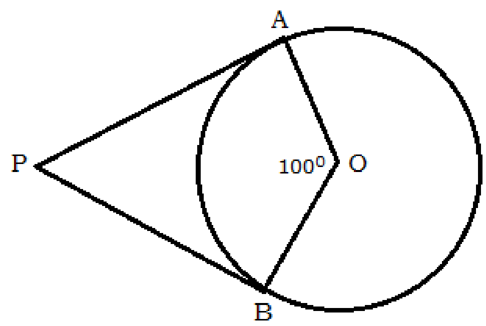
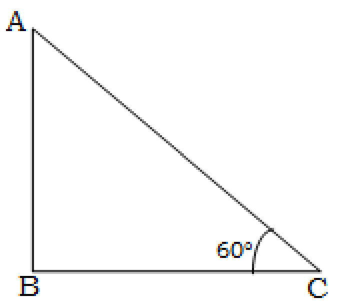
Goa Board
Class X Mathematics
Term II
Sample Paper 5 - Solution

Section A
(Questions 1 to 8 carry 1 mark each)

1. Correct answer: C
We know that radius is perpendicular to the point of contact of tangents.
$m\angle PAO = 90°$ and $m\angle PBO = 90°$
Using angle sum property of quadrilaterals, we get,
$m\angle P + m\angle O = 180°$
$m\angle P + 100° = 180°$
$\therefore m\angle P = 80°$



2. Correct answer: A
Let AB be the tower.
BC = Length of shadow = 30 m
$\tan 60° = \frac{AB}{BC}$
$\sqrt{3} = \frac{AB}{30}$
$AB = 30\sqrt{3}$
Thus, height of the tower = $30\sqrt{3}$ m



3. Correct answer: C
Let A(0, 4), B(0, 0), and C(3, 0) be the given points.
Using distance formula, we have:
$AB^2 = (0 - 0)^2 + (0 - 4)^2 \Rightarrow AB = \sqrt{16} = 4$
$BC^2 = (3 - 0)^2 + (0 - 0)^2 \Rightarrow BC = \sqrt{9} = 3$
$AC^2 = (3 - 0)^2 + (0 - 4)^2 \Rightarrow AC = \sqrt{25} = 5$
Thus, perimeter = AB + BC + AC = 4 + 3 + 5 = 12 units

4. Correct answer: B
For the given terms to be in A.P.
2(2a + 1) = (a + 1) + (4a – 1)
4a + 2 = 5a
a = 2

5. Correct answer: D

Using section formula,

$$\text{Co-ordinates of Point} = \left(\frac{1\times 3+2\times 7}{1+2}, \frac{1\times 4+2\times(-6)}{1+2}\right)$$

$$= \left(\frac{17}{3}, -\frac{8}{3}\right)$$

The point lies in the fourth quadrant.

6. Correct answer: C

Total possible outcomes = 89

Number of cards bearing two-digit numbers = 81

P(drawing two-digit numbered card) = $\frac{81}{89}$

7. Correct answer: B

For equation $kx^2 - 6x - 2 = 0$ to have equal roots:

$$b^2 - 4ac = 0$$

$$\Rightarrow (-6)^2 - 4(k)(-2) = 0$$

$$\Rightarrow 36 + 8k = 0$$

$$\Rightarrow 8k = -36$$

$$\Rightarrow k = -\frac{9}{2}$$

8. Correct answer: A

Given: $2\pi r = 22$ cm

$$\Rightarrow r = \frac{7}{2}\text{ cm}$$

$$\text{Area of quadrant} = \frac{\theta}{360^o}\times \pi r^2 = \frac{90^o}{360^o}\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2} = \frac{77}{8}\text{cm}^2$$

