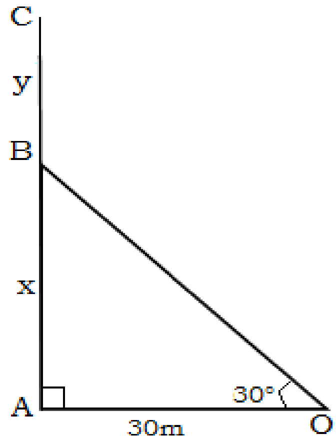
Goa Board
Class X Mathematics
Term II
Sample Paper 8 - Solution

Section A
(Questions 1 to 8 carry 1 mark each)

1. Correct answer: A
Two tangents of a circle are parallel if they are drawn at the end points of a diameter.
Therefore, distance between them is the diameter of the circle = 2 x 5 cm = 10 cm

2. Correct answer: D
The range of P(A) = $0 \leq P(A) \leq 1$.

3. Correct answer: B
Let AB be the tree broken at a point C such that the broken part CB takes the position CO and touches the ground at O. OA = 30 m, m$\angle$AOC = 30°. Let AC = x and BC = CO = y.
In ΔAOC,



$\tan 30^o = \frac{AC}{OA}$

$\frac{1}{\sqrt{3}} = \frac{x}{30}$

$x = 10\sqrt{3}$

Again, in ΔAOC,

$\cos 30^o = \frac{OA}{OC}$

$\frac{\sqrt{3}}{2} = \frac{30}{y}$

$y = 20\sqrt{3}$

Height of the tree = (x + y) = $10\sqrt{3} + 20\sqrt{3}$ = $30\sqrt{3}$ metres

4. Correct answer: B
AB is divided into 3 equal parts at P and Q.

$\therefore$ AB : AQ = 3 : 2

$\Rightarrow AB = \frac{3}{2}\ AQ$

Thus, the value of x is $\frac{3}{2}$.

5. Correct answer: A

Given, $2\pi r = \pi r^2$

$r = 2$

Thus, the radius of the circle is 2 units.

6. Correct answer: D

Let x be the edge of the cube. Then, x is also the diameter of the sphere.

Ratio of the volume of the cube to that of the sphere

$= x^3 : \frac{4}{3} \times \pi \times \frac{x^3}{8}$

$= 1 : \frac{4\pi}{24} = 6 : \pi$

7. Correct answer: B

Given that the first and last terms of an A.P. are 1 and 11, i.e., a = 1 and l = 11.

Let the sum of its n terms is 36, then,

$S_n = \frac{n}{2} \times (a + l)$

$36 = \frac{n}{2} \times (1 + 11)$

$n = \frac{36}{6} = 6$

Thus, the number of terms in the A.P. is 6.

8. Correct answer: D

The points A and B lie on the x and y-axis respectively. Let the co-ordinates of A and B be (x, 0) and (0, y) respectively.

It is given that (4, −3) is the mid-point of AB. By mid-point formula,

$4 = \frac{x+0}{2}$ and $-3 = \frac{0+y}{2}$

$x = 8$ and $y = -6$

Thus, the co-ordinates of points A and B are (8, 0) and (0, −6) respectively.

Section B

9. The given quadratic equation is $2x^2 - \sqrt{5}x - 2 = 0$.

$b^2 - 4ac = 5 - 4 \times 2 \times (-2) = 21$

The roots of the given equation are given as:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{\sqrt{5} \pm \sqrt{21}}{4}$$

Thus, the roots of the given equation are $\frac{\sqrt{5}+\sqrt{21}}{4}$ and $\frac{\sqrt{5}-\sqrt{21}}{4}$

10. In given figure, O is the centre of the circle.

Therefore, $m\angle OQL = m\angle ORM = 90°$

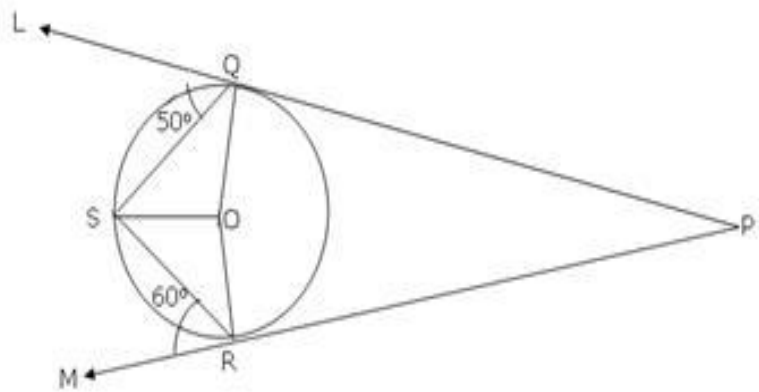
(Radius is perpendicular to tangent at the point of contact)

