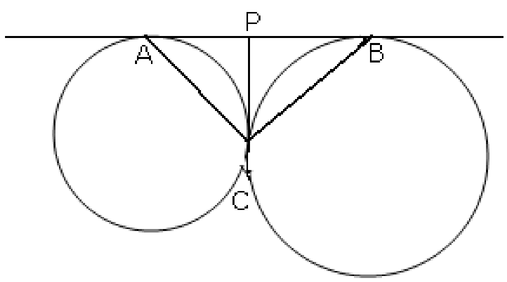
Goa Board
Class X Mathematics
Term II
Sample Paper 2 - Solution

Section A
(Questions 1 to 8 carry 1 mark each)

1. Correct answer: C
$a + 8d = 449$ and,
$a + 448d = 9$
On solving we get, $440d = -440$
$d = -1$
Therefore, $a - 8 = 449$
$a = 457$
Let its n^{th} term be zero.
$a_n = a+(n-1)d$
$0 = 457 + (n-1)(-1)$
$457 = n - 1$
or $n = 458$

2. Correct answer: D



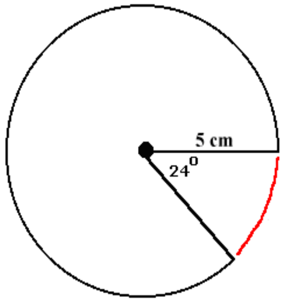
Lengths of tangents drawn from an external point to a circle are equal.
PA = PC (tangents from P)
Therefore, $\angle PAC = \angle PCA = x$ (say)
Also, PC = PB ($\because$ tangents from external point P are congruent)
$\angle PBC = \angle PCB = y$
In ΔABC,
$m\angle ABC + m\angle ACB + m\angle BAC = 180^o$
$y + (x + y) + x = 180^o$
$2(x + y) = 180^o$
$x + y = 90^o$
$m\angle ACB = 90^o$

3. Correct answer: A

The length of an arc that subtends an angle of 24° at the centre of a circle of 5 cm radius is

$$\frac{24}{360} x 2\pi \times 5 = \frac{2}{30} x 2\pi \times 5 = \frac{2\pi}{3}$$

The length of the arc is $\frac{2\pi}{3}$ cm.



4. Correct answer: C

Let AC be the height of the tower of h metres and ED be the observer of height 1.5 m at a distance of DC = 28.5 m from the tower AC.

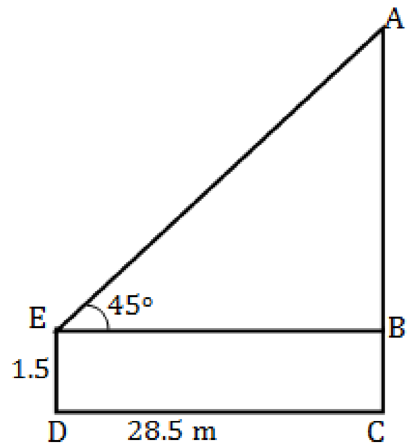
In right ΔAED,

$\tan 45 = \frac{AB}{EB}$

$1 = \frac{AB}{28.5}$ (as EB = DC = 28.5 m)

AB = 28.5 m

Height of the tower =h = AB + BC = AB + DE = 28.5 + 1.5 = 30 m



5. Correct answer: B

Total number of outcomes = 12

Numbers which are divisors of 12 are 1, 2, 3, 4, 6, and 12.

Number of favourable outcomes = 6

P(divisor of 12) = $\frac{6}{12} = \frac{1}{2}$

6. Correct answer: B

Area of a triangle = 0

$$\frac{1}{2}\left|x(1-5)+2(5+1)+4(-1-1)\right| = 0$$

$$\frac{1}{2}\left|-4x+12-8\right| = 0$$

$$-4x+4=0$$

$$x=1$$

7. Correct answer: D

Since one root of the equation $2x^2 - 10x + p = 0$ is 2, therefore,

$$2(2)^2 - 10(2) + p = 0 \Rightarrow 8 - 20 + p = 0 \Rightarrow p = 12$$

8. Correct answer: B

Here, $\frac{r_1}{r_2}=\frac{2}{3}$ and $\frac{h_1}{h_2}=\frac{5}{3}$

Therefore, $\frac{V_1}{V_2}=\frac{\pi r_1^2 h_1}{\pi r_2^2 h_2}=\left(\frac{r_1}{r_2}\right)^2\left(\frac{h_1}{h_2}\right)=\left(\frac{2}{3}\right)^2\left(\frac{5}{3}\right)=\frac{20}{27}$

Hence, $V_1 : V_2 = 20:27$

Section B

9. Here, n = 60, a = 7 and t_{60} = 125

Therefore,

7 + 59d = 125

⇒ d = 2

Therefore, 32^{nd} term (t_{32}) = a + 31d = 7 + 31(2) = 69

10. Let the required numbers be x and (x + 1)

Given $x^2 + (x + 1)^2 = 25$

$2x^2 + 2x - 24 = 0$

$x^2 + x - 12 = 0$

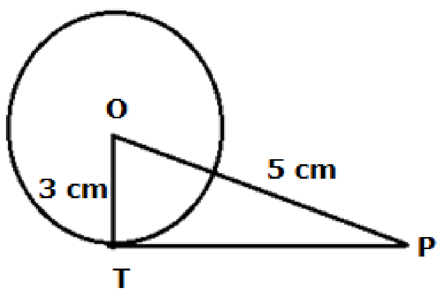
$(x + 4)(x - 3) = 0$

x = −4 or x = 3

Reject x = −4

Therefore the given consecutive positive integers are x = 3 and x + 1 = 3 + 1 = 4

11. In ΔOTP, m∠OTP = 90° [∵ angle between tangent and radius of a circle is 90°]



By Pythagoras theorem,

$OP^2 = OT^2 + PT^2$

∴ $PT^2 = OP^2 - OT^2 = 25 - 9 = 16$

Thus, PT = 4 cm

Hence the length of the tangent is 4 cm.

12. Let $P(x_1, y_1) = P(a, b)$, $Q(x_2, y_2) = Q(b, c)$ and $R(x_3, y_3) = R(c, a)$ be the vertices of ΔPQR.

We know that the co-ordinates of the centroid of a triangle is given by

$$\left[\left(\frac{x_1+x_2+x_3}{3}\right),\left(\frac{y_1+y_2+y_3}{3}\right)\right] \text{ i.e.}$$

$$\left(\frac{a+b+c}{3},\frac{b+c+a}{3}\right)$$

Also, given that centroid is at origin, i.e., its coordinates are (0,0).

So, $\left(\frac{a+b+c}{3}\right)=0 \Rightarrow a+b+c=0$

13. Let the line segment joining the points A(−4, 5) and B(3, −7) be divided by the y-axis at the point P(0, y) in the ratio k : 1.

Now, by section formula,

x-coordinate of point P is: $\frac{k(3)+1(-4)}{k+1}$.

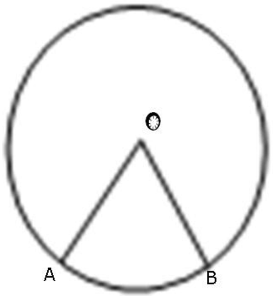
But x-coordinate of P is 0.

So, $\frac{k(3)+1(-4)}{k+1}=0$

$\Rightarrow 3k-4=0 \Rightarrow k=\frac{4}{3}$

Hence, the required ratio is 4 : 3.

