Goa Board
Class X Mathematics
Term II
Sample Paper 1 - Solution

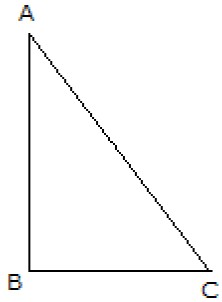
Section A
(Questions 1 to 8 carry 1 mark each)

1. Correct answer: D

In the word 'PROBABILITY', there are 11 letters out of which 4 are vowels (O, A, I, I).

P(getting a vowel) = $\frac{4}{11}$

2. Correct answer: B



Let AB be the height h of the pole.

Length of the shadow = BC = $\sqrt{3}$ h

If θ denotes the angle of elevation of the sun, then tanθ

$= \frac{h}{\sqrt{3}h} = \frac{1}{\sqrt{3}}$

$\Rightarrow \theta = 30^o$

3. Correct answer: A

Let R be the mid-point of PQ, then, the co-ordinates of R are $\left[\frac{(-2-6)}{2}, \frac{(8-4)}{2}\right]$ = (−4, 2)

4. Correct answer: A

It is given that $\frac{1}{2}$ is a root of the equation $x^2 + kx - \frac{5}{4} = 0$.

$\therefore \left(\frac{1}{2}\right)^2 + kx\left(\frac{1}{2}\right) - \frac{5}{4} = 0$

1 + 2k − 5 = 0 or k = 2

Putting the value of k in the given equation we get,

$4x^2 + 8x - 5 = 0$

$\Rightarrow (2x + 5)(2x - 1) = 0$

$\Rightarrow x = \frac{1}{2} \text{ or } -\frac{5}{2}$

Hence, the other root of the given equation is $-\frac{5}{2}$.

5. Correct answer: D

Given, AR = 4 cm, BR = 3 cm and AC = 11 cm

We know that the lengths of the tangents drawn to the circle from an external point are equal.

Therefore, AR = AQ = 4 cm, BR = BP = 3 cm and

PC = QC = AC – AQ = 11 cm – 4 cm = 7 cm

BC = BP + PC = 3 cm + 7 cm = 10 cm

6. Correct answer: A

Let R be the radius of the original steel ball and r be the radius of each of the new balls formed after melting.

$\frac{4}{3}\pi R^3 = 8 \times \frac{4}{3}\pi r^3 \Rightarrow R^3 = 8r^3 \Rightarrow r = \frac{1}{2}R$

Hence, the radius of each new ball is $\frac{1}{2}$ times the radius of the original ball.

7. Correct answer: C

Common difference = $\sqrt{48} - \sqrt{27} = 4\sqrt{3} - 3\sqrt{3} = \sqrt{3}$

$\therefore$ Required next term $= \sqrt{75} + \sqrt{3} = 5\sqrt{3} + \sqrt{3} = 6\sqrt{3} = \sqrt{36 \times 3} = \sqrt{108}$

8. Correct answer: A

Diameter of the sphere = Side of the cube = 7 cm

Volume of the sphere = $\frac{4}{3} \times \pi \times \left(\frac{7}{2}\right)^3$ = 179.67 cc

Section B

9. The corresponding quadratic polynomial can be factorised as below.

$6x^2 - \sqrt{2}x - 2 \Rightarrow 6x^2 - 3\sqrt{2}x + 2\sqrt{2}x - 2$

$\Rightarrow 3x(2x - \sqrt{2}) + \sqrt{2}(2x - \sqrt{2})$

$\Rightarrow (3x + \sqrt{2})(2x - \sqrt{2})$

Now, $6x^2 - \sqrt{2}x - 2 = 0$ gives $(3x + \sqrt{2})(2x - \sqrt{2}) = 0$

i.e., $(3x + \sqrt{2}) = 0$ or $(2x - \sqrt{2}) = 0$

Therefore, the roots of the given quadratic equation are $-\frac{\sqrt{2}}{3}$ and $\frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$.

10. It is known that radius is $\perp$ to the tangent at the point of contact.

Therefore, $m\angle OAT = 90^o$.

In ΔOAT,

$$\frac{AT}{OT} = \cos 30^o = \frac{\sqrt{3}}{2}$$

$$\Rightarrow AT = \frac{\sqrt{3}}{2}(OT) = \frac{\sqrt{3}}{2}(4) = 2\sqrt{3} \text{ cm}$$

11. Given $a_n = 10 - 3n \Rightarrow a_{n+1} = 10 - 3(n + 1)$

$\therefore a_{n+1} - a_n = \{10 - 3(n + 1)\} - (10 - 3n) = -3$, which is independent of n and hence a constant.

Therefore, the given sequence $\{a_n\}$ is an A.P.

12. Given points are A(8, 2), B(5, −3) and C(0, 0).

Using the distance formula, we get,

$$AC = \sqrt{(8-0)^2 + (2-0)^2} = \sqrt{68}$$

$$BC = \sqrt{(5-0)^2 + (-3-0)^2} = \sqrt{34}$$

$$AB = \sqrt{(5-8)^2 + (-3-2)^2} = \sqrt{34}$$

Since, BC = AB, ΔABC is an isosceles triangle.

13. There are 10 ribs in an umbrella. The area between two consecutive ribs subtends an angle of $\frac{360°}{10} = 36°$ at the centre of the assumed flat circle.

Area between two consecutive ribs of circle $= \frac{36°}{360°} \text{ x } \pi r^2$

$$= \frac{36°}{360°} \text{ x } \frac{22}{7} \text{ x } (40)^2$$

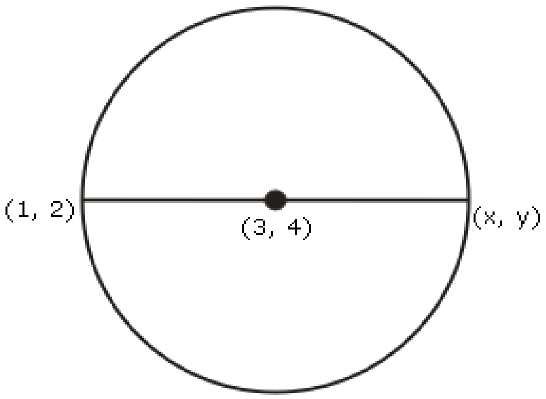
$$= \frac{1}{10} \text{ x } \frac{22}{7} \text{ x } 40 \text{ x } 40 = 502.86 \text{ cm}^2$$

14. Let the other end of the diameter be (x, y). As center is the mid-point of the diameter, we get,

$$x = \frac{x_1 + x_2}{2} \Rightarrow \frac{1+x}{2} = 3 \Rightarrow x = 5$$

$$y = \frac{y_1 + y_2}{2} \Rightarrow \frac{2+y}{2} = 4 \Rightarrow y = 6$$

Thus, the required point is (5, 6).



Section C

15. Discriminant = $b^2 - 4ac = 49 - 4 \text{ x } 6 \text{ x } 2 = 1 > 0$

So, the given equation has two distinct real roots.

Now, $6x^2 - 7x + 2 = 0$

$\Rightarrow 36x^2 - 42x + 12 = 0$

$$\Rightarrow \left(6x - \frac{7}{2}\right)^2 + 12 - \left(\frac{7}{2}\right)^2 = 0$$

$$\Rightarrow \left(6x - \frac{7}{2}\right)^2 - \left(\frac{1}{2}\right)^2 = 0$$

$$\Rightarrow \left(6x - \frac{7}{2}\right)^2 = \left(\frac{1}{2}\right)^2$$

The roots are given by $6x - \frac{7}{2} = \pm \frac{1}{2}$

$\Rightarrow 6x = 4, 3$

$\Rightarrow x = \frac{2}{3}, \frac{1}{2}$

16. When two unbiased coins are tossed the possible outcomes are HH, HT, TH, TT.

Total number of outcomes = 4

i. Number of favourable outcomes of getting exactly two heads = 1 (HH)

P(exactly two heads) = $\frac{1}{4}$

ii. Number of favourable outcomes of getting at least two tails = 1 (TT)

P(at least two tails) = $\frac{1}{4}$

iii. Number of favourable outcomes of getting no tail = 1(HH)

P(no tail) = $\frac{1}{4}$

17. Let a and d respectively be the first term and the common difference of the A.P. respectively.

According to the given conditions,

$a + (m - 1)d = n$ …………(i)

$a + (n - 1)d = m$ ………..(ii)

On solving (i) and (ii), we get,

$d = -1; a = m + n - 1$

Therefore, r^{th} term = $a + (r - 1)d = (m + n - 1) -(r - 1) = m + n - r$

18. It is known that the angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

$\therefore \angle ECB .= 2\angle EDB$

$\Rightarrow m\angle ECB = 2 \times 25° = 50°$

It is also known that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

$\therefore \angle ABC = 90°$

Applying angle sum property of triangles in ΔABC:

$m\angle ABC + m\angle ACB + m\angle BAC = 180°$

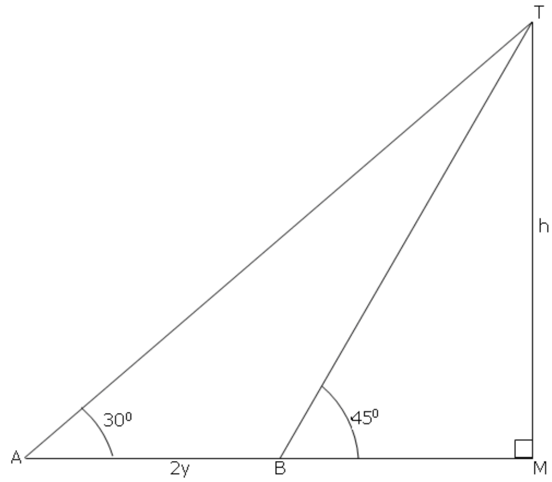
$\Rightarrow 90° + 50° + m\angle BAC = 180°$

$\Rightarrow 140° + m\angle BAC = 180°$

$\Rightarrow m\angle BAC = 180° - 140° = 40°$

Thus, the measure of $\angle BAC$ is 40°.

19. Let TM be the tower of height h. It is given that AB =2y



In ΔBMT,

$\tan 45° = \frac{h}{BM} \Rightarrow h = BM$ .....(i)

In ΔAMT,

$\tan 30° = \frac{h}{2y + BM}$

$\Rightarrow 2y + BM = h\sqrt{3}$

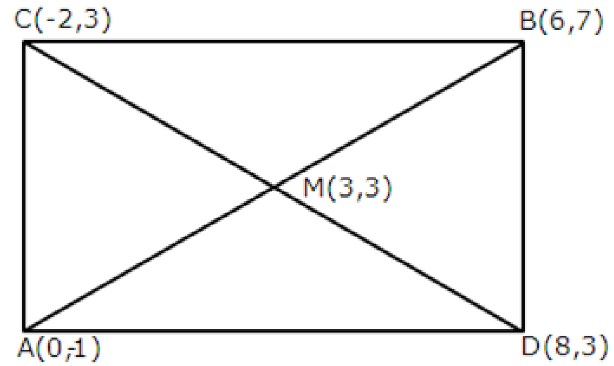
$\Rightarrow h\left(\sqrt{3} - 1\right) = 2y$

$\Rightarrow h = y\left(\sqrt{3} + 1\right)$ m

Thus, the height of the tower is $y(\sqrt{3} + 1)$ metres.

20. The co-ordinates of the mid-point of AB is given by

$\left(\frac{0+6}{2}, \frac{-1+7}{2}\right) = (3, 3)$



The co-ordinates of the mid-point of CD are given by

$\left(\frac{-2+8}{2}, \frac{3+3}{2}\right) = (3, 3)$

$\therefore$ Diagonals AB and CD bisect each other at the point M(3, 3).

By distance formula:

$AD^2 = (8 - 0)^2 + (3 + 1)^2 = 64 + 16 = 80$

$DB^2 = (6 - 8)^2 + (7 - 3)^2 = 4 + 16 = 20$

Also, $AB^2 = (6 - 0)^2 + (7 + 1)^2 = 36 + 64 = 100$

Clearly $AD^2 + DB^2 = AB^2$

Hence the park is rectangular. Its area

$= AD \times DB = \sqrt{80} \times \sqrt{20} = \sqrt{1600} = 40$ sq.km

Yes, as the P.M. of my country, I will try my best to make a policy of creating green parks and gardens in every village. This will help in reducing global warming and help slow down climatic change.

21. Amount of water required to fill the conical vessel = volume of the conical vessel

$= \frac{1}{3}\pi(20)^2 \times 24 = 3200\pi$ cc ....(i)

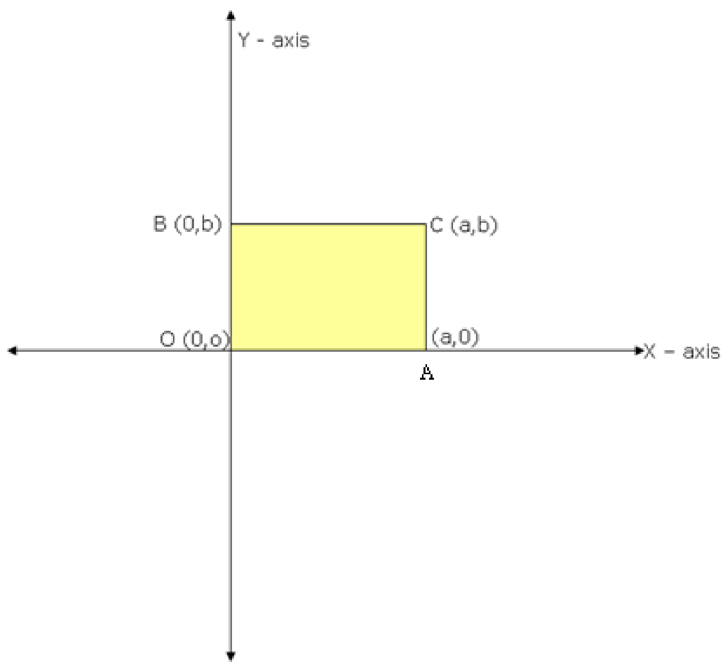
Amount of water which flows out of the cylindrical pipe in 1 minute

$= \pi x \left(\frac{5}{20}\right)^2 \times 10 \times 100 = 62.5\pi$ cc ....(ii)

From (i) and (ii) we get,

Time taken to fill the vessel = $\frac{3200\pi}{62.5\pi}$ = 51.2 minutes

22.



Let OACB be a rectangle such that OA is along the x-axis and OB is along the y-axis.

Let OA = a and OB = b.

Then, the co-ordinates of A and B are (a, 0) and (0, b) respectively.

Since, OACB is a rectangle. Therefore,

AC = OB and OA = BC

Thus, we have:

OA = a and AC = b

So, the coordinates of C are (a, b).

The co-ordinates of the mid-point of OC are $\left(\frac{a+0}{2}, \frac{b+0}{2}\right) = \left(\frac{a}{2}, \frac{b}{2}\right)$

Also, the coordinates of the mid-point of AB are $\left(\frac{a+0}{2}, \frac{b+0}{2}\right) = \left(\frac{a}{2}, \frac{b}{2}\right)$

Clearly, co-ordinates of the mid-point of OC and AB are same.
Hence, OC and AB bisect each other.

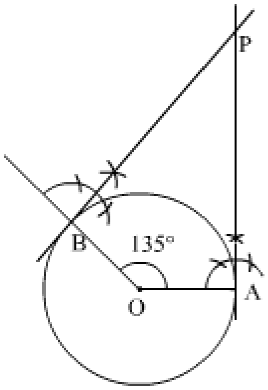
Also, OC = $\sqrt{\left(a^2+b^2\right)}$ and AB $=\sqrt{(a-0)^2+(0-b)^2}=\sqrt{\left(a^2+b^2\right)}$

Therefore, OC = AB.
Hence, the diagonals of a rectangle bisect each other and are equal.

23. Steps of construction:

1) Draw a circle of radius 4 cm and centre O.
2) Take a point A on the circle. Join OA.
3) Draw a perpendicular to OA at A.
4) Draw a radius OB, making an angle of 135^o $(180^o – 45^o)$ with OA.
4) Draw a perpendicular to OB at point B. Let these perpendiculars intersect at P.

PA and PB are the required tangents inclined at angle of 45°.



24. First term = a = 3
Common difference = d = 15 – 3 = 12
We know that the n^{th} term of an A.P. is given by:
$a_n = a + (n – 1)d$
$\therefore a_{60} = 3 + (60 – 1)\ 12 = 3 + 708 = 711$

Now, we need to find the term which is 132 more than 60^{th} term.

132 + 711 = 843

Let 843 be the n^{th} term.

$\therefore 843 = 3 + (n-1)\,12$
$\Rightarrow n-1 = \frac{840}{12} = 70$
$\Rightarrow n = 71$

Section D

25. Let the total number of camels be x.

Number of camels seen in the forest $= \frac{x}{4}$

Number of camels gone to the mountains = $2\sqrt{x}$

Remaining number of camels on the bank of river =15

Total number of camels = $15 + 2\sqrt{x} + \frac{x}{4}$

$15 + 2\sqrt{x} + \frac{x}{4} = x$

$3x - 8\sqrt{x} - 60 = 0$

Let $x = y^2$, we get,

$3y^2 - 8y - 60 = 0$

$3y^2 - 18y + 10y - 60 = 0$

$3y(y - 6) + 10(y - 6) = 0$

$(3y + 10)(y - 6) = 0$

$y = 6$ or $y = -\frac{10}{3}$

Now, $y = -\frac{10}{3} \Rightarrow x = \frac{100}{9}$, which is not possible

Therefore, y = 6

$\Rightarrow x = 36$

Hence, the number of camels is 36.

26. Here, a = 52° and d = 8°

Let the polygon have n sides. Then, the sum of interior angles of the polygon is (n – 2)180°

$\Rightarrow S_n = (n - 2)180$

$\Rightarrow \frac{n}{2}[2a + (n - 1)d] = (n - 2)180$

$\Rightarrow \frac{n}{2}[104 + (n-1)8] = (n - 2)180$

$\Rightarrow \frac{n}{2}[8n + 96] = (n - 2)180$

$\Rightarrow 8n^2 + 96n = 360n - 720$

$\Rightarrow 8n^2 - 264n + 720 = 0$

$\Rightarrow n^2 - 33n + 90 = 0$

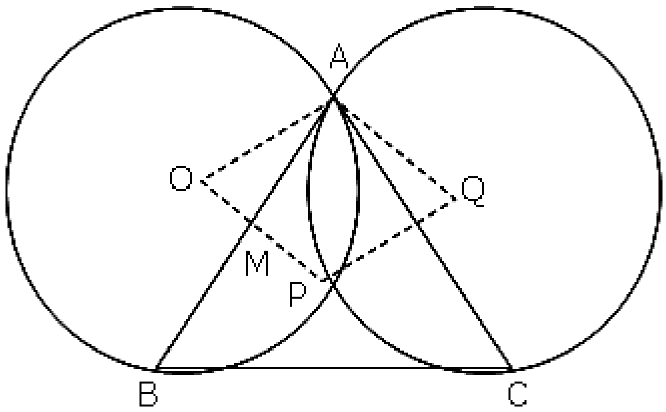
$\Rightarrow (n - 30)(n - 3) = 0$

$\Rightarrow n = 3, 30$

Hence, the number of sides of the polygon can be either 3 or 30.

27. It is known that the radius through the point of contact is ⊥ to the tangent.

$\therefore$ AQ ⊥ AB



Also, AQ || OP (Opposite sides of a ||gm are parallel)

$\therefore$ OP ⊥ AB ($\because$ AQ || OP and AQ ⊥ AB)

Let OP intersects AB at M.

$\therefore$ OM ⊥ AB

It is also known that the perpendicular from the centre of a circle to a chord bisects the chord.

$\therefore$ AM = MB

Thus, OM and hence OP is the perpendicular bisector of AB. Similarly, PQ is the perpendicular bisector of AC.

Now in ΔABC,

OP is the perpendicular bisector of side AB.

$\therefore$ PA = PB

($\because$ Any point of the perpendicular bisector is equidistant from the fixed points)

Similarly, PA = PC.

Hence, PA = PB =PC

Thus, P is equidistant from the three vertices of ΔABC.

Now, the circle with P as centre and its distance from any vertex as radius passes through the three vertices of ΔABC and thus the point P is the circumcentre of ΔABC.

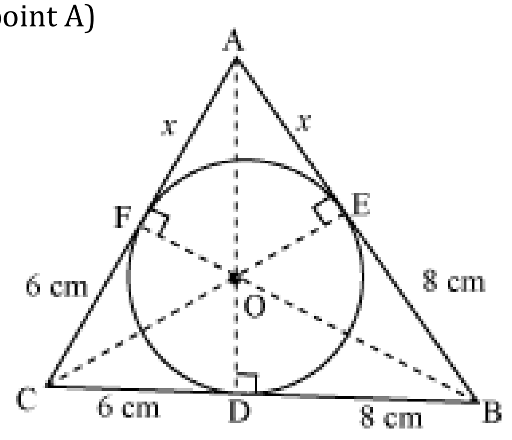
28. Let given circle touch the sides AB and AC of the triangle at point E and F respectively and the length of line segment AF be x.

Now in ΔABC we may observe that:

CF = CD = 6cm (tangents on circle from point C)

BE = BD = 8cm (tangents on circle from point B)

AE = AF = x (tangents on circle from point A)



AB = AE + EB = x + 8

BC = BD + DC = 8 + 6 = 14

CA = CF + FA = 6 + x

2s = AB + BC + CA

= x + 8 + 14 + 6 + x

= 28 + 2x

s = 14 + x

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$$=\sqrt{\{14+x\}\{(14+x)-14\}\{(14+x)-(6+x)\}\{(14+x)-(8+x)\}}$$

$$=\sqrt{(14+x)(x)(8)(6)}$$

$$=4\sqrt{3(14x+x^2)}$$

Area of $\Delta OBC = \frac{1}{2}\times OD\times BC = \frac{1}{2}\times 4\times 14 = 28$

Area of $\Delta OCA = \frac{1}{2}\times OF\times AC = \frac{1}{2}\times 4\times(6+x) = 12 + 2x$

Area of $\Delta OAB = \frac{1}{2}\times OE\times AB = \frac{1}{2}\times 4\times(8+x) = 16 + 2x$

Area of ΔABC = Area of ΔOBC + Area of ΔOCA + Area of ΔOAB

$$4\sqrt{3(14x+x^2)} = 28 + 12 + 2x + 16 + 2x$$

$$4\sqrt{3(14x+x^2)} = 56+4x$$

$$\sqrt{3(14x+x^2)} = 14+x$$

$$3(14x+x^2) = (14+x)^2$$

$$42x+3x^2 = 196+x^2+28x$$

$$2x^2+14x-196=0$$

$$x^2+7x-98=0$$

$$x^2+14x-7x-98=0$$

$$x(x+14)-7(x+14)=0$$

$$(x+14)(x-7)=0$$

Hence, either $x + 14 = 0$ or $x - 7 = 0$
So, $x = -14$ and 7
But $x = -14$ is not possible as length of sides cannot be negative.
So, $x = 7$
Hence $AB = x + 8 = 7 + 8 = 15$ cm
$CA = 6 + x = 6 + 7 = 13$ cm

29. Let r = 4 cm be the radius of the hemisphere and the cone and h = 4 cm be the height of the cone.

Volume of the toy = volume of the hemisphere + volume of the cone

$$= \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$$

$$= \frac{2\times22}{3\times7} x 4^3 + \frac{1\times22}{3\times7}\times4^2\times4 = \frac{1408}{7}\ cm^3$$

A cube circumscribes the given solid. Therefore, edge of the cube should be 8 cm.

Volume of the cube = 8^3 cm^3 = 512 cm^3

Difference in the volumes of the cube and the toy

$= 512 - \frac{1408}{7} = 310.86\ cm^3$

Total surface area of the toy = curved surface area of cone + curved surface area of hemisphere

$= \pi rl + 2\pi r^2$, where l = $\sqrt{h^2 + r^2} = 4\sqrt{2}$

$= \pi r(l + 2r)$

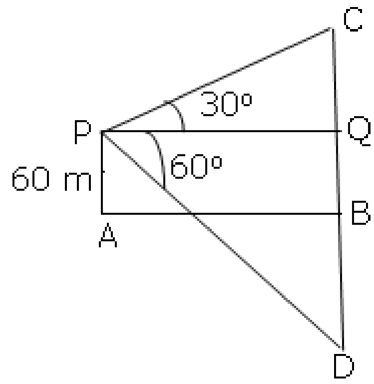
$= \frac{22}{7} \times 4 \times (4\sqrt{2} + 2\times4)$

$= \frac{88}{7}(4\sqrt{2}+8) = 171.67\ cm^2$

30. Let C be the cloud and D be its reflection. Let the height of the cloud be H metres.

BC = BD = H

BQ = AP = 60 m. Therefore CQ = H − 60 and DQ = H + 60



In ΔCQP,

$\frac{PQ}{CQ} = \cot 30°$

$\Rightarrow \frac{PQ}{H-60} = \sqrt{3} \Rightarrow PQ = (H - 60)\sqrt{3}$ m ....(i)

In ΔDQP,

$\frac{PQ}{DQ} = \cot 60°$

$\Rightarrow \frac{PQ}{H+60} = \frac{1}{\sqrt{3}} