Goa Board
Class X Mathematics
Term II
Sample Paper 9 - Solution

Section A
(Questions 1 to 8 carry 1 mark each)

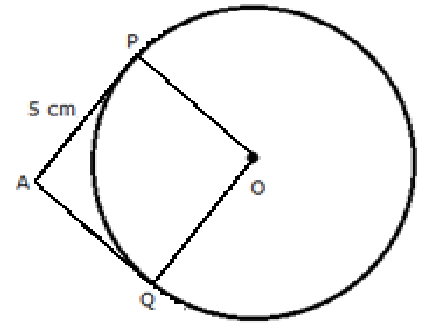
1. Correct answer: A
$2(l + b) = 20$
$l + b = 10$
Area of rectangle = 24
$lb = 24$
If l and b are the roots of the quadratic equation, then the required quadratic equation is
$x^2 - 10x + 24 = 0$

2. Correct answer: C
Let R divide the line segment joining P (−2, 2) and Q (2, 8) into two equal parts, i.e., R is the mid-point of PQ.
Co-ordinates of the mid-point of PQ, i.e., R is given by
$\left(\frac{-2+2}{2}, \frac{2+8}{2}\right) = \left(\frac{0}{2}, \frac{10}{2}\right) = (0, 5)$

3. Correct answer: B
$S_n = 3n^2 + 5n$
$a_1 = 8$
$a_1 + a_2 = 12 + 10 = 22$
So, $a_2 = 22 - 8 = 14$
$\therefore d = 6$
Let the n^{th} term of the A.P. be 164.
Then, $a_n = 164 = a + (n - 1)d$
$164 = 8 + (n - 1)6$
$\Rightarrow n - 1 = 26$
$\Rightarrow n = 27$
Thus, 164 is the 27^{th} term of the AP.

4. Correct answer: C

Join O to P and Q.



It is known that the radius is perpendicular to the tangent at the point of contact.

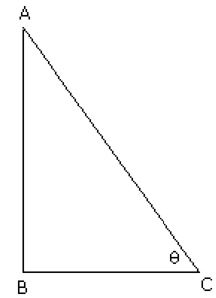
$\therefore m\angle OPA = m\angle OQA = 90°$

Thus, APOQ is a square.

$\therefore OP = OQ = 5$ cm

Thus, the radius of the circle is 5 cm.

5. Correct answer: C



In the figure, AB is the vertical tower of height 20 m. C denotes the position of the man.

Let $\angle ACB = \theta$ denotes the angle of elevation. Then $\cos\theta = 0.5 = \frac{1}{2}$, i.e.,

$\theta = 60°$

Therefore, distance of the man from the foot of the tower = BC

$$\tan 60^\circ = \frac{AB}{BC}$$

$$\sqrt{3} = \frac{20}{BC}$$

$$BC = \frac{20}{\sqrt{3}} \text{ m}$$

6. Correct answer: C

P(winning) + P(losing) = 1

0.3 + P(losing) = 1

P(losing) = 1 – 0.3 = 0.7

7. Correct answer: B

$r = \frac{14}{2}$ cm = 7 cm

Perimeter of semicircular protractor = $\pi r + 2r$

$= \frac{22}{7} \times 7 + 2 \times 7$

= 36 cm

8. Correct answer: B

Let the internal diameter of the shell be d cm. Then, its internal radius = $\frac{d}{2}$

Volume of lead obtained on melting the spherical shell = $\frac{4}{3}\pi\left[9^3-\left(\frac{d}{2}\right)^3\right]$

Volume of cylinder = $\pi \times 36 \times 8$

Hence, we have:

$$\frac{4}{3}\pi\left[9^3-\left(\frac{d}{2}\right)^3\right]=288\pi$$

$729 - \frac{d^3}{8} = 216$

$\Rightarrow \frac{d^3}{8} = 513$

$\Rightarrow d^3 = 4104$

$\Rightarrow d = 6(19)^{\frac{1}{3}}$ cm

Section B

9. Distance travelled by a wheel in 450 complete revolutions = 0.99 km = 990 m

Distance covered in one revolution = circumference

$2\pi r = \frac{990}{450}$

$2 \times \frac{22}{7} \times r = \frac{990}{450}$

$r = \frac{11\times7}{5\times2\times22} = \frac{7}{20}$ m = 35 cm

10. Points (a, 0), (0, b) and (1, 1) are collinear. So, area of triangle formed by these points will be 0.

$\therefore \frac{1}{2}[x_1(y_2 - y_3)+x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$