14. Let OAB be the given sector. Then, perimeter of sector OAB = 16.4 cm



OA + OB + arc AB = 16.4 cm

5.2 + 5.2 + arc AB = 16.4

Arc AB = 6 cm, i.e., l = 6 cm

Area of sector = $\frac{1}{2} \times l \times r = \frac{1}{2} \times 6 \times 5.2 = 15.6 \text{ cm}^2$

Section C

15. Let $\frac{x}{x+1} = y \Rightarrow \frac{x+1}{x} = \frac{1}{y}$

$\therefore y + \frac{1}{y} = \frac{34}{15} \Rightarrow \frac{y^2+1}{y} = \frac{34}{15}$

$\Rightarrow 15y^2 - 34y + 15 = 0$

$\Rightarrow 15y^2 - 9y - 25y + 15 = 0$

$\Rightarrow 3y(5y-3) - 5(5y-3) = 0$

$\Rightarrow (5y-3)(3y-5) = 0$

$\Rightarrow 5y - 3 = 0$ or $3y - 5 = 0$

$\therefore y = \frac{3}{5}$ or $y = \frac{5}{3}$

If $y = \frac{5}{3}$ then $\frac{x}{x+1} = \frac{5}{3}$

$\Rightarrow 5x + 5 = 3x \Rightarrow 2x = -5 \Rightarrow x = -\frac{5}{2}$

If $y = \frac{3}{5}$ then $\frac{x}{x+1} = \frac{3}{5}$

$\Rightarrow 3x + 3 = 5x \Rightarrow -2x = -3 \Rightarrow x = \frac{3}{2}$

Hence, $x = -\frac{5}{2}$ and $x = \frac{3}{2}$

16. The two-digit natural numbers divisible by 4 are:

12, 16, 20, ..., 96

This is an A.P. with

a = 12 and d = 16 – 12 = 4

We know that:

$t_n = a + (n-1)d$

But $t_n = 96$

$\Rightarrow 96 = 12 + (n - 1)\,4$

$\Rightarrow 96 = 12 + 4n - 4$

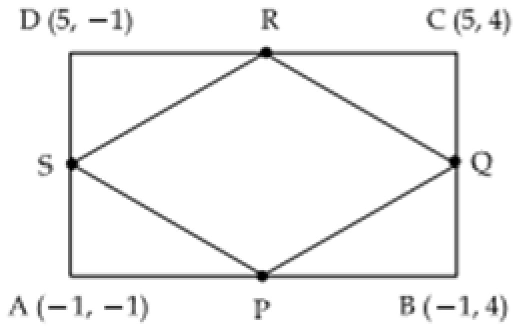
$\Rightarrow 4n = 88 \Rightarrow n = 22$

Now, $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_n = \frac{22}{2}[2 \times 12 + (22-1)4]$

$S_n = 11 \times 108 = 1188$

17. Using mid-point formula, we have:



Co-ordinates of P are $(-1, \frac{3}{2})$

Co-ordinates of Q are (2, 4)

Co-ordinates of R are $(5, \frac{3}{2})$

Co-ordinates of S are (2, −1)

Now using distance formula,

$$PQ = \sqrt{(2+1)^2 + \left(4 - \frac{3}{2}\right)^2} = \sqrt{9 + \frac{25}{4}} = \sqrt{\frac{61}{4}}$$

$$QR = \sqrt{(5-2)^2 + \left(\frac{3}{2} - 4\right)^2} = \sqrt{9 + \frac{25}{4}} = \sqrt{\frac{61}{4}}$$

$$RS = \sqrt{(2-5)^2 + \left(-1 - \frac{3}{2}\right)^2} = \sqrt{9 + \frac{25}{4}} = \sqrt{\frac{61}{4}}$$

$$SP = \sqrt{(2+1)^2 + \left(-1 - \frac{3}{2}\right)^2} = \sqrt{9 + \frac{25}{4}} = \sqrt{\frac{61}{4}}$$

Since all 4 sides are equal, PQRS is a rhombus.

