# Goa Board
# Class X Mathematics
# Term II
# Sample Paper 10 - Solution

## Section A
## (Questions 1 to 8 carry 1 mark each)

**1.** Correct answer: C

$$\frac{\text{Volume of cylinder}}{\text{Volume of cone}} = \frac{(\pi r^2 h)}{\frac{1}{3}\pi r^2 h} = \frac{3}{1}$$

Hence, the required ratio is 3:1.

**2.** Correct answer: B

Consider the equation $-x^2 + 3x - 3 = 0$.

Here, $a = -1$, $b = 3$ and $c = -3$

Sum of the roots = $-\frac{b}{a} = -\frac{3}{(-1)} = 3$

**3.** Correct answer: D

By distance formula,

Required distance = $\sqrt{(8-3)^2 + (-6-4)^2} = \sqrt{25+100} = \sqrt{125} = 5\sqrt{5}$ units

**4.** Correct answer: A

In $\triangle ABO$,

$$\frac{h}{25} = \tan 60° = \sqrt{3}$$

$$\Rightarrow h = 25\sqrt{3}\text{ m}$$

**5.** Correct answer: C

As $\frac{4}{5}$, a and 2 are in A.P.,

Therefore, $a - \frac{4}{5} = 2 - a$

$2a = 2 + \frac{4}{5}$

$a = \frac{14}{5} \times \frac{1}{2}$

Hence, $a = \frac{7}{5}$

**6.** Correct answer: A
It is known that the lengths of the tangents drawn from an external point are equal.
PB = PA = 10 cm and CQ = CA = 2 cm
PC = PA – CA = 10 cm – 2 cm = 8 cm

**7.** Correct answer: D
Total number of cards = 52
Number of face cards = 12
P (face card) = $\frac{12}{52}=\frac{3}{13}$

**8.** Correct answer: B
The co-ordinates of the point P(x, y) which divides the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio $m_1 : m_2$ are

$$\left[\frac{m_1x_2+m_2x_1}{m_1+m_2},\frac{m_1y_2+m_2y_1}{m_1+m_2}\right]$$

Let the ratio be m : 1.
Where $m_1$ = m, and $m_2$ = 1.

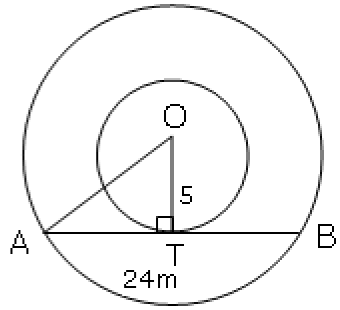
$$\frac{-2m-5}{m+1}=-3\Rightarrow m=2$$

Hence the ratio is 2 : 1.

**Section B**

**9.** Three terms p, q and r are in A.P. if 2q = p + r.
Now, first term + third term = (a – b) + (a + b)
= 2a
= $2\times$(second term)
Thus, (a – b), a and (a + b) are consecutive terms of an A.P.

**10.** Let O be the centre of circle and AB be the chord of larger circle and OT be radius of smaller circle.
So OT $\perp$ AB since tangent is $\perp$ to radius at its point of contact.
AT = TB = 12 cm
(Since perpendicular from centre to the chord bisects it)
So, in triangle OAT,
$OA^2 = OT^2 + AT^2$
$OA^2 = 5^2 + 12^2$
So, OA = 13 cm
Thus, the radius of the larger circle is 13 cm.



**11.** Sum of roots (S) = $\frac{-1}{3}+\frac{5}{2}=\frac{13}{6}$

Product of roots (P) = $\frac{-1}{3}\times\frac{5}{2}=\frac{-5}{6}$

Required equation is $x^2 - Sx + P = 0$

$$x^2-\frac{13}{6}x+\left(\frac{-5}{6}\right)=0$$

$6x^2 - 13x - 5 = 0$

**12.** Let n be the required number of spheres.

Since, the spheres are melted to form a cylinder. So, the volume of all the n spheres will be equal to the volume of the cylinder.

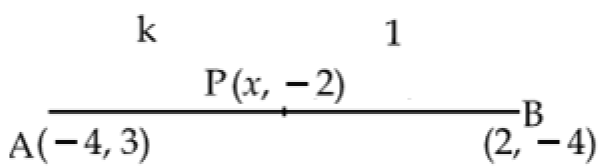
$$n\times\frac{4}{3}\times\pi\times3\times3\times3=\pi\times2\times2\times45$$

$\therefore$ n = 5

Thus, the required number of spheres which are melted to form the cylinder is 5.

**13.** Let AP : PB = k : 1

By section formula,




$$-2=\frac{-4k+3\times(1)}{k+1}$$

$\Rightarrow -2k-2=-4k+3$

$\Rightarrow$ 2k = 5

$\Rightarrow \frac{k}{1}=\frac{5}{2}$

Required ratio = 5 : 2

Now, x = $\frac{2k-4}{k+1}=\frac{2\times\frac{5}{2}-4}{\frac{5}{2}+1}=\frac{10-8}{5+2}=\frac{2}{7}$

**14.** For the resulting cuboid:

$l$ = 8 cm, b = 4 cm, h = 4 cm

Surface area = $2(lb + bh + lh)$

= $2[8\times4+4\times4+8\times4]$

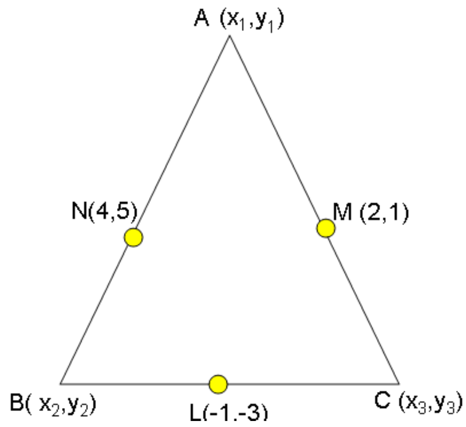
= 160 $cm^2$

**Section C**

**15.** Let the vertices of the triangle be $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$

Let the given mid-points of the sides BC, CA and AB be L(–1, –3), M(2, 1) and N(4, 5).

Now, by mid-point formula,




$$-1 = \left(\frac{x_2 + x_3}{2}\right); 4 = \left(\frac{x_1 + x_2}{2}\right); 2 = \left(\frac{x_1 + x_3}{2}\right)$$

$x_2 + x_3 = -2$; $x_1 + x_2 = 8$; $x_1 + x_3 = 4$

Adding these equations,

$2(x_1 + x_2 + x_3) = 10$

$x_1 + x_2 + x_3 = 5$

On solving, we get:

$x_1 = 7, x_2 = 1, x_3 = -3$

Similarly,

$$-3 = \left(\frac{y_2 + y_3}{2}\right); \ 1 = \left(\frac{y_1 + y_3}{2}\right); \ 5 = \left(\frac{y_1 + y_2}{2}\right)$$

$y_2 + y_3 = -6$; $y_1 + y_3 = 2$; $y_1 + y_2 = 10$

Adding these equations,

$2(y_1 + y_2 + y_3) = 6$

$y_1 + y_2 + y_3 = 3$

On solving, we get:

$y_1 = 9, y_2 = 1, y_3 = -7$

Hence, the vertices of the triangle are (7, 9), (1, 1) and (–3, –7).

**16.** In a leap year, there are 366 days.

52 weeks = 364 days

1 leap year = 52 weeks and 2 days

These extra two days can be Sun-Mon or Mon-Tue or Tue-Wed or Wed-Thu or Thu-Fri, or Fri-Sat or Sat-Sun.

Total number of outcomes = 7

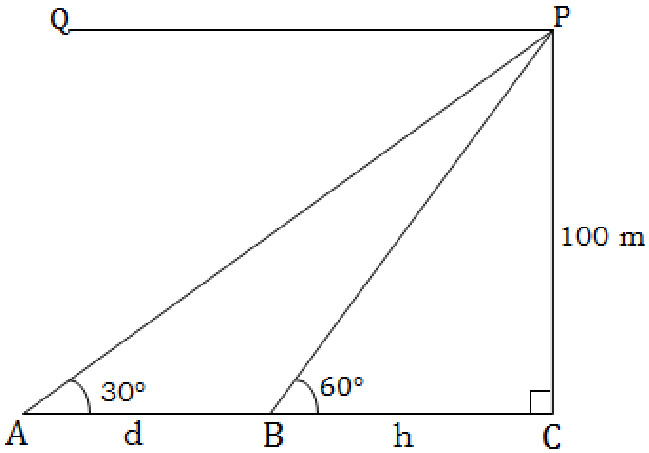
Number of favourable outcomes =2

P(53 Thursdays) = $\frac{2}{7}$

**17.** Let A and B be the two positions of the ship. Let d be the distance travelled by the ship during the period of observation, i.e., AB = d metres.

Suppose that the observer is at the point P. It is given that PC = 100 m.

Let h be the distance (in metres) from B to C.




In right ΔPCA,

$$\frac{d+h}{100} = \cot 30^o = \sqrt{3}$$

$d+h = 100\sqrt{3}$ ... (i)

In triangle PCB,

$$\frac{h}{100} = \cot 60^o = \frac{1}{\sqrt{3}}$$

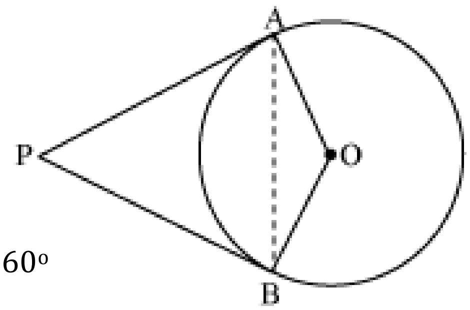
$$\Rightarrow h = \frac{100}{\sqrt{3}} \text{ metres}$$

Putting the value of h in (i), we get,

$$d = 100\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right) = \frac{200}{\sqrt{3}} = 115.47 \text{ (approx.)}$$

Thus, the distance travelled by the ship from A to B is 115.47 metres (approx).

**18.** Let us consider a circle with centre O. Let P be a external point from which two tangents PA and PB are drawn to circle which touch the circle at points A and B respectively and AB is the line segment, joining point of contacts A and B together such that it subtends $\angle$AOB at centre O of circle.




Now we may observe that:

OA (radius) $\perp$ PA (tangent)

So, m$\angle$OAP = 90°

Similarly, OB (radius) $\perp$ PB (tangent)

m$\angle$OBP = 90°

Now in quadrilateral OAPB, sum of all interior angles = 360°

m$\angle$OAP + m$\angle$APB + m$\angle$PBO + m$\angle$BOA = 360°

90° + m$\angle$APB + 90° + m$\angle$BOA = 360°

m$\angle$APB + m$\angle$BOA = 180°

Hence, the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

**19.** –5, –8, –11, ... –230 forms an A.P. with

a = –5, d = –8 – (–5) = –3

Let –230 = $a_n$ = a+(n – 1)d = –5 + (n – 1)( –3)

$\Rightarrow$ –230 + 5 = (n – 1)(–3)

$\Rightarrow n - 1 = \frac{-225}{-3}$

$\Rightarrow$ n – 1 = 75

$\Rightarrow$ n = 76

$S_{76} = \frac{n}{2}(a+l)$

$= \left(\frac{76}{2}\right)[(-5) + (-230)]$

= 38(–235)

= –8930

**20.** The volume of the metal in the metallic spherical shell $= \left[\frac{4}{3}\pi(5)^3 - \frac{4}{3}\pi(3)^3\right]$

$= \left[\frac{4}{3}\pi(125-27)\right]$

$= \frac{392\pi}{3}$ cu cm

Let the diameter of the base of the cylinder be r cm.

Since the volume of the hollow spherical shell will be equal to the volume of the cylinder casted.

$\therefore \frac{392\pi}{3} = \pi\left(\frac{r}{2}\right)^2\left(\frac{32}{3}\right)$

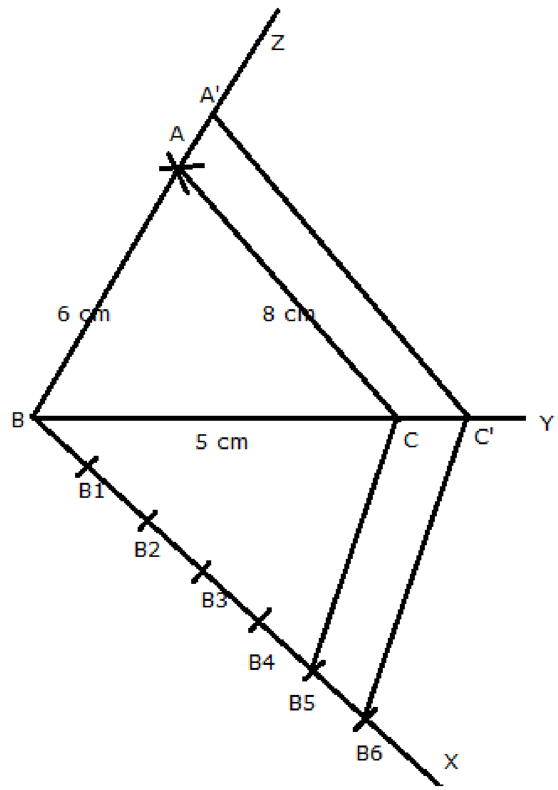
$\Rightarrow \left(\frac{r}{2}\right)^2 = \frac{392}{32} = \frac{49}{4}$

$\Rightarrow \frac{r}{2} = \frac{7}{2}$

$\Rightarrow$ r = 7

Hence, the diameter of the base of the cylinder is 7 cm.

**21. Steps of construction:**




1. Draw a line segment, BC, of length 5 cm. With B and C as centres and radii 6 cm and 8 cm respectively draw two arcs intersecting each other at A. Join AB and AC. Now, ΔABC is the required triangle with sides 5 cm, 6 cm and 8 cm.

2. Draw an acute ∠XBC on the side opposite to vertex A.

3. On BX, mark six points $B_1$, $B_2$, $B_3$, $B_4$, $B_5$, and $B_6$ such that:

$BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5 = B_5B_6$

4. Join $B_5$ to C.

5. From $B_6$, draw $B_6C'$ parallel to $B_5C$, intersecting BC produced in C'.

6. From C' draw C'A' parallel to CA, meeting AB produced in A'.

ΔA'BC' is the required triangle.

**22.** Given equation: $px^2 + (p - 1)x + (p - 1) = 0$

For repeated root, D = 0

$\Rightarrow b^2 - 4ac = 0$

$\Rightarrow (p - 1)^2 - 4p(p - 1) = 0$

$\Rightarrow (p - 1)(p - 1 - 4p) = 0$

$\Rightarrow (p - 1)(-3p - 1) = 0$

$\Rightarrow p - 1 = 0 \text{ or } -3p - 1 = 0$

$\Rightarrow p = 1 \text{ or } p = -\frac{1}{3}$

**23.** Given: Radius ($r$) of the cylindrical bucket = 18 cm

Height (h) of the cylindrical bucket = 32 cm

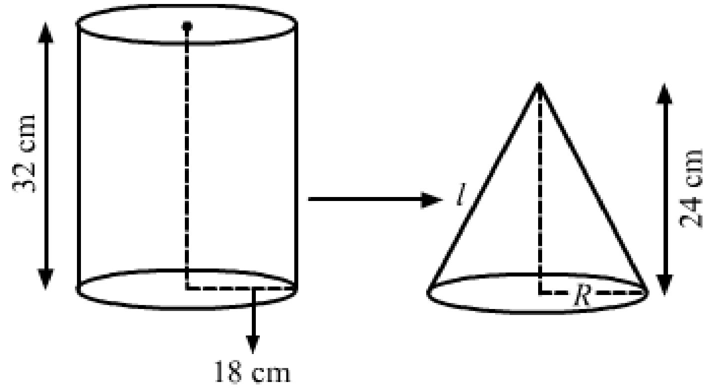
Volume of the cylindrical bucket = $\pi r^2 h$

= $(\pi \times 18 \times 18 \times 32)\ \text{cm}^3$

Let R be the radius of the conical heap.

Height (H) of the conical heap = 24 cm

Volume of the conical heap = $\frac{1}{3}\pi R^2 H$




= $(\frac{1}{3}\pi R^2 \times 24)\ \text{cm}^3 = 8\pi R^2\ \text{cm}^3$

According to the question:

Volume of the cylindrical bucket = Volume of the conical heap

$\Rightarrow \pi \times 18 \times 18 \times 32 = 8 \times \pi \times R^2$

$\Rightarrow R^2 = \frac{18 \times 18 \times 32}{8} = 18 \times 18 \times 4 = 1296$

$\Rightarrow R = 36$

Let l be the slant height of the conical heap.

$\therefore l^2 = R^2 + H^2 = 36^2 + 24^2 = 1296 + 576 = 1872$

$\Rightarrow l = 43.3$

Thus, the slant height (l) of the conical heap is 43.3 cm.

**24.** Let a and d respectively be the first term and the common difference of the AP.

Then, we have:

$a + (m - 1)d = n$ ............(i)

And, $a + (n - 1)d = m$ ...........(ii)

On solving (i) and (ii), we get,

$d = -1$ ; $a = m + n - 1$

$\therefore a_{m+n} = a + (m + n - 1)d$

$= m + n - 1 + (m + n - 1)(-1)$

$= m + n - 1 - m - n + 1 = 0$

## Section D

**25.** Let the speed of the stream be x km/ hr

Speed downstream = (15 + x) km/hr

Speed upstream = (15 – x) km/hr

Distance downstream = 30 km

Distance upstream = 30 km

Total time taken = 4 hrs 30 mins = $4\frac{30}{60}$ hrs $= \frac{9}{2}$ hrs

$\therefore \left(\frac{30}{15+x}\right)+\left(\frac{30}{15-x}\right)=\frac{9}{2}$

$\Rightarrow 10\,[15-x+15+x] \times 2 = 3\,(15+x)(15-x)$

$\Rightarrow 200=225 - x^2$

$\Rightarrow x^2 = 25$

$\Rightarrow x = 5$ or $-5$

Rejecting x = –5, we have x = 5

Thus, the speed of stream is 5 km/hr.

**26.** Let the radius of the hemispherical dome be 'r' metres and the total height of the building be 'h' metres.

Since, the base diameter of the dome is equal to $\frac{2}{3}$ of the total height, $2r = \frac{2h}{3}$.

$\therefore r = \frac{h}{3}$

Let H metres be the height of the cylindrical portion.

Therefore, H= $h-\frac{h}{3}=\frac{2h}{3}$ metres

Volume of the air inside the building

= volume of air inside the dome+ volume of the air inside the cylinder

$= \frac{2}{3}\pi\left(\frac{h}{3}\right)^3+\pi\left(\frac{h}{3}\right)^2\frac{2h}{3}=\frac{8}{81}\pi h^3$

Volume of the air inside the building is $67\frac{1}{21}$ m$^3$ = $\frac{1408}{21}$ m$^3$.

$\therefore \frac{8}{81}\pi h^3 = \frac{1408}{21}$

$\Rightarrow h^3 = 216$

$\Rightarrow h = 6$

Thus, the height of the building is 6 m.

**27.** Here, 1, 4, 7, 10... x form an AP with a=1, d=3, $a_n$=x.

$a_n = a+(n-1)d$

$\Rightarrow x=1+(n-1)(3) = 3n-2$

Also, $S_n = \frac{n}{2}(a+l)$

$\Rightarrow 287 = \frac{n}{2}(1+x)$

$\Rightarrow 287 = \frac{n}{2}(1+3n-2)$

$\Rightarrow 574 = n(3n-1)$

$\Rightarrow 3n^2 - n - 574 = 0$

$\Rightarrow (3n+41)(n-14) = 0$

$\Rightarrow n = 14, \frac{-41}{3}$

Rejecting $n = \frac{-41}{3}$, we have n = 14.

Therefore, $x = 3n - 2 = 3 \times 14 - 2 = 40$.

**28.** Total probability = 36

Favourable outcomes with the sum as 11 are (5, 6) and (6, 5).

Favourable outcome with the sum as 12 is (6, 6).

P(sum 11) = $\frac{2}{36}$

P(sum 12) = $\frac{1}{36}$

P(sum $\leq$ 10) = 1 – [P(sum 11) + P(sum 12)]

$$= 1 - \left[\frac{2}{36} + \frac{1}{36}\right]$$

$$= 1 - \frac{3}{36}$$

$$= 1 - \frac{1}{12}$$

$$= \frac{11}{12}$$

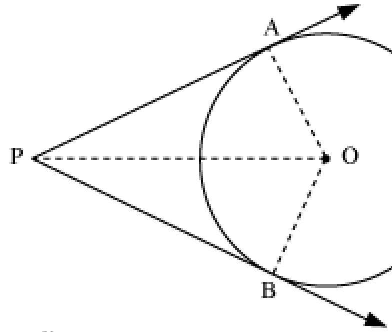
**29.** Given: A circle with centre O; PA and PB are two tangents to the circle drawn from an external point P.

To prove: PA = PB

Construction: Join OA, OB, and OP.

It is known that a tangent at any point of a circle is perpendicular to the radius through the point of contact.

$\therefore$ OA $\perp$ PA and OB $\perp$ PB ... (1)




In $\triangle$ OPA and $\triangle$ OPB,

$\angle$OAP = $\angle$OBP (Using (1))

OA = OB (Radii of the same circle)

OP = PO (Common side)

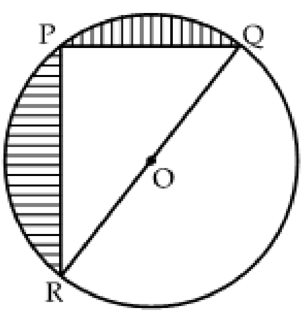
Therefore, $\triangle OPA \cong \triangle OPB$ (RHS congruency criterion)

$\therefore$ PA = PB

(Corresponding parts of congruent triangles are equal)

Thus, it is proved that the lengths of the two tangents drawn from an external point to a circle are equal.

**30.** Using Pythagoras theorem,




$QR^2 = PQ^2 + PR^2 = (7)^2 + (24)^2 = 49 + 576 = 625 = 25^2$

QR = 25 cm

$\Rightarrow r = \frac{25}{2}$ cm

Area of shaded region = Area of semicircle − area of $\triangle$ PQR

$$= \frac{1}{2}\pi\left(\frac{25}{2}\right)^2 - \frac{1}{2}\times 24\times 7$$

$$= \frac{1}{2}\times\frac{22}{7}\times\frac{625}{4} - 84$$

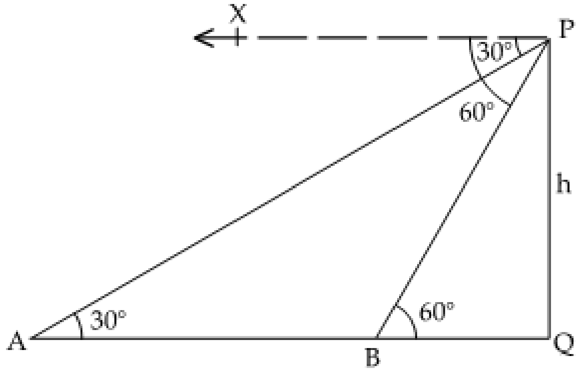
$$= 245.5 - 84 = 161.5\ cm^2$$

**31.** Let PQ = h metres be the height of the tower. P is the top of the tower.

The first and second positions of the car are at A and B respectively.

$m\angle APX = 30° \Rightarrow m\angle PAQ = 30°$

$m\angle BPX = 60° \Rightarrow m\angle PBQ = 60°$




Let the speed of the car be x m/ second

Then, distance AB = 6x meters

Let the time taken from B to Q be 'n' seconds

$\therefore$ BQ = nx metres

In $\triangle$ PAQ,

$$\frac{h}{6x+nx} = \tan 30^{o} = \frac{1}{\sqrt{3}}$$

$$\therefore h = \frac{(n+6)x}{\sqrt{3}} \qquad ----(1)$$

In $\triangle$ PBQ,

$$\frac{h}{nx} = \tan 60^{o} = \sqrt{3}$$

$$\therefore h = nx\left(\sqrt{3}\right) \qquad ----(2)$$

From (1) and (2),

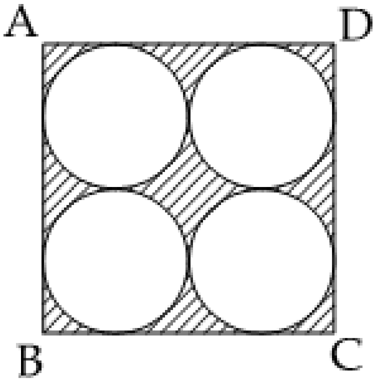
$$\frac{(n+6)x}{\sqrt{3}} = nx\left(\sqrt{3}\right)$$

$$nx + 6x = 3nx$$

$\therefore$ n = 3

Hence, the time taken by the car to reach the foot of the tower from B is 3 seconds.

**32.** Area of square = $(14\text{ cm})^2 = 196\text{ cm}^2$




Radius of each circle = $\frac{7}{2}$ cm

Area of 4 circles = $\cancel{4} \times \frac{22}{\cancel{7}} \times \frac{\cancel{7}}{\cancel{2}} \times \frac{7}{\cancel{2}} = 154\text{ cm}^2$

Area of shaded region = Area of square − Area of 4 circles

= 196 − 154 = 42 $\text{cm}^2$

**33.** Diameter of sphere = 6 cm

Radius of sphere = r = 3 cm

Radius of cylinder = R = 6 cm

Let height of water raised be h cm.

Then, volume of water thus raised = $\pi R^2 h$

$\therefore$ Volume of water raised = volume of sphere

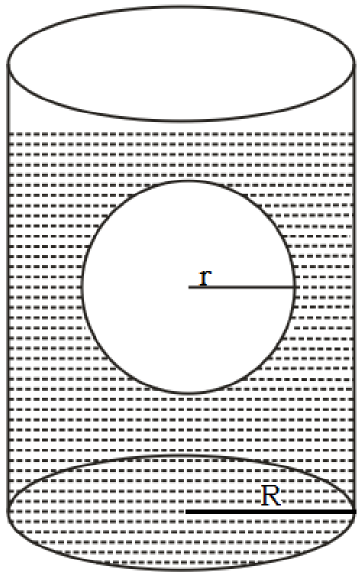
$\Rightarrow \pi R^2 h = \frac{4}{3}\pi r^3$

$\Rightarrow R^2 h = \frac{4}{3} r^3$

$\Rightarrow 36h = \frac{4}{3} \times 27$

$\Rightarrow h = 1$ cm

Therefore, water will be raised by 1 cm.




**34.** Since, OT is perpendicular bisector of PQ, therefore

PR = RQ ... (1)

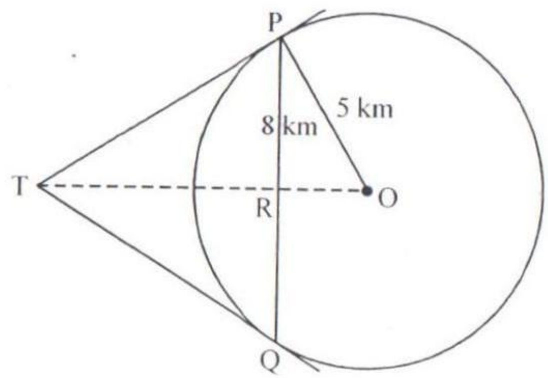
But PQ = 8 km (given)

$\Rightarrow$ PR + RQ = 8

$\Rightarrow$ PR + PR = 8 [using (1)]

$\Rightarrow$ PR = 4

$\Rightarrow$ RQ = PR = 4 km ... (2) [using (1)]




In right triangle ORP, we have:

$OP^2 = OR^2 + PR^2$

$\Rightarrow OR^2 = OP^2 - PR^2$

$\Rightarrow OR^2 = 25 - 16 = 9$

$\Rightarrow$ OR = 3 km ... (3)

Since, TP is tangent to the circle with centre O and OP is its radius, therefore,

$OP \perp TP$

[$\because$ The tangent at any point of a circle is perpendicular to the radius through the point of contact]

$\therefore m\angle OPT = 90°$

In right $\Delta$OPT, we have:

$OT^2 = PT^2 + OP^2$

$\Rightarrow (TR + OR)^2 = PT^2 + 25$

$\Rightarrow (TR + 3)^2 = PT^2 + 25$ ... (4) [Using (3)]

In right triangle PRT, we have:

$PT^2 = TR^2 + PR^2$

$\Rightarrow PT^2 = TR^2 + 16$ ... (5) [using (2)]

From (4) and (5), we have:

$(TR + 3)^2 = (TR^2 + 16) + 25$

$\Rightarrow TR^2 + 9 + 6TR = TR^2 + 41$

$\Rightarrow 6TR = 32$

$\Rightarrow TR = \frac{16}{3}$ ... (6)

Now, from (5) and (6), we get

$$PT^2 = \left(\frac{16}{3}\right)^2 + 16 = \frac{256}{9} + 16$$

$$\Rightarrow PT^2 = \frac{256+144}{9} = \frac{400}{9}$$

$$\Rightarrow TP = \frac{20}{3}$$

We know tangents drawn from an external point to a circle are equal in length.

So, TP = TQ

$\therefore$ Total length of the roads TP and TQ = $\frac{40}{3}$ km

Total cost of constructing roads = $\frac{40}{3} \times 12,000$ = Rs. 1,60,000

In order to improve the quality of roads and to control the cost, the contractors, concerned engineers and officers should be honest and persons of integrity and strong character. There should be no political interference and demands except honesty and fair control on the quality of construction. Only then will the country reach new heights.