$[a(b - 1) + 0(1 - 0) + 1(0 - b)] = 0$

$ab - a - b = 0$

$ab = a + b$

On dividing throughout by ab, we get,

$$1=\frac{1}{a}+\frac{1}{b}$$

11. m∠ABC=90°

Since AB being diameter is perpendicular to tangent BC at the point of contact.

So m∠ABP + m∠PBC = 90° (i)

Also m∠APB = 90° (angle in the semi-circle)

So m∠BAP + m∠ABP = 90° (ii) (using angle sum property of triangles)

From (i) and (ii),

∠PBC = ∠BAP

12. Here, a = 5 and d = 10

a_{31} = a + 30d = 5 + 30 x 10 = 305

Let the required term be the n^{th} term. Then,

a_n= 130 + a_{31}

⇒ a + (n − 1)d = 130 + 305

⇒ 5 + (n − 1)10 = 435

⇒ (n − 1)10 = 430

⇒ n − 1 = 43

⇒ n = 44

Hence, the 44^{th} term of the given A.P. is 130 more than its 31^{st} term.

13. The given points are A(4, −1), B(6, 0), C(7, 2) and D(5, 1).

Using distance formula,

Diagonal AC = $\sqrt{(4-7)^2+(-1-2)^2}=\sqrt{9+9}=\sqrt{18}$

Diagonal BD = $\sqrt{(6-5)^2+(0-1)^2}=\sqrt{1+1}=\sqrt{2}$

Since, AC≠ BD, ABCD is not a square.

14.

$x^2+2\sqrt{2}x-6$

$=x^2+3\sqrt{2}x-\sqrt{2}x-6$

$=x\left(x+3\sqrt{2}\right)-\sqrt{2}\left(x+3\sqrt{2}\right)$

$=\left(x+3\sqrt{2}\right)\left(x-\sqrt{2}\right)$

Now, $x^2+2\sqrt{2}x-6=0$ gives

$\left(x+3\sqrt{2}\right)\left(x-\sqrt{2}\right)=0$

$\left(x+3\sqrt{2}\right)=0$ or $\left(x-\sqrt{2}\right)=0$

$x=-3\sqrt{2}$ or $\sqrt{2}$

Therefore, the roots of the given equation are $-3\sqrt{2}, \sqrt{2}$.

Section C

15. Let the distance between the two towers AB and CD be 140m.

$\Rightarrow$ DE = CB = 140 m

Height of the second tower CD = 60 m

Let the height of first tower, AB, be h m.

CD = BE = 60 m

$\Rightarrow$ AE = (h − 60) m

In Δ AED,

$$\frac{AE}{DE} = \tan 30^o$$

$$\frac{h-60}{140} = \frac{1}{\sqrt{3}}$$

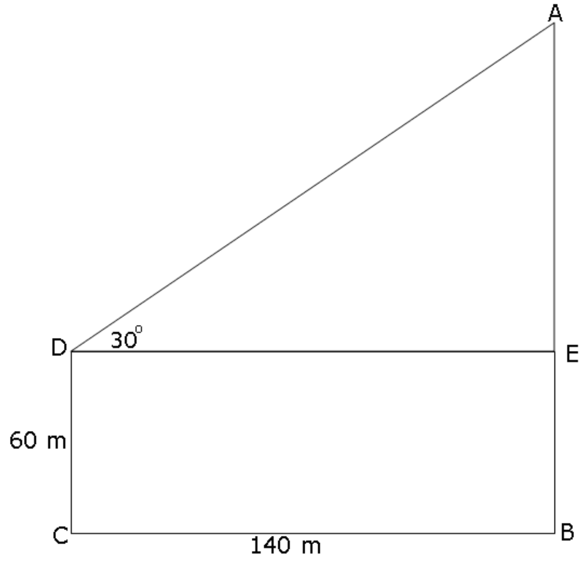
$$\Rightarrow \sqrt{3}h - 60\sqrt{3} = 140$$

$$\Rightarrow \sqrt{3}h = 140 + 60\sqrt{3}$$

$$\Rightarrow h = \frac{140+60\sqrt{3}}{\sqrt{3}} = \frac{140}{\sqrt{3}} + 60$$

$\Rightarrow$ h = 80.83 + 60 = 140.83 m

Thus, the height of the first tower is 140.83 m.



16. Radius of the well r = 1.5 m, h = 14 m

Volume of earth taken out of the well = $\pi r^2 h$

$$= \frac{22}{7} \times 1.5 \times 1.5 \times 14$$

$= 99 \text{ m}^3$

Outer radius of embankment = R = 1.5 + 4 = 5.5 m

Area of embankment = Outer area − Inner area

$= \pi(R^2 - r^2)$

$$= \frac{22}{7} \times [5.5^2 - 1.5^2]$$

$$= \frac{22}{7} \times 28$$

$= 88 \text{ m}^2$

Height of embankment = $\frac{\text{Volume}}{\text{Area}} = \frac{99}{88} = \frac{9}{8}$ = 1.125 m.

17. Join OT.

It is known that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

$\therefore m\angle OTP = 90°$

In a triangle, the measure of an exterior angle is equal to the sum of the measures of its opposite interior angles.

Therefore, in Δ PAT,

$\angle OAT = \angle APT + \angle ATP$

$\therefore \angle OAT = x + y$

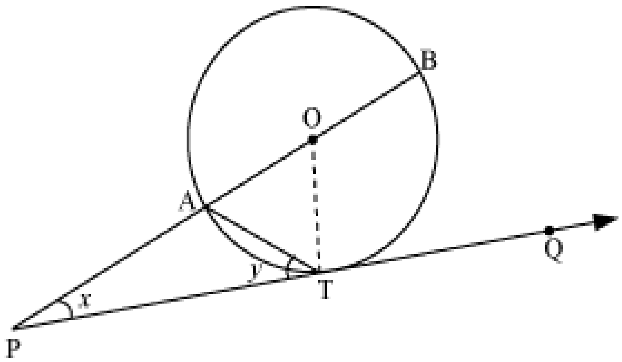
In Δ OAT, OA = OT (Radii of the same circle)

$\therefore \angle OAT = \angle OTA$

$\Rightarrow x + y = \angle OTP - \angle ATP$

$\Rightarrow x + y = 90° - y$

$\Rightarrow x + 2y = 90°$

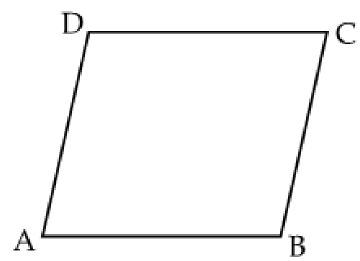


18. Area of rhombus = $\frac{1}{2}$ product of diagonals = $\frac{1}{2} \times AC \times BD$

$AC = \sqrt{(3+1)^2 + 4^2} = 32 = 4\sqrt{2}$

$BD = \sqrt{(4+2)^2 + (5+1)^2} = 6\sqrt{2}$

Area of rhombus = $\frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2} = 24$ sq. cm



19. Given, sum of first m terms of an A.P. is n.

$\therefore \frac{m}{2}[2a + (m-1)d] = n$

$\Rightarrow [2am + m(m-1)d] = 2n$ ... (1)

Also, sum of first n terms is m.

$\frac{n}{2}[2a + (n-1)d] = m$

$\Rightarrow [2an + n(n-1)] = 2m$ ... (2)

Subtracting (2) from (1),

$2a\,(m - n) + [(m^2 - n^2) - (m-n)]d = 2(n-m)$

$\Rightarrow 2a + (m + n - 1)d = -2$

$\Rightarrow \frac{m+n}{2}[2a + (m+n-1)d] = -2 \times \frac{m+n}{2} = -(m+n)$

Hence, the sum of its first (m + n) terms is −(m + n).

20. Steps of construction:

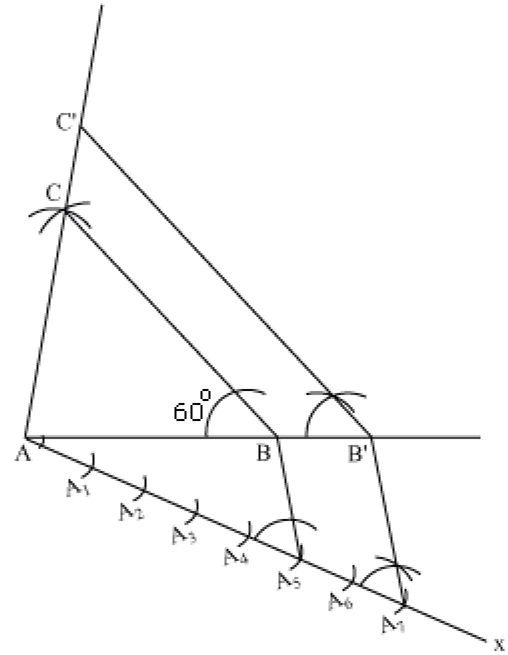
i. Draw a Δ ABC with AB = 6 cm, BC = 5 cm , m∠ABC = 60°.

ii. Draw a ray AX making an acute angle with AB on the opposite side of vertex C.

iii. Locate 7 points, A_1, A_2, A_3, A_4 A_5, A_6, A_7 (as 7 is greater between 5and 7), on line AX such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5 = A_5A_6 = A_6A_7$.

iv. Join BA_5 and draw a line through A_7 parallel to BA_5 to intersect the extended line segment AB at point B'.

v. Draw a line through B' parallel to BC intersecting the extended line segment AC at C'. Δ AB'C' is the required triangle.



21. Multiples of 6 are 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84, 90, 96, 102, 108, 114, 120, 126, 132, 138, 144, 150, 156, 162, 168, 174, 180, 186, 192, and 198.

Multiples of 8 are 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96, 104, 112, 120, 128, 136, 144, 152, 160, 168, 176, 184, 192, and 200.

Number of multiples of 6 or 8 = 33 + 25 – 8 = 50

P (chosen integer is a multiple of 6 or 8) = $\frac{50}{200} = \frac{1}{4}$

22. $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$, where x ≠ −4, 7

$\Rightarrow \frac{x-7-x-4}{(x+4)(x-7)} = \frac{11}{30}$

$\Rightarrow \frac{-11}{(x+4)(x-7)} = \frac{11}{30}$

$\Rightarrow -30 = x^2 - 7x + 4x - 28$

$\Rightarrow x^2 - 3x + 2 = 0$

$\Rightarrow (x-2)(x-1)=0$

$\therefore x = 2, 1$

Thus, the roots of given equation are 1, 2.

23. There are three sections of each class and it is given that the number of trees planted by any class is equal to the class number.

The number of trees planted by class I = number of sections $\times$ 1 = 3 $\times$ 1 = 3

The number of trees planted by class II = number of sections $\times$ 2 = 3 $\times$ 2 = 6

The number of trees planted by class III = number of sections $\times$ 3 = 3 $\times$ 3 = 9

Therefore, we have the sequence: 3, 6, 9, ..., (12 terms)

This sequence is an A.P.

To find the total number of trees planted by all the students, we need to find the sum of the 12 terms of the sequence.

First term = a = 3

Common difference = d= 6 − 3 = 3

n = 12

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{12} = \frac{12}{2}[6 + (12-1)(3)] = 6(6 + 33) = 6 \times 39 = 234$$

Thus, in total 234 trees will be planted by the students.

Values inferred are environment friendly and social awareness.

24. If the area of the triangle formed by joining the given points is zero, then the points will be collinear.

$$\text{Area of triangle} = \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$$

Here, $x_1 = a$, $y_1 = a^2$; $x_2 = b$, $y_2 = b^2$; $x_3 = c$, $y_3 = c^2$

Substituting the values in the formula, we get,

$$\text{Area of triangle} = \frac{1}{2}\left[a(b^2 - c^2) + b(c^2 - a^2) + c(a^2 - b^2)\right]$$

$$= \frac{1}{2}[ab^2 - ac^2 + bc^2 - a^2b + a^2c - cb^2]$$

$$= \frac{1}{2}[-a^2(b - c) + a(b^2 - c^2) - bc(b - c)]$$

$$= \frac{1}{2}[(b - c)\{-a^2 + a(b + c) - bc\}]$$

$$= \frac{1}{2}[(b - c)(-a^2 + ab + ac - bc)]$$

$$= \frac{1}{2}[(b - c)\{a(-a + b) + c(a - b)]$$

$$= \frac{1}{2}[(b - c)(a - b)(c - a)]$$

It is given that $a \neq b \neq c$, therefore, area of the triangle $\neq 0$

Hence, the given points can never be collinear.

Section D

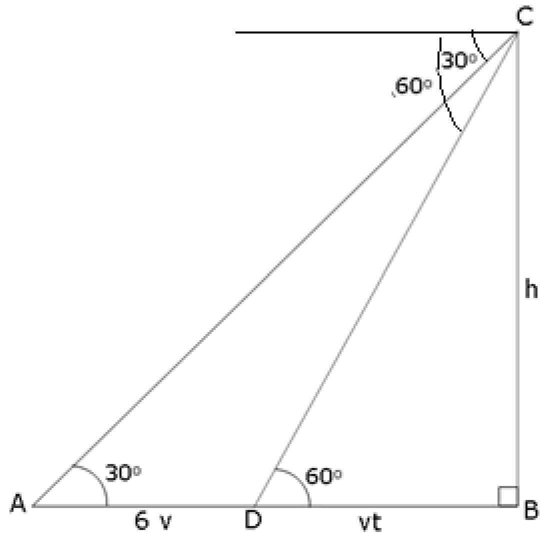
25. Let A and D be the positions of the boat at the two instants.

Let the speed of the boat be v m/ min.

Therefore, AD= 6v metres

Let the boat takes t minutes to reach from D to B.

Thus, BD= vt metres



In Δ DBC,

$$\frac{BC}{DB} = \tan 60^{o}$$

$$\Rightarrow \frac{h}{vt} = \sqrt{3} \Rightarrow h = \sqrt{3}\ vt \ ....(i)$$

In Δ ABC,

$$\frac{BC}{AB} = \tan 30^{o}$$

$$\Rightarrow \frac{h}{6v+vt} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow h = \frac{6v+vt}{\sqrt{3}} \ ....(ii)$$

From (i) and (ii),

$$\sqrt{3}\ vt = \frac{6v+vt}{\sqrt{3}} \Rightarrow 3vt = 6v + vt$$

$\Rightarrow$ 2vt = 6v

$\Rightarrow$ t = 3

Thus, the time taken to reach the shore is 3 minutes.

26. Since tangents drawn from an external point to a circle are equal.

DR = DS = 5 cm

Now, AR = AD – DR = 23 – 5 = 18 cm

But, AR = AQ

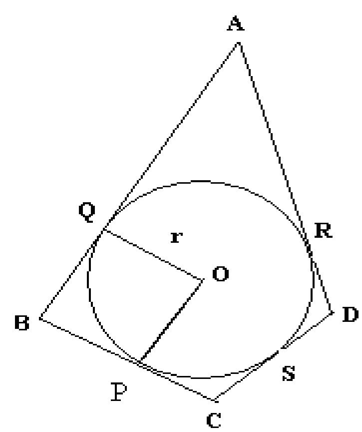
$\therefore$ AQ = 18 cm

Also, BQ = AB – AQ = 29 – 18 = 11 cm

But, BP = BQ

$\therefore$ BP = 11 cm

Also, m $\angle$ Q = m $\angle$ P = 90°.



In quadrilateral OQBP,

m$\angle$QOP + m$\angle$P + m$\angle$Q + m$\angle$B = 360°

m$\angle$QOP = 360° – ($\angle$P + $\angle$Q + $\angle$B)

= 360° – (90° + 90° + 90°) = 90°

Hence, OQBP is a square.

$\therefore$ BQ = OQ = OP = BP = 11 cm

Hence, radius of the circle is 11 cm.

27. Here, R = 33 cm, r = 27 cm and l = 10 cm

$$\therefore h = \sqrt{l^2 - \left(R^2 - r^2\right)}\ \text{cm} = \sqrt{(10)^2 - (33-27)^2}\ \text{cm}$$

$$= \sqrt{(10)^2 - (6)^2} = \sqrt{64}\,\text{cm} = 8\,\text{cm}$$

Capacity of the frustum

$$= \frac{1}{3}\pi h\left(R^2 + r^2 + Rr\right)\text{cm}^3$$

$$= \frac{1}{3}\times\frac{22}{7}\times 8\left[(33)^2 + (27)^2 + 33\times 27\right]\text{cm}^3$$

$$= (8.38\times 2709)\text{cm}^3 = 22701.4\ \text{cm}^3$$

Total surface area = $\left[\pi R^2 + \pi r^2 + \pi l(R+r)\right]\text{cm}^2$

$$= \pi\left[R^2 + r^2 + l(R+r)\right]\text{cm}^2$$

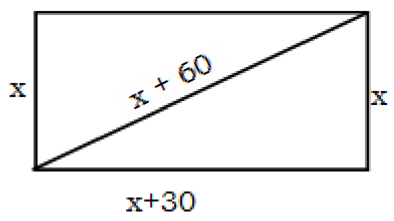
$$= \frac{22}{7}\left[(33)^2 + (27)^2 + 10\times(33+27)\right]\text{cm}^2$$

$$= \left(\frac{22}{7}\times 2418\right)\text{cm}^2 = 7599.43\,\text{cm}^2$$

28. Let the shorter side be x.

Diagonal = x + 60

Longer side = x + 30



By applying Pythagoras theorem,

$(x + 30)^2 + x^2 = (x + 60)^2$

$\Rightarrow x^2 + 60x + 900 + x^2 = x^2 + 3600 + 120x$

$\Rightarrow x^2 - 60x - 2700 = 0$

D= 3600 − 4(1)(−2700) = 3600 + 10800 = 14400

$\sqrt{D} = 120$

$$\therefore x = \frac{60 \pm 120}{2}$$

$\Rightarrow x = 90, -30$

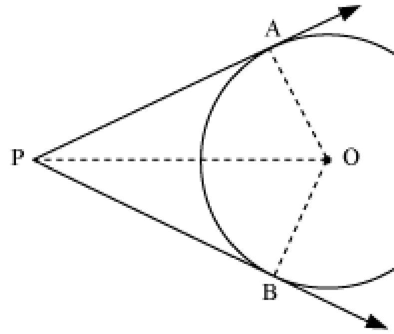
But, x cannot be negative. So, x = 90

Thus, shorter side = 90 m, longer side= 120 m

29. Given: A circle with centre O; PA and PB are two tangents to the circle drawn from an external point P.

To prove: PA = PB

Construction: Join OA, OB, and OP.



Proof:

It is known that a tangent at any point of a circle is perpendicular to the radius through the point of contact.

$\therefore$ OA $\perp$ PA and OB $\perp$ PB

$\angle$OAP = $\angle$OBP = 90° ... (1)

In ΔOPA and ΔOPB,

$\angle$OAP = $\angle$OBP (Using (1))

OA = OB (Radii of the same circle)

OP = PO (Common side)

Therefore, ΔOPA $\cong$ ΔOPB (RHS congruency criterion)

$\therefore$ PA = PB (Corresponding parts of congruent triangles are equal)

Thus, it is proved that the lengths of the two tangents drawn from an external point to a circle are equal.