Section B

9. $10ax^2 - 6x + 15ax - 9 = 0$

$2x(5ax - 3) + 3(5ax - 3) = 0$

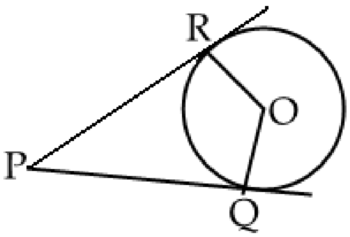
$(5ax - 3)(2x + 3) = 0$

$5ax - 3 = 0$ or $2x + 3 = 0$

$5ax = 3$ or $2x = -3$

$x = \frac{3}{5a}$ or $x = \frac{-3}{2}$

10. It can be observed that:



OR (radius) $\perp$ PR (tangent)

Therefore, $m\angle ORP = 90°$

Similarly, OQ (radius) $\perp$ PQ (tangent)

$m\angle OQP = 90°$

In quadrilateral ORPQ,

Sum of all interior angles = 360°

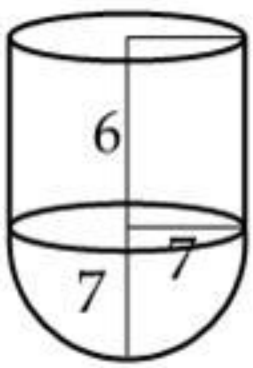
$m\angle ORP + m\angle RPQ + m\angle PQO + m\angle QOR = 360°$

$90° + m\angle RPQ + 90° + m\angle QOR = 360°$

$m\angle RPQ + m\angle QOR = 180°$

Hence, PROQ is a cyclic quadrilateral.

11. Radius of hemisphere = Radius of cylinder = 7 cm



Total height of the vessel = 13 cm

Height of cylinder = 13 cm – 7 cm = 6 cm

Surface Area = $2\pi r^2 + 2\pi rh$

= $2\pi r[r + h]$

= $2 \times \frac{22}{7} \times 7(7 + 6)$

= $44 \times 13 = 572$ cm^2

12. Let P(x, y) be equidistant from the points A(7, 1) and B(3, 5).

Given, AP = BP

$AP^2 = BP^2$

By distance formula,

$(x - 7)^2 + (y - 1)^2 = (x - 3)^2 + (y - 5)^2$

$x^2 + 49 - 14x + y^2 + 1 - 2y = x^2 + 9 - 6x + y^2 + 25 - 10y$

$-8x + 8y + 16 = 0$

$-x + y + 2 = 0$

$x - y = 2$

This is the required relation between x and y.

13. $r = \frac{1}{2}$cm , l = 8 cm

Volume of rod = $\pi\left(\frac{1}{2}\right)^2 \times 8$

Length of wire = 18 m = 1800 cm

Let r be the radius of the cross-section of the wire.

Volume = $\pi \times r^2 \times 1800$

$\pi\left(\frac{1}{2}\right)^2 \times 8 = \pi \times r^2 \times 1800 \ \therefore r^2 = \frac{1}{900} \Rightarrow r = \frac{1}{30}$

Thus, diameter of cross-section, i.e., thickness of the wire is

$2r = \frac{1}{15}$cm

14. First 40 positive integers divisible by 6 are 6, 12, 18, ...

This is an A.P. with a = 6 and d = 6

$S_{40} = \frac{n}{2}\left[2a + (n-1)d\right]$

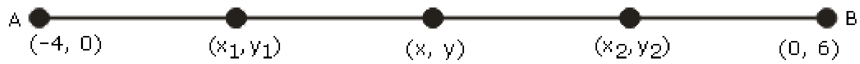
S_{40}= 20[2(6) + (40 – 1)6] = 20[12 + 234] = 4920

Section C

15. Given points are A(−4, 0) and B(0, 6). Let the mid-point of A and B be (x, y).

Then, $x=\frac{-4+0}{2}=-2$ and $y=\frac{0+6}{2}=3$ = −2

Therefore, the co-ordinates of the mid-point of AB is (−2, 3).



Now, the co-ordinates of the mid-point of (-4, 0) and (-2, 3) will be

$x_1=\frac{-4-2}{2}=-3$ and $y_1=\frac{0+3}{2}=\frac{3}{2}$ i.e. $\left(-3, \frac{3}{2}\right)$

Again, the co-ordinates of the mid-point of (−2, 3) and (0, 6) will be

$x_2=\frac{-2+0}{2}=-1$ and $y_2=\frac{3+6}{2}=\frac{9}{2}$ i.e. $\left(-1, \frac{9}{2}\right)$

Therefore, the co-ordinates of the three points which divides AB in four equal parts are

$\left(-3, \frac{3}{2}\right)$, (−2, 3) and $\left(-1, \frac{9}{2}\right)$.

16. Let a and d respectively be the first term and common difference of the AP.

$S_n=\frac{n}{2}[2a+(n-1)d]$

$42 = \frac{6}{2}[2a + (6 - 1)d]$ (Given, $S_6 = 42$)

42 = 3(2a + 5d)

2a + 5d = 14 ... (i)

It is given that $\frac{a_{10}}{a_{30}}=\frac{1}{3}$

$\frac{a+(10-1)d}{a+(30-1)d}=\frac{1}{3}$ (As $a_n = a + (n - 1)d$)

$\frac{a+9d}{a+29d}=\frac{1}{3}$

3a + 27d = a + 29d

2a = 2d

a = d ... (ii)

Putting the value of d in (i),

2a + 5a = 14 ⇒ 7a = 14

a = 2

Therefore, first term = 2 and common difference = 2

$a_{13} = a + (13 - 1)d = a + 12d = 2 + 12 \times 2$

$\therefore a_{13} = 26$

17. Let E be the window which is 60 m above the ground, AC be the house that is in observation.

Consider the right ΔCDE

$\tan 45^\circ = \frac{DE}{CD}$

$\Rightarrow 1 = \frac{60}{y}$

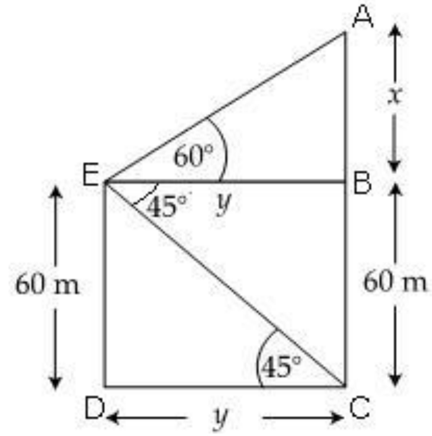
$\Rightarrow y = 60 \text{ m}$

Similarly, consider the right ΔABE:

$\tan 60^\circ = \frac{AB}{BE}$

$\Rightarrow \sqrt{3} = \frac{x}{60}$

$\Rightarrow x = 60\sqrt{3} \text{ m}$



$\therefore$ Height of the opposite house = x+60 = 60 + $60\sqrt{3} = 60\left(1+\sqrt{3}\right)$ m

18.

$$\left(\frac{4x-3}{2x+1}\right) - 10\left(\frac{2x+1}{4x-3}\right) = 3$$

Let $\frac{4x-3}{2x+1} = y$

The equation becomes

$y - \frac{10}{y} = 3$

$\Rightarrow y^2 - 3y - 10 = 0$

$\Rightarrow (y-5)(y+2) = 0$

$\Rightarrow y = 5, \ -2$

$\therefore \frac{4x-3}{2x+1} = 5$ or $\frac{4x-3}{2x+1} = -2$

10x + 5 = 4x - 3 or 4x - 3 = -4x - 2

6x = -8 or 8x = 1

$x = -4/3$ or $x = 1/8$

19. Given, volume of hemisphere = $2425\frac{1}{2}\ \text{cm}^3 = \frac{4851}{2}\ \text{cm}^3$

Let the radius of the hemisphere be '*r*' cm.

Volume of hemisphere $= \frac{2}{3}\pi r^3$

$$\Rightarrow \frac{2}{3}\pi r^3 = \frac{4851}{2}$$
$$\Rightarrow \frac{2}{3}\times\frac{22}{7}r^3 = \frac{4851}{2}$$
$$\Rightarrow r^3 = \frac{4851\times3\times7}{2\times2\times22}$$
$$\Rightarrow r^3 = \frac{441\times21}{2\times2\times2}$$
$$\Rightarrow r = \sqrt[3]{\frac{21\times21\times21}{2\times2\times2}}$$
$$\Rightarrow r = \frac{21}{2} \qquad ----(1)$$

∴Curved surface area of hemisphere = $2\pi r^2$

$$= 2\times\frac{22}{7}\times\left(\frac{21}{2}\right)^2 \qquad \text{[by (1)]}$$
$$= 2\times\frac{22}{7}\times\frac{21\times21}{4}$$
$$= 693\ \text{cm}^2$$

20. Total number of outcomes = 22

Let A be the event of getting a prime number.

Prime numbers are 5, 7, 11, 13, 17, 19, and 23.

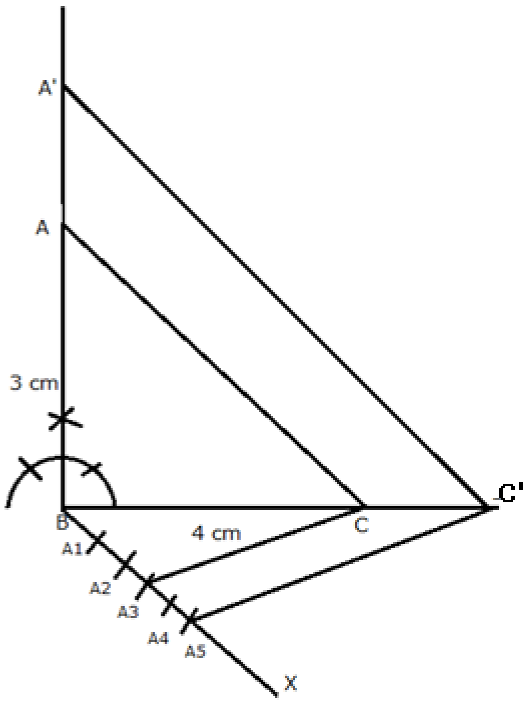
Number of favorable outcomes = 7

$$\therefore P(A) = \frac{7}{22}$$

21. Steps of construction:

1. Draw a right angle triangle ABC right angled at B with AB = 3 cm and BC = 4 cm.
2. Draw an acute angle CBX with BC and mark five points on it A_1, A_2, A_3, A_4 and A_5 such that $BA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5$.
3. Join A_3 to C.
4. Through A_5 draw a parallel line to A_3C. This line intersects BC produced at point C'.
5. Through C', draw a line parallel to AC. This line intersects the line AB produced at point A'.

Thus, ΔA'BC' is the required triangle similar to ΔABC.



22. $a_n = 3 + 2n$

Now, put n = 1,2,3,....

$a_1 = 3 + 2(1) = 5$

$a_2 = 3 + 2(2) = 7$

$a_3 = 3 + 2(3) = 9$

Thus, the terms of the AP are 5, 7, 9,.......

Here, a = 5 and d = 2

$S_{24} = \frac{24}{2}[2 \times 5 + (24 - 1)2] = 12[10 + 46] = 12 \times 56 = 672$

23. ABCD is a square. A quadrant of circle of radius 1 cm is drawn at each vertex of the square.

The quadrant of a circle is a sector of angle 90°.

Area of each quadrant = $\pi \times \frac{90°}{360°} r^2 = 3.14 \times \frac{1}{4} \times 1 = 0.785$ cm^2

Area of square = $(\text{Side})^2 = (4 \text{ cm})^2 = 16$ cm^2

Diameter of the circle = 2 cm

∴ Radius of the circle = 1 cm

∴ Area of circle = $\pi r^2 = 3.14 \times (1 \text{ cm})^2 = 3.14$ cm^2

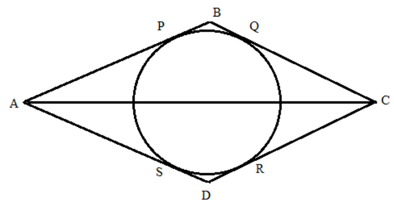
Area of shaded region

= Area of square − Area of circle − (4 × Area of each quadrant)

= 16 cm^2 − 3.14 cm^2 − (4 × 0.785 cm^2)

= 9.72 cm^2

24. Since ABCD is a parallelogram,



AB = CD ...(1)
BC = AD ...(2)
It can be observed that:
DR = DS (Tangents on the circle from point D)
CR = CQ (Tangents on the circle from point C)
BP = BQ (Tangents on the circle from point B)
AP = AS (Tangents on the circle from point A)
Adding all these equations, we obtain:
DR + CR + BP + AP = DS + CQ + BQ + AS
(DR + CR) + (BP + AP) = (DS + AS) + (CQ + BQ)
CD + AB = AD + BC
From (1) and (2), we obtain:
2AB = 2BC
AB = BC ...(3)
From (1), (2), and (3), we obtain:
AB = BC = CD = DA
Hence, ABCD is a rhombus.

Section D

25. Let a and d respectively be the first term and common difference of the A.P.

$S_{10} = S_{10} = \frac{10}{2}[2a + 9d] = -80$

2a + 9d = −16 ... (1)

$S_{20} = \frac{20}{2}[2a + 19d] = -280 + (-80) = -360$

2a + 19d = −36 ... (2)

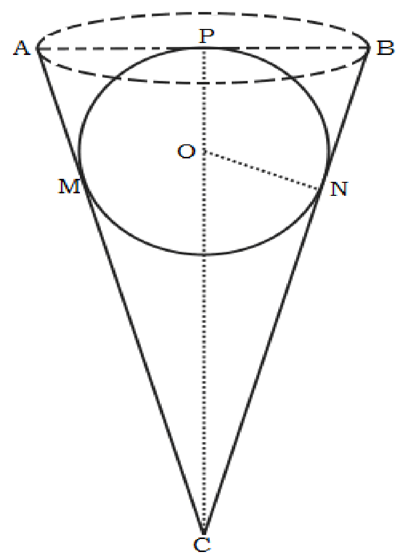
Solving (1) and (2), we get,
d = −2
So, from (1),
2a − 18 = −16
$\Rightarrow$ a = 1
Thus, the AP is
1, −1, −3,...

26. The sphere is just immersed in the cone filled of water. Hence, uppermost point of the sphere and the centre of the base of the cone will be same, i.e. lie on point 'P'.

Let ABC represent the cone and O be the centre of sphere touching the sides at M and N of the cone.

Therefore, PB = radius of the base of the cone = 6 cm

and PC = height of the cone = h = 8 cm



Now, in Δ PBC, we have,

$BC^2 = PB^2 + PC^2 = 6^2 + 8^2 = 100$

$\Rightarrow$ BC = 10 cm

We know that two tangents to a circle from the same point are equal.

Therefore, BP = BN = 6 cm

$\Rightarrow$ NC = BC – BN = 10 – 6 = 4 cm

Let the radius of the sphere (circle) = ON = x cm

$\Rightarrow$ OP = ON = x cm

Therefore, OC = PC – OP = (8 – x) cm

Now, in Δ ONC, we have,

$OC^2 = ON^2 + NC^2$

$(8 - x)^2 = x^2 + 4^2 \Rightarrow x = 3$ cm

Therefore, radius of the sphere is 3 cm.

Volume of sphere = $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi(3)^3$ cu. cm

Volume of the cone = $\frac{1}{3}\pi R^2 h = \frac{1}{3}\pi(6)^2 \times 8$

$\therefore$ Volume of remaining water

= volume of cone – volume of sphere

$= \frac{1}{3}\pi(6)^2 \times 8 - \frac{4}{3}\pi(3)^3$

$= \frac{1}{3}\pi[36 \times 8 - 4 \times 27]$

$= \frac{1}{3} \times \frac{22}{7} \times 180$

= 188.57 cm^3

27. Radius of the quadrant = 14 cm

BC = $14\sqrt{2}$ cm

$\therefore$ Area of shaded region =

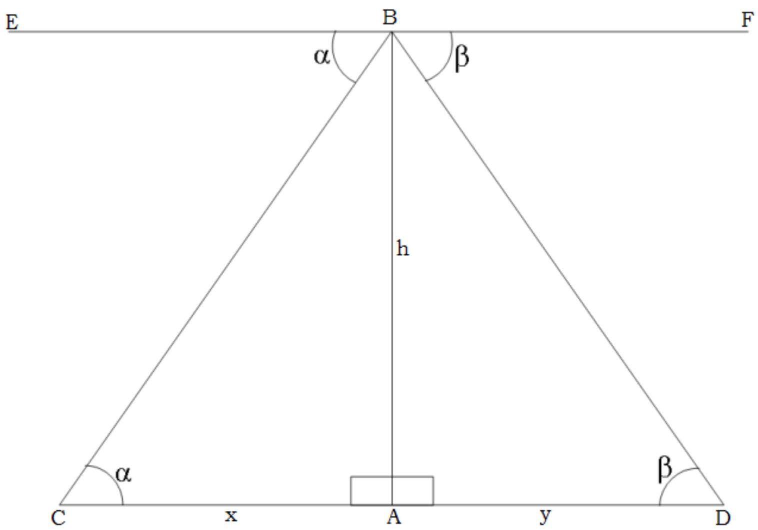
Area of semicircle with diameter BC – area of quadrant of radius 14 + area of ΔABC

$$=\frac{1}{2}\times\frac{22}{7}\times\left(\frac{14\sqrt{2}}{2}\right)^2-\frac{1}{4}\times\frac{22}{7}\times14\times14+\frac{1}{2}\times14\times14$$

= (154 – 154 + 98) cm^2

= 98 cm^2

28. Let AB be the lighthouse of height h metres. Let AC = x and AD = y.



In $\triangle$ CAB,

$$\frac{AB}{AC}=\tan\alpha$$

$$\tan\alpha=\frac{h}{x}\Rightarrow x=\frac{h}{\tan\alpha}\quad\text{...............(i)}$$

In $\triangle$ DAB,

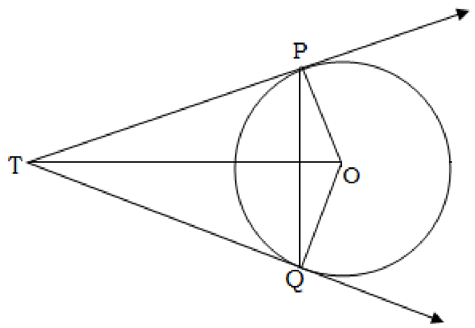
$$\frac{AB}{AD}=\tan\beta$$

$$\frac{h}{y}=\tan\beta\Rightarrow y=\frac{h}{\tan\beta}\quad\text{................ (ii)}$$

Distance between the ships = x + y = $\left(\frac{h}{\tan\alpha}+\frac{h}{\tan\beta}\right)$ = h $\left(\frac{\tan\alpha+\tan\beta}{\tan\alpha\tan\beta}\right)$ metres

29. We know that lengths of tangents drawn from an external point to a circle are equal.

$\therefore$ TP = TQ



$\Rightarrow$ $\triangle$ TPQ is an isosceles triangle.

$\Rightarrow \angle TPQ = \angle TQP$

In $\triangle$ TPQ, we have,

$\Rightarrow m\angle TPQ + m\angle TQP + m\angle PTQ = 180°$

$\Rightarrow 2\angle TPQ = 180° - \angle PTQ$

$\Rightarrow m\angle TPQ = 90° - \frac{1}{2} m\angle PTQ$

$\Rightarrow \frac{1}{2} m\angle PTQ = 90° - m\angle TPQ$ ...(i)

Since, OP is perpendicular to TP, $m\angle OPT = 90°$

$\Rightarrow m\angle OPQ + m\angle TPQ = 90°$

$\Rightarrow m\angle OPQ = 90° - m\angle TPQ$ ....(ii)

From (i) and (ii), we get:

$\frac{1}{2}\angle PTQ = \angle OPQ$

$\angle PTQ = 2\angle OPQ$

30. R = 7 cm, r = 3.5 cm

Area of shaded region

$$= \pi\left[R^2 - r^2\right]\frac{\theta}{360^o}$$

$$= \frac{22}{7}\left[7^2 - (3.5)^2\right]\frac{30^o}{360^o}$$

$$= \frac{22}{7}\left[(7+3.5)(7-3.5)\right]\frac{30^o}{360^o}$$

$$= \frac{22}{7}\times 10.5 \times 3.5 \times \frac{1}{12}$$

$$= 9.625 \text{ cm}^2$$

31. Diameter = 5 cm
Radius = 2.5 cm
Height = 10 cm

Volume of glass of type A = $\pi r^2 h$
$= 3.14 \times 2.5 \times 2.5 \times 10$
$= 196.25 \text{ cm}^3$

For glass of type B:

Volume of hemisphere $= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times 3.14 \times 2.5 \times 2.5 \times 2.5$

$= 32.71 \text{ cm}^3$

$\therefore$ Volume of glass of type B = $196.25 \text{ cm}^3 - 32.71 \text{ cm}^3$
$= 163.54 \text{ cm}^3$

For glass of type C:

Volume of cone = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times 3.14 \times 2.5 \times 2.5 \times 1.5$

N $= 3.14 \times 2.5 \times 2.5 \times 0.5$

$= 9.81 \text{ cm}^3$

Volume of glass of type C = $(196.25 - 9.81) \text{ cm}^3$
$= 186.44 \text{ cm}^3$

(a) The volume of glass of type A = 196.25 cm^3.

(b) The glass of type B has the minimum capacity of 163.54 cm^3.

(c) Ramesh displays honesty in his business.

32. S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1) (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

Total number of outcomes when two dice are thrown = 36

Let A be the event '5 will not come up either time'.

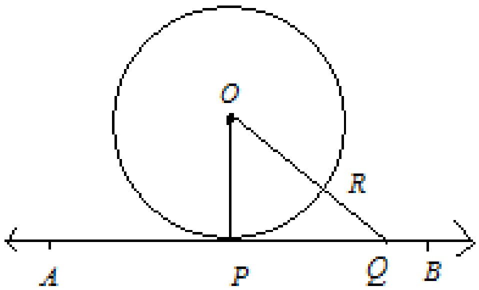
Number of favourable outcomes = 25

$\therefore$ P (A) = $\frac{25}{36}$

33. Given: A circle C(O, r) and a tangent AB at a point P.

To Prove: OP is perpendicular to AB.

Construction: Take any point Q, other than P, on the tangent AB. Join OQ.



Since, Q is a point on the tangent AB, other than the point of contact P, so Q will be outside the circle.

Let OQ intersect the circle at R.

Then, OQ = OR + RQ

$\Rightarrow$ OQ > OR

$\Rightarrow$ OQ > OP (OR=OP=radius)

Thus, OP < OQ, i.e., OP is shorter than any other segment joining O to any point on AB.

But, we know that among all the line segments joining the point O to a point on AB, the shortest one is the perpendicular from O on AB.

Hence, OP is perpendicular to AB.

34. Let the number of children be x.

It is given that Rs. 250 is divided amongst x children.

So, money received by each child = Rs. $\frac{250}{x}$

If there were 25 children more, then

Money received by each child = Rs. $\frac{250}{x+25}$

From the given information,

$$\frac{250}{x}-\frac{250}{x+25}=\frac{50}{100}$$

$$\frac{250x+6250-250x}{x(x+25)}=\frac{1}{2}$$

$$\frac{6250}{x^2+25x}=\frac{1}{2}$$

$$x^2+25x-12500=0$$

$$(x+125)(x-100)=0$$

$$x=-125,100$$

Since, the number of children cannot be negative, so, x = 100.

Hence, the number of children is 100.