$m\angle OSQ = m\angle OQS = 90° - 50° = 40°$

$m\angle RSO = m\angle SRO = 90° - 60° = 30°$

Thus, $m\angle QSR = m\angle OSQ + m\angle OSR$

$= 40° + 30° = 70°$



11. We know that three terms p, q and r form consecutive terms of an A.P. if and only if $2q = p + r$

Thus, $2k + 7$, $6k - 2$ and $8k + 4$ will form consecutive terms of an A.P. if

$2(6k - 2) = (2k + 7) + (8k + 4)$

$\Rightarrow 12k - 4 = 10k + 11$

$\Rightarrow 2k = 15$

$\Rightarrow k = \frac{15}{2}$

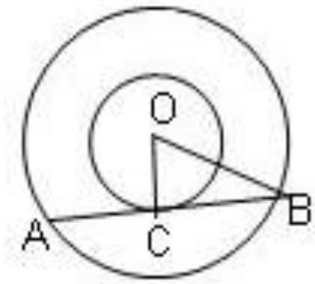
12. Let AB = c be a chord of the larger circle of diameter d_2, which touches the other circle at C. Then ΔOCB is a right triangle.

By Pythagoras theorem,

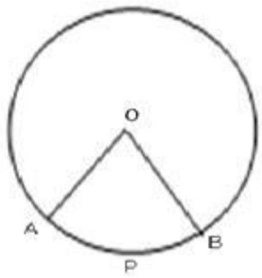
$OC^2 + BC^2 = OB^2$

i.e., $\left(\frac{1}{2}d_1\right)^2 + \left(\frac{1}{2}c\right)^2 = \left(\frac{1}{2}d_2\right)^2$ (as C bisects AB)

Therefore, $d_2^2 = c^2 + d_1^2$



13. Area of sector OAPB = $\frac{\pi r^2\theta}{360} = \frac{22}{7} \times \frac{10x10x90}{360} = \frac{550}{7}$



Area of major sector = area of circle – area of sector OAPB

$= \pi r^2 - \frac{550}{7}$

$= \frac{22}{7}$ x 10 x 10 – $\frac{550}{7}$

$= \left(\frac{2200}{7} - \frac{550}{7}\right) = \frac{1650}{7}$ cm^2

14. Required shaded region = area of two quadrants – area of square

$= 2\left(\frac{1}{4} \times \pi \times 8^2\right) - 8 \times 8$

$= \frac{1}{2}\left(\frac{22}{7} \times 64\right) - 64$

$= \frac{4}{7}$ x 64 = $\frac{256}{7}$ cm^2

Section C

15. The given quadratic equation is $2x^2 - 10x + k = 0$.
Here, a = 2, b = −10 and c = k
Therefore, D = $b^2 - 4ac = (-10)^2 - 4 \times 2 \times k = 100 - 8k$
The equation will have real and equal roots, if

$D = 0 \Rightarrow 100 - 8k = 0 \Rightarrow k = \frac{100}{8} = \frac{25}{2}$

16. In a non-leap year, there are 365 days, i.e., 52 weeks.
52 weeks = 364 days
1 year = 52 weeks and 1 day
This extra one day can be Mon, Tue, Wed, Thu, Fri, Sat or Sun.
Total number of outcomes = 7
Number of favourable outcomes = 1

P(having 53 Thursdays) = $\frac{1}{7}$

17. If three points A, B and C are collinear, then area of ΔABC = 0.

$\Rightarrow \frac{1}{2}[7(1 - k) + 5(k + 2) + 3(-2 - 1)] = 0$

$\Rightarrow 7 - 7k + 5k + 10 - 9 = 0$

$\Rightarrow 2k + 8 = 0$

$\Rightarrow k = -4$

Thus, the given points are collinear for k = −4.

18. Co-ordinates of the mid-point of the line segment joining A (3, 4) and B (k, 6) =

$\left(\frac{3+k}{2}, \frac{4+6}{2}\right) = \left(\frac{3+k}{2}, 5\right)$

$\therefore \left(\frac{3+k}{2}, 5\right) = (x, y)$

$\left(\frac{3+k}{2}\right) = x$ and 5 = y

Since, x + y − 10 = 0

So, $\frac{3+k}{2} + 5 - 10 = 0$

3 + k = 10

k = 7

19. Let a − d, a and a + d be three terms in A.P.

According to the question,

a − d + a + a + d = 3

3a = 3 or a = 1

(a − d) (a) (a + d) = −8

$a(a^2 - d^2) = -8$

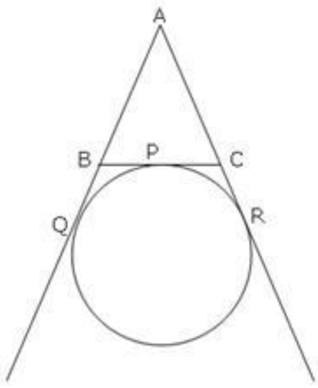
Putting the value of a = 1, we get,

$1 - d^2 = -8$

$d^2 = 9$ or d = ±3

Thus, the required three terms are −2, 1, 4 or 4, 1, −2.

20. BQ = BP (tangents drawn from an external point to a circle are equal)



Similarly, CP = CR, and AQ = AR

2AQ = AQ + AR

= (AB + BQ) + (AC + CR)

= AB + BP + AC + CP

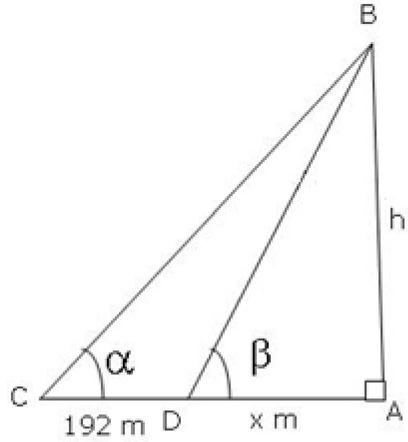
= (BP + CP) + AC + AB

2AQ = BC + CA + AB

$AQ = \frac{1}{2}(BC + CA + AB)$

21. Let AB be the tower of height h metres. Let AD = x metres, CD = 192 metres.

$\tan\alpha = \frac{5}{12}$, $\tan\beta = \frac{3}{4}$



In ΔBAC,

$\tan\alpha = \frac{AB}{AC} \Rightarrow \frac{5}{12} = \frac{h}{(x+192)} =$ .... (i)

In ΔDAB,

$\tan\beta = \frac{AB}{AD} \Rightarrow \frac{3}{4} = \frac{h}{x}$ or $x = \frac{4h}{3}$ ....(ii)

Using (ii) in (i)

$$\frac{5}{12} = \left(\frac{h}{192 + \frac{4h}{3}}\right)$$

$$5\left(192 + \frac{4h}{3}\right) = 12h$$

2880 + 20h = 36h

16h = 2880

h = 180

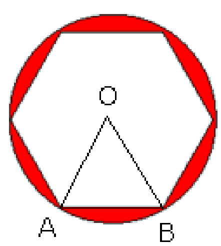
Hence, the height of the tower is 180 metres.

22. AOB is an equilateral triangle.

Area of one design = $\frac{60}{360} \times \pi \times 28^2$ – area of Δ OAB

$$= \pi \times \frac{28^2}{6} - \frac{\sqrt{3}}{4} \times 28^2$$

$$= 28^2\left(\frac{11}{21} - \frac{\sqrt{3}}{4}\right) cm^2$$



Total cost of making the design = Rs. $6 \times 28^2 \left(\frac{11}{21} - \frac{\sqrt{3}}{4}\right) \times 0.35$

= Rs. 6 × 28 × 28 × 0.091 × 0.35= Rs. 149.8224

23. The cone of maximum size that is carved out from a cube of edge 14 cm cm will be of base radius 7 cm and the height 14 cm.