30. Total number of cards = 52

Cards removed = 2 + 2 + 2 + 2 = 8

$\therefore$ Remaining number of cards = 52 – 8 = 44

i. Number of black queens = 2

$\therefore$ Required probability = $\frac{2}{44} = \frac{1}{22}$

ii. Number of red cards left = 26 – 8 = 18

$\therefore$ Required probability = $\frac{18}{44} = \frac{9}{22}$

iii. Number of black jacks = 2

$\therefore$ Required probability = $\frac{2}{44} = \frac{1}{22}$

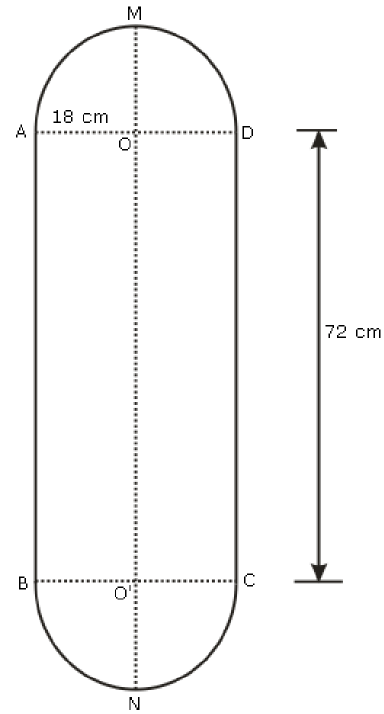
iv. Number of picture cards left = 2 + 2 + 2 = 6

$\therefore$ Required probability = $\frac{6}{44} = \frac{3}{22}$

31. Let r = radius of the cylinder = radius of hemispherical ends = 18 cm

h = height of the cylinder = 108 cm − 18 cm − 18 cm

= 72 cm



Total surface area of the solid = Curved surface area of the cylinder + Curved surface area of the hemispherical ends = $(2\pi rh + 2x\ 2\pi r^2)$

$= 2\pi r(h + 2r)$

$= 2 \times \frac{22}{7} \times 18 \times (72 + 36)$

$= 2 \times \frac{22}{7} \times 18 \times 108 = 12219.43\ \text{cm}^2$

Rate of polishing = 7 paise per sq cm

$\therefore$ Cost of polishing = Rs. $\left(12219.43 \times \frac{7}{100}\right)$

= Rs. 855.36

32. AB = AL + LB = 4 + 3 = 7, AC = AM + MC = 4 + 3 = 7

The area of the right ΔABC is given by $\frac{1}{2} \times AB \times AC = \frac{1}{2} \times 7 \times 7 = 24.5\ \text{cm}^2$

The area of the quadrant ALM

$= \frac{1}{4} \times \pi \times (4)^2$

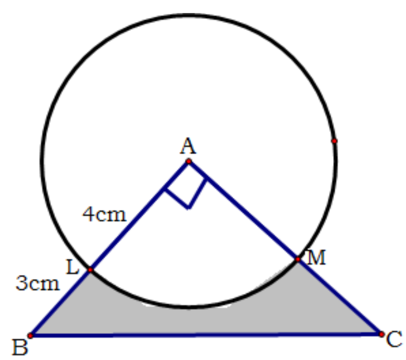
$= \frac{1}{4} \times \frac{22}{7} \times 16 = 12.57\ \text{cm}^2$

Area(LBCM) = Ar(Δ ABC) − Ar(quadrant ALM)

= 24.5 − 12.57 = 12.93 cm^2

At the rate of Rs. 20 per cm^2, the total cost of silver plating

= Rs. 20 × 12.93 = Rs. 258.6



33. Here, A.P. (in rupees) is 6500, 6390, 6280, ……

The show will cease to be profitable on the night when the receipts are just Rs 1000. Let it happen on the n^{th} night. It means $a_n \leq 1000$.

Here, a = 6500, d = −110

Now, $a_n = a + (n - 1)d = 6500 - 110(n - 1)$

Equating it with 1000, we get,

6500 − 110(n − 1) = 1000

⇒ 110(n − 1) = 5500

⇒ n − 1 = 50

⇒ n = 51

Hence, on the 51^{st} night the show will cease to be profitable.

34. Area of sectors = $\frac{\angle A}{360^o}\times \pi r^2+\frac{\angle B}{360^o}\times \pi r^2+\frac{\angle C}{360^o}\times \pi r^2$

$=\frac{3.14\times 25}{360^o}\left(\angle A+\angle B+\angle C\right)$

$=\frac{3.14\times 25}{360^o}\times 180^o$

= 39.25 cm^2

Since, 14 cm, 48 cm and 50 cm marks a Pythagorean triplet, it is a right triangle.

Area of ΔABC = $\frac{1}{2}\times 48\times 14=336\ cm^2$

∴ Area of shaded portion = (336 – 39.25) cm^2

= 296.75 cm^2