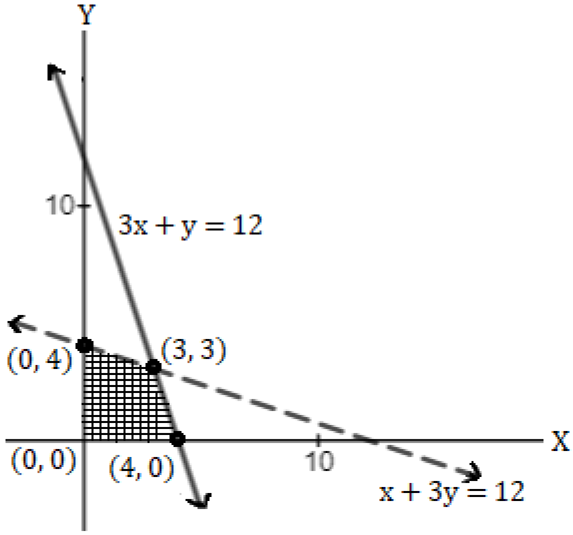
$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{3}{5} \\ -\frac{2}{5} & \frac{4}{25} & \frac{11}{25} \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix} A \qquad \text{[By performing } R_1 \rightarrow R_1 - 10R_3\text{]}$$

Hence, we obtain

$\Rightarrow$ B is inverse of A by definition.

Hence, $A^{-1} = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{3}{5} \\ -\frac{2}{5} & \frac{4}{25} & \frac{11}{25} \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix}$

36. Let the number of packages of bolts = x and,
Number of packages of nuts = y
To maximize profit Z = 19.6x + 9y
As per the question,
$3y + x \le 12$ and,
$y + 3x \le 12$
$x \ge 0, y \ge 0$
The feasible region is given below:



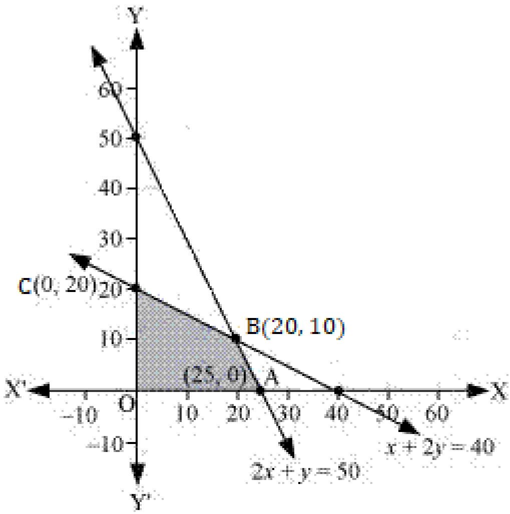
Here, the points lying on x + 3y = 12 are (0, 4) and (12, 0)
Also, the points lying on 3x + y = 12 are (0, 12) and (4, 0)
The feasible region contains (0, 0) as it satisfies both the inequality.
The point of intersection of both the equation is (3, 3)
We have the corner points of the feasible region are (0, 0), (4, 0), (0, 4) and (3, 3)
Z = 19.6x + 9y
At (0, 0), Z = 19.6 × 0 + 9 × 0 = 0
At (4, 0), Z = 19.6 × 4 + 9 × 0 = 78.4
At (0, 4), Z = 19.6 × 0 + 9 × 4 = 36
At (3, 3), Z = 19.6 × 3 + 9 × 3 = 58.8 + 27 = 85.8
Therefore, maximum profit = Rs. 85.8 at (3, 3)
Hence, the profit is maximum when the number of packets of bolts and that of nuts is 3 each.