Total Surface area of the cone = $\pi rl + \pi r^2$

$$= \frac{22}{7} \times 7 \times \sqrt{7^2 + 14^2} + \frac{22}{7}(7)^2$$

$$= \frac{22}{7} \times 7 \times \sqrt{245} + 154$$

$$= (154\sqrt{5} + 154)$$

$$= 154(\sqrt{5} + 1)\text{cm}^2$$

Surface area of the cube = 6 x $(14)^2$ = 6 x 196 = 1176 cm^2

So, surface area of the remaining solid left out after the cone is carved out = Surface area of the cube - surface area of the circle + Curved surface area of the cone = 1176 – 154 + $154\sqrt{5}$ = 1022 + $154\sqrt{5}$ cm^2

24. Width of the canal = 300 cm = 3 m

Depth of the canal = 120 cm = 1.2 m

Volume of water that flows through the canal in one hour

= width of the canal x depth of the canal x speed of the canal water

= 3 x 1.2 x 20 x 1000 m^3 = 72000 m^3

In 20 minutes, the volume of water in the canal

= 72000 x $\frac{20}{60}$ m^3 = 24000m^3

Area irrigated in 20 minutes, if 8 cm, i.e., 0.08 m standing water is required

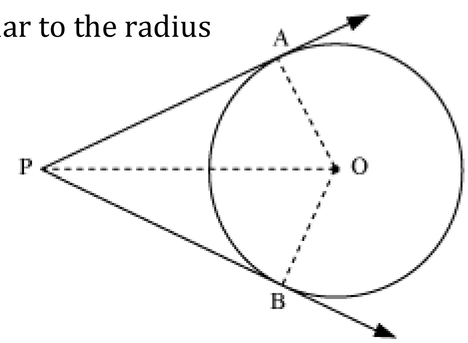
= $\frac{24000}{0.08}$ m^2= 300000 m^2 = 30 hectares

Section D

25. Let the larger tap fill the tank in x hours.
Then, the smaller tap will fill the tank in (x + 10) hours.

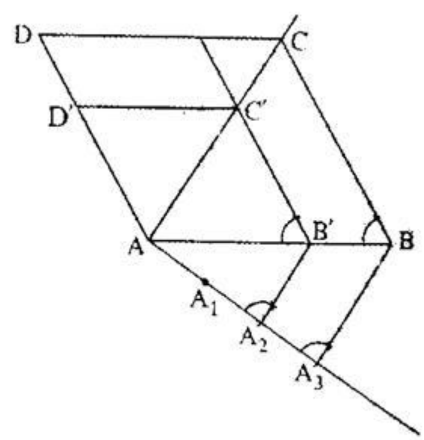
$$\therefore \frac{1}{x}+\frac{1}{x+10}=\frac{8}{75}$$
$$\Rightarrow \frac{2x+10}{x^2+10x}=\frac{8}{75}$$
$$\Rightarrow (2x+10)75=8(x^2+10x)$$
$$\Rightarrow 8x^2-70x-750=0$$
$$\Rightarrow 4x^2-35x-375=0$$
$$x=\frac{35\pm\sqrt{1225+6000}}{8}=\frac{35\pm 85}{8}$$
= 15 (rejecting negative value)
$\therefore$ The larger tap can fill the tank in 15 hours and the smaller tap can fill it in 25 hours.

26. Given: A circle with centre O; PA and PB are two tangents to the circle drawn from an external point P.
To prove: PA = PB
Construction: Join OA, OB, and OP.



We know that a tangent at any point of a circle is perpendicular to the radius through the point of contact.
$OA \perp PA$ and $OB \perp PB$... (1)
In ΔOPA and ΔOPB:
$m\angle OAP = m\angle OBP = 90^{o}$ (Using (1))
OP = PO (Common side)
OA = OB (Radii of the same circle)
Therefore, $\Delta OPA \cong \Delta OPB$ (RHS congruency criterion)
$\therefore$ PA = PB (Corresponding parts of congruent triangles are equal)
Thus, it is proved that the lengths of the two tangents drawn from an external point to a circle are equal.

27. From the figure:



$$\frac{AB'}{AB}=\frac{2}{3}=\frac{AC'}{AC}$$
Also, $\frac{AC'}{AC}=\frac{C'D'}{CD}=\frac{AD'}{AD}=\frac{2}{3}$

Therefore, AB' = B'C' = C'D' = AD' = $\frac{2}{3}$AB

Thus, AB'C'D' is a rhombus.

28. Let number of visitors to the zoo on 1st November be x. Then the daily visitors in November in the zoo are: x, x + 20 ...

This will form an A.P. with the first term x and common difference 20.

Total no. of visitors in Nov. = 12300 (Given)

$\therefore S_{30} = 12300$

$$S_{30} = \frac{n}{2}[2a+(n-1)d] = \frac{30}{2}\left[2x+(29)(20)\right]$$

$12300 = 15\,(2x + 29 \times 20)$

$= 30x + 8700$

$x = 120$

$\therefore$ Number of visitors on 1st Nov. 2009 = 120

29. Capacity (or volume) of the bucket = $\frac{\pi h}{3}[r^2_1 + r^2_2 + r_1 r_2]$

Here, h = 30 cm, r_1 = 20 cm and r_2 = 10 cm

So, the capacity of bucket = $3.14 \times \frac{30}{3}[20^2 + 10^2 + 20 \times 10]$ cm^3

= 21.980 liters

Cost of 1 litre of milk = Rs. 25

Cost of 21.980 litres of milk = Rs. 21.980 x 25 = Rs 549.50

Surface area of the bucket = Curved surface area of the bucket + Surface area of the bottom

$= \pi l(r_1 + r_2) + \pi r^2_2$

Now, $l = \sqrt{h^2 + (r_1 - r_2)^2}$

$l = \sqrt{900 + 100}$ cm = 31.62 cm

Therefore, surface area of the bucket

$= 3.14 \times 31.62\,(20+10) + 3.14 \times (10)^2$

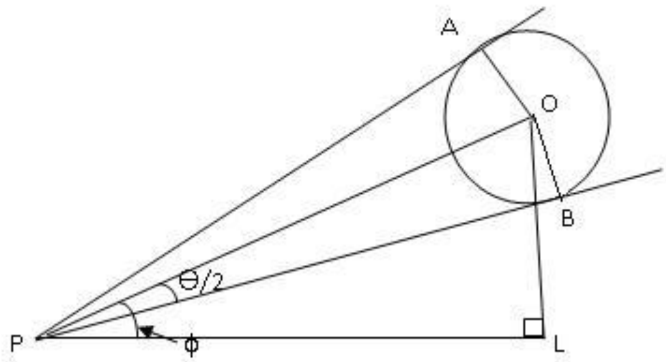
$= 3.14 \times 1048.6$ cm^2 = 3292.6 cm^2(approx.)

30. Let O be the centre of the balloon of radius r and P the eye of the observer. Let PA and PB be tangents from P to the balloon. $\angle APB = \theta$.

Therefore, $\angle APO = \angle BPO = \frac{\theta}{2}$

Let OL be perpendicular from O to the horizontal.

$\angle OPL = \phi$



In ΔOAP,

$\sin\frac{\theta}{2} = \frac{OA}{OP} = \frac{r}{OP}$

$\Rightarrow OP = r\,\text{cosec}\frac{\theta}{2}$ ...(i)

In ΔOPL,

$\sin\phi = \frac{OL}{OP}$

$\Rightarrow OL = OP\sin\phi$

$\Rightarrow OL = r\sin\phi\ \text{cosec}\frac{\theta}{2}$ (from (i))

Thus, the height of the centre of the balloon is $r\sin\phi\ \text{cosec}\frac{\theta}{2}$.

31. Let unit's place digit = x and ten's place digit = $\frac{18}{x}$

$\therefore$ Number = (1) (x) + (10) $\left(\frac{18}{x}\right) = x + \frac{180}{x}$ ...(i)

Interchanged Number = $10x + 1\left(\frac{18}{x}\right)$

Given,

$\left(x+\frac{180}{x}\right) - 63 = 10x + \frac{18}{x}$

$\Rightarrow \frac{x^2+180-63x}{x} = \frac{10x^2+18}{x}$

$\Rightarrow \frac{x^2+180-63x}{x} - \frac{10x^2+18}{x} = 0$

$\Rightarrow x^2 + 180 - 63x - 10x^2 - 18 = 0$

$\Rightarrow -9x^2 - 63x + 162 = 0$

$\Rightarrow x^2 + 7x - 18 = 0$

$\Rightarrow (x + 9)(x - 2) = 0 \Rightarrow x = -9$ or 2

$\Rightarrow x = 2$ (Rejecting $x = -9$ as digit is not –ve)

From (i), Number = $x + \frac{180}{x} = 2 + \frac{180}{2} = 2 + 90 = 92$

32. Length of the cylinder = 24 cm

Diameter of the copper wire = 4 mm

One round of the wire will cover the surface of cylinder by 4 mm

Therefore, the number of rounds of wire to cover the length of cylinder

$$=\frac{\text{length of cylinder}}{\text{thickness of wire}}=\frac{24\text{cm}}{4\text{mm}}=\frac{240}{4}=60$$

Now, diameter of cylinder = 20 cm

Therefore, length of wire in one round

= circumference of base of the cylinder = πd

$$=\frac{22}{7}\times 20=\frac{440}{7}\text{cm}$$

Length of wire for covering the whole surface of cylinder = length of wire in 60 rounds = $60\times\frac{440}{7}=3771.428$ cm

Radius of copper wire = $\frac{0.4}{2}$ = 0.2 cm

Therefore, volume of wire = $\pi r^2 h$

$$=\frac{22}{7}\times(0.2)^2\times 3771.428$$

= 474.122 cu cm

Weight of wire = Volume × Density = 474.122 × 8.68 = 4115.37 gm = 4.11537 kg

33.

a) Probability of safe landing= $\frac{\text{Area of Safe Land A + Area of Safe Land B}}{\text{Total area}}$

$$=\frac{\pi(1\text{m})^2+3\text{m}\times 2\text{m}}{6\text{m}\times 9\text{m}}$$

$$=\frac{\pi+6}{54}$$

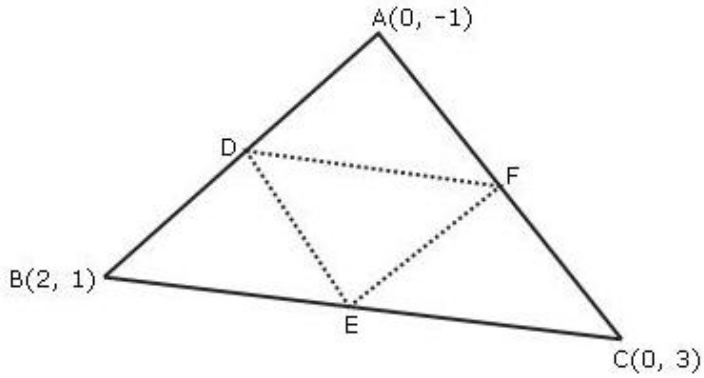
b) Probability of landing in jungle= $\frac{\text{Area of Jungle}}{\text{Total area}}$

$$=\frac{54\text{m}^2\left[\pi(1\text{m})^2+3\text{m}\times 2\text{m}\right]}{6\text{m}\times 9\text{m}}$$

$$=\frac{54-[\pi+6]}{54}$$

c) Leadership

34. D, E and F are the mid-points of AB, BC and AC respectively.



Co-ordinates of D = $\left(\frac{0+2}{2}, \frac{-1+1}{2}\right) = (1, 0)$

Co-ordinates of E = $\left(\frac{2+0}{2}, \frac{1+3}{2}\right) = (1, 2)$

Co-ordinates of F = $\left(\frac{0+0}{2}, \frac{3-1}{2}\right) = (0, 1)$

Since, area of a triangle = $\frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$

Therefore, area $\triangle$ABC

$= \frac{1}{2}\left[0(1-3) + 2(3+1) + 0(-1-1)\right]$

$= \frac{1}{2}[0+8+0]$

$= 4$ sq units

Now, area $\triangle$DEF

$= \frac{1}{2}\left[1(2-1) + 1(1-0) + 0(0-2)\right]$

$= \frac{1}{2}\left[1+1+0\right]$

$= 1$ sq unit

Therefore, $\frac{\text{area } \Delta\text{DEF}}{\text{area } \Delta\text{ABC}} = \frac{1}{4} = 1:4$