18. Let the A.P. be a, a + d, a +2d,...

Now $a_4 = 3a$

$\Rightarrow a + 3d = 3a \Rightarrow d = \frac{2}{3}a$ ... (1)

Also, $a_7 = 2a_3 + 1$

$\Rightarrow a + 6d = 2(a + 2d) + 1$

$\Rightarrow a + 6d = 2a + 4d + 1$

$\Rightarrow a - 2d + 1 = 0$

Using (1), we get

$$a - 2\times\left(\frac{2}{3}a\right) + 1 = 0$$

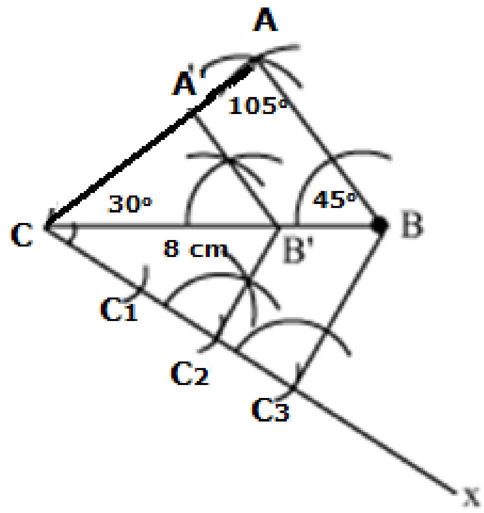
$\Rightarrow 3a - 4a + 3 = 0$

$\Rightarrow a = 3$

And, $d = \frac{2}{3} \times 3 = 2$

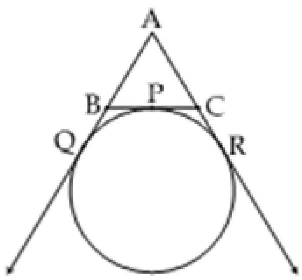
19. The steps of construction are as follows:

i. Draw ΔABC with the given measures.
ii. Draw a ray CX making acute angle with line CB on opposite side of vertex A.
iii. Locate 3 points C_1, C_2, C_3 on line CX such that $CC_1 = C_1C_2 = C_2C_3$.
iv. Join BC_3 and draw a line through C_2 parallel to BC_3 to intersect CB at point B'.
v. Draw a line through B' parallel to the line BA to intersect AC at A'.
vi. ΔAB'C' is the required triangle.



20. Since tangents drawn from an external points are equal, therefore

AQ = AR ....(1)
BQ = BP ....(2)
CP = CR ....(3)



Now, perimeter of ΔABC = AB + BC + CA
= AB + BP + PC + CA
= AB + BQ + CR +CA [From (2) and (3)]
= AQ + AR [AB + BQ = AQ, CR + AC = AR]
= AQ + AQ (From (1))
= 2AQ

$\therefore$ AQ = $\frac{1}{2}$ [perimeter of ΔABC]

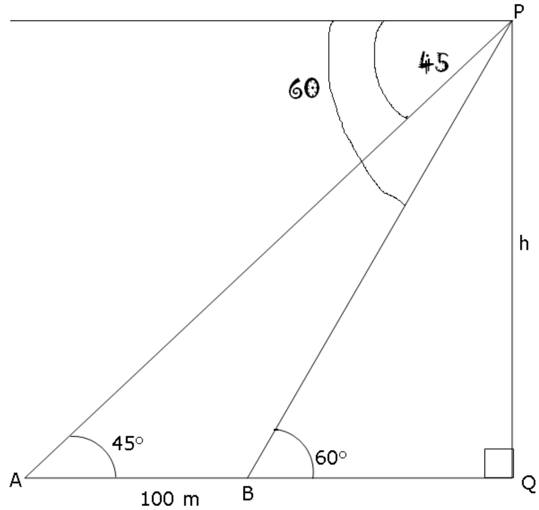
21. Total no. of equally likely outcomes for visiting the shop in same week = 6 x 6 = 36

i. Same days are (M, M), (T, T), (W, W), (Th, Th), (F, F), (Sat, Sat)
Number of favourable outcomes = 6
Required probability = $\frac{6}{36}=\frac{1}{6}$

ii. Number of different days = 36 – 6 = 30
Number of favourable outcomes = 30
Required probability = $\frac{30}{36}=\frac{5}{6}$

iii. Favourable outcomes for (I customer, II customer)
= (M, T), (T, W), (W, Th), (Th, F), (F, Sat) and for
(II customer, I customer)
= (M, T), (T, W), (W, Th), (Th, F), (F, Sat)
Number of favourable outcomes = 10
Required probability = $\frac{10}{36}=\frac{5}{18}$

22. Let the height of the balloon at P be h metres. Let A and B be the two cars. Thus, AB = 100 m.



From right ΔPAQ, AQ = PQ = h (as tan 45° = 1)

From right ΔPBQ,

$\frac{PQ}{BQ} = \tan 60^\circ = \sqrt{3}$ or $\frac{h}{h-100} = \sqrt{3}$

$h = \sqrt{3}(h-100)$

Therefore, $h = \frac{100\sqrt{3}}{\sqrt{3}-1} = 50(3+\sqrt{3})$

Thus, the height of the balloon is $50(3+\sqrt{3})$ m.

23. Diameter of the cylinder = 12 cm

Radius of the cylinder = 6 cm

Height of the cylinder = 15 cm

Volume of ice-cream in the cylinder = $\pi r^2 h = \pi \times 36 \times 15 = 540\pi$

Diameter of cone = 6 cm

Radius of cone = 3 cm

Height of cone = 12 cm

Volume of one ice cream = volume of ice cream cone + volume of hemispherical top of ice cream = $\frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$

$= \frac{1}{3}\pi(9)(12) + \frac{2}{3}\pi(27)$

$= 36\pi + 18\pi$

$= 54\pi$

So, the number of ice-cream cones = $\frac{\text{Volume of cylinder}}{\text{Volume of 1 cone}} = \frac{540\pi}{54\pi} = 10$

Hence, number of cones which can be filled with the ice-cream from the cylinder is 10.

24. Since P and Q are trisecting AB and P is nearer to A, so the point P divides AB in the ratio 1:2. Let the coordinates of P are (x, y).

Therefore, using section formula:

$P\left(\frac{1 \times 5 + 2 \times 2}{1+2}, \frac{1 \times -8 + 2 \times 1}{1+2}\right) = P(3, -2)$

Now P(3, −2) lies on the line 2x – y + k = 0

$\Rightarrow 2 \times 3 - (-2) + k = 0$

$\Rightarrow 6 + 2 + k = 0$

$\Rightarrow k = -8$

Section D

25. Suppose 200 logs are stacked in 'n' rows.

There are 20 logs in the first row, 19 logs in the second row, 18 logs in the third row, and so on.

So, number of logs in various rows form an A.P. whose first term is a = 20 and common difference is d(19 – 20) = −1

Given: Sum of n terms (rows of an A.P. with a = 20, d = −1) is equal to 200

$\Rightarrow \frac{n}{2}\left[2\times20+(n-1)\times(-1)\right]=200 \qquad \left[\because S_n=\frac{n}{2}\left[2a+(n-1)d\right]\right]$

$\Rightarrow$ 40n – n(n – 1) = 400

$\Rightarrow$ $40n - n^2 + n = 400$

$\Rightarrow$ $n^2 - 41n + 400 = 0$

$\Rightarrow$ $n^2 - 25n - 16n + 400 = 0$

$\Rightarrow$ n(n – 25) – 16(n – 25) = 0

$\Rightarrow$ (n – 25) (n – 16) = 0

$\Rightarrow$ n – 25 = 0 or n – 16 = 0

$\Rightarrow$ n = 25 or n = 16

When n = 25, then number of logs in the 25^{th} row

= 25^{th} term of an A.P.

= a + (25 – 1)d

= 20 + 24(−1)

= −4, not possible

When n = 16, then number of logs in the 16^{th} row

= 16^{th} term of an A.P.

= a + (16 – 1)d

= 20 + 15(−1)

= 20 – 15

= 5

Hence, there are 16 rows in which 200 logs are placed and 5 logs are in the top row.

Value depicted: Space saving, Creative, Reasoning, Balancing

26. Let the marks obtained by P in Mathematics be x.

Therefore marks obtained by P in Science = 28 – x

New marks in Mathematics = x + 3

New marks in Science = 28 − x − 4 = 24 − x

According to question,

$(x + 3)(24 - x) = 180$

$24x + 72 - x^2 - 3x = 180$

$-x^2 + 21x = 180 - 72$

$x^2 - 21x + 108 = 0$

$x^2 - 12x - 9x + 108 = 0$

$x(x - 12) - 9(x - 12) = 0$

$(x - 12)(x - 9) = 0$

$x = 12, 9$

Therefore,

Marks in Mathematics = 12, Marks in Science = 28 − 12 = 16

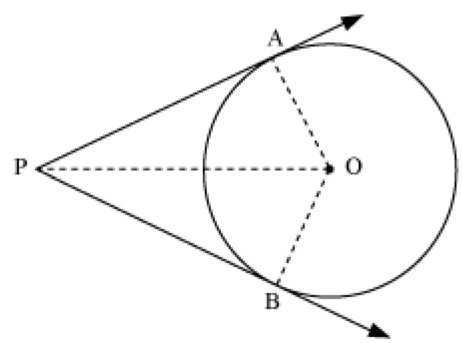
Or, Marks in Mathematics = 9, Marks in Science = 28 − 9 = 19

27. Given: A circle with centre O; PA and PB are two tangents to the circle drawn from an external point P

To prove: PA = PB

Construction: Join OA, OB, and OP

Proof: It is known that a tangent at any point of a circle is perpendicular to the radius at the point of contact.



$\therefore$ OA $\perp$ PA and OB $\perp$ PB

$m\angle OAP = m\angle OBP = 90^o$ ... (1)

In ΔOPA and ΔOPB:

$\angle OAP = \angle OBP$ (Using (1))

OA = OB (Radii of the same circle)

OP = PO (Common side)

Therefore, $\Delta OPA \cong \Delta OPB$ (RHS congruency criterion)

$\therefore$ PA = PB (Corresponding parts of congruent triangles are equal)

Thus, it is proved that the lengths of two tangents drawn from an external point to a circle are equal.

28. Total number of balls = 5 + 7 + 4 + 2 = 18

i. Favourable outcomes = 5 + 2 = 7

$\therefore$ Required probability = $\dfrac{\text{Favourable outcomes}}{\text{Total outcomes}} = \dfrac{7}{18}$

ii. Favourable outcomes = 7 + 4 = 11

$\therefore$ Required probability = $\dfrac{\text{Favourable outcomes}}{\text{Total outcomes}} = \dfrac{11}{18}$

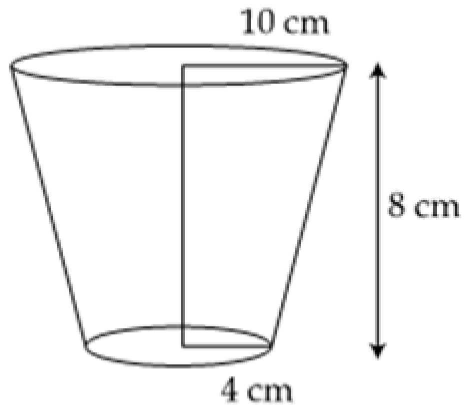
iii. Number of balls which are not white = 18 – 5 = 13

$\therefore$ Required probability = $\dfrac{\text{Favourable outcomes}}{\text{Total outcomes}} = \dfrac{13}{18}$

iv. Number of balls which are neither white nor black = Number of balls which are red or blue = 7 + 2 = 9

$\therefore$ Required probability = $\dfrac{\text{Favourable outcomes}}{\text{Total outcomes}} = \dfrac{9}{18} = \dfrac{1}{2}$

29. Volume of the container $= \dfrac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$



$= \dfrac{1}{3} \times 3.14 \times 8(100 + 16 + 40)$

$= 3.14 \times 8 \times \dfrac{156}{3} = 1306.24 \text{ cm}^3$

Thus, volume of container is 1306.24 cm^3.

But, 1 $\text{cm}^3 = \dfrac{1}{1000}$ l

So, the capacity of container is

$1306.24 \times \dfrac{1}{1000}$ l $= 1.30624$ l

$\therefore$ Cost of oil = Rs. (50 × 1.30624) = Rs. 65.31

Now, surface area of the metal sheet = $\pi(r_1 + r_2)l + \pi r_1^2$

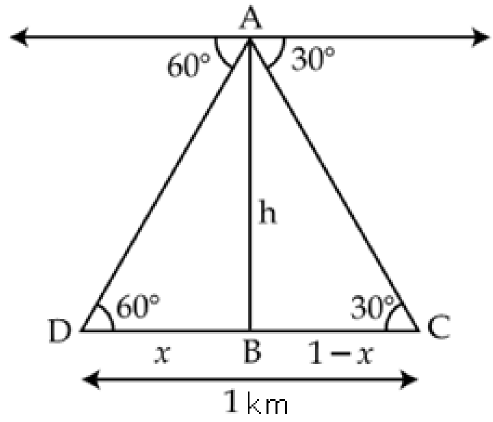
Where, $l = \sqrt{h^2 + (r_1 - r_2)^2} = \sqrt{8^2 + 6^2} = 10\text{cm}$

$\therefore$ Surface area = $3.14 \times (10+4) \times 10 + 3.14 \times 4^2 = 3.14 \times 156$

= 489.84 sq cm

$\therefore$ Cost of the sheet = $\dfrac{5}{100} \times 489.84$ = Rs. 24.49

30. In the figure, AB denotes the height of the airplane. Points C and D denotes the two stones which are 1 km apart.



In Δ ABD,

$$\frac{h}{x} = \tan 60° = \sqrt{3}$$

$$\Rightarrow x = \frac{h}{\sqrt{3}} \qquad ----(1)$$

In Δ ABC,

$$\frac{h}{1-x} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$\sqrt{3}h = 1 - x \Rightarrow x = 1 - \sqrt{3}h \qquad \ldots (2)$$

From (1) and (2),

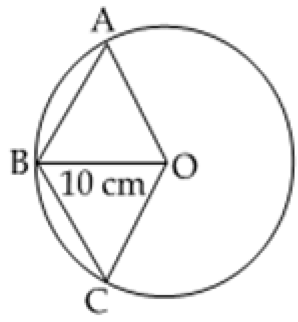
$$\frac{h}{\sqrt{3}} = 1 - \sqrt{3}h$$

$$\Rightarrow \sqrt{3}h + \frac{h}{\sqrt{3}} = 1$$

$$\Rightarrow \frac{4h}{\sqrt{3}} = 1 \Rightarrow h = \frac{\sqrt{3}}{4}\text{km}$$

Thus, the height of airplane is $\frac{\sqrt{3}}{4}$ km.

31. Since, OA = OB = OC = 10 cm (being radii of same circle)



Now OABC is a rhombus,

$\therefore$ AB = BC = OA = OC = 10 cm

$\therefore$ OAB and OBC are equilateral triangles of side 10 cm

So the area rhombus $= 2$(Area of triangle OAB)

$$= 2\left[\frac{\sqrt{3}}{4}(\text{side})^2\right]$$

$$= 2\left[\frac{\sqrt{3}}{4} \times 100\right] \text{cm}^2$$

$$= 50\sqrt{3} \text{ cm}^2$$

Hence, the area of rhombus is $50\sqrt{3}$ cm^2.

32. Let radius of hemisphere be r

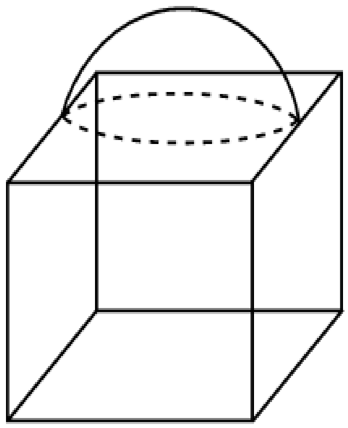
Total surface area of a cube = $6 \times 5 \times 5\ cm^2 = 150\ cm^2$

Base area of hemisphere = πr^2

C.S.A. of the hemisphere = $2\pi r^2$

S.A. of the decorative block

$$= 150 - \pi r^2 + 2\pi r^2$$
$$= 150 + \pi r^2$$
$$= 150 + \frac{22}{7} \times 2.1 \times 2.1$$
$$= 163.86 cm^2$$



33. Since, the radius is perpendicular to the tangent at the point of contact, m∠OPO' = 90°.

O'O = $\sqrt{4^2 + 3^2}$ = 5 cm (Using Pythagoras theorem)

Let O'L = x, then OL = 5 – x

$\therefore PL^2 = 4^2 - x^2 = 3^2 - (5 - x)^2$

$\Rightarrow 16 - x^2 = 9 - (25 + x^2 - 10x)$

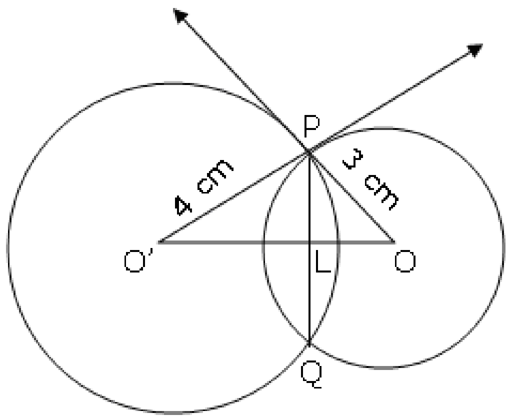
$\Rightarrow$ 16 = 9 – 25 + 10x

$\Rightarrow$ 10x = 32

$\Rightarrow x = \frac{32}{10}$ = 3.2 cm

$\therefore PL = \sqrt{4^2 - (3.2)^2} = \sqrt{16 - 10.24} = \sqrt{5.76} = 2.4$

$\therefore$ PQ = 2 × 2.4 cm = 4.8 cm



34. It is given that, height (h) of cylindrical part = height (h) of the conical part = 7 cm

Diameter of the cylindrical part = 12 cm

$\therefore$Radius (r) of the cylindrical part = $\frac{12}{2} = 6\text{cm}$

$\therefore$ Radius of conical part = 6 cm

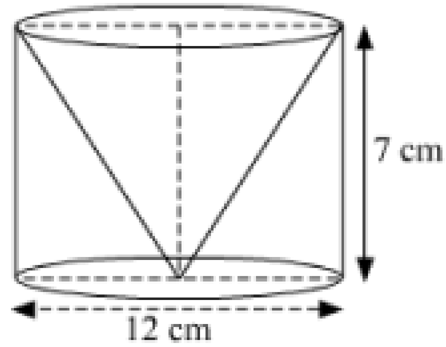
Slant height (l) of conical part

$= \sqrt{r^2 + h^2}$ cm

$= \sqrt{6^2 + 7^2}$ cm

$= \sqrt{85}$ cm

$= 9.22$ cm(approx.)



Total surface area of the remaining solid

= CSA of cylindrical part + CSA of conical part + Base area of the circular part

$= 2\pi rh + \pi rl + \pi r^2$

$= 2 \times \frac{22}{7} \times 6 \times 7 + \frac{22}{7} \times 6 \times 9.22 + \frac{22}{7} \times 6 \times 6$

$= 264 + 173.86 + 113.14$

$= 551 \text{ cm}^2$