OR

Let there be x chocolates of one kind and y chocolates of other kind.
Therefore, $x \geq 0, y \geq 0$
The given information is as follows:

	Flour(g)	Fat (g)
Chocolates of 1st kind, x	16	3
Chocolates of 2nd kind, y	8	6
Availability	400	120

$16x + 8y \leq 400 \Rightarrow 2x + y \leq 50$
$3x + 6y \leq 120 \Rightarrow x + 2y \leq 40$
Total number of chocolates is $Z = x + y$
The mathematical formation of the given problem is
Maximize $Z = x + y$ ... (i)
Subject to constraints
$2x + y \leq 50$ ... (ii)
$x + 2y \leq 40$ ... (iii)
$x, y \geq 0$ ... (iv)
The feasible region is as follows:



The corner points are A(25, 0), B(20, 10), C(0, 20) and O(0, 0).
$Z = x + y$
At A(25, 0), Z = 25
At B(20, 10), Z = 30
At C(0, 20), Z = 20
At O(0, 0), Z = 0
Thus the maximum number of chocolates that can be made is 30 (20 of one kind and 10 of other kind).

37. Each student has the same chance of being chosen.

Therefore, the probability of each student to be selected $= \frac{1}{15}$

Now, the given information is as follows:

X	14	15	16	17	18	19	20	21
f	2	1	2	3	1	2	3	1

$$P(X=14)=\frac{2}{15}, P(X=15)=\frac{1}{15}, P(X=16)=\frac{2}{15}, P(X=17)=\frac{3}{15},$$
$$P(X=18)=\frac{1}{15}, P(X=19)=\frac{2}{15}, P(X=20)=\frac{3}{15}, P(X=21)=\frac{1}{15}$$

Thus, the probability distribution of random variable X is as follows:

X	14	15	16	17	18	19	20	21
P	$\frac{2}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$

Mean of X = E(X) = $\sum X_i P(X_i)$

$$=14\times\frac{2}{15}+15\times\frac{1}{15}+16\times\frac{2}{15}+17\times\frac{3}{15}+18\times\frac{1}{15}+19\times\frac{2}{15}+20\times\frac{3}{15}+21\times\frac{1}{15}$$
$$=\frac{1}{15}(28+15+32+51+18+38+60+21)$$
$$=\frac{263}{15}$$

E(X^2) = $\sum X_i^2 P(X_i)$

$$=14^2\times\frac{2}{15}+15^2\times\frac{1}{15}+16^2\times\frac{2}{15}+17^2\times\frac{3}{15}+18^2\times\frac{1}{15}+19^2\times\frac{2}{15}+20^2\times\frac{3}{15}+21^2\times\frac{1}{15}$$
$$=\frac{1}{15}(392+225+512+867+324+722+1200+441)$$
$$=\frac{4683}{15}$$
$$=312.2$$

Variance(X) = $E(X^2)-[E(X)]^2$

$$=312.2-\left(\frac{263}{15}\right)^2$$
$$=312.2-307.4177$$
$$=4.7823$$
$$\approx 4.78$$

OR

Let E_1, E_2 and E_3 be the respective events representing a scooter driver, a car driver and a truck driver.

Let A be the event that the person meets with an accident.

Total number of drivers = 2000 + 4000 + 6000 = 12000

$P(E_1)$ = P(Driver is a scooter driver) $= \frac{2000}{12000} = \frac{1}{6}$

$P(E_2)$ = P(Driver is a car driver) $= \frac{4000}{12000} = \frac{1}{3}$

$P(E_3)$ = P(Driver is a truck driver) $= \frac{6000}{12000} = \frac{1}{2}$

$P(A|E_1)$ = P(Scooter driver met with an accident) $= 0.01 = \frac{1}{100}$

$P(A|E_2)$ = P(Car driver met with an accident) $= 0.03 = \frac{3}{100}$

$P(A|E_3)$ = P(Scooter driver met with an accident) $= 0.15 = \frac{15}{100}$

The probability that the driver is a scooter driver, given that he met with an accident is given by $P(E_1|A)$

Using Bayes' theorem, we get

$$P(E_1|A) = \frac{P(E_1)\cdot P(A|E_1)}{P(E_1)\cdot P(A|E_1) + P(E_2)\cdot P(A|E_2) + P(E_3)\cdot P(A|E_3)}$$

$$= \frac{\frac{1}{6}\cdot\frac{1}{100}}{\frac{1}{6}\cdot\frac{1}{100} + \frac{1}{3}\cdot\frac{3}{100} + \frac{1}{2}\cdot\frac{15}{100}}$$

$$= \frac{\frac{1}{6}}{\frac{52}{6}}$$

$$= \frac{1}{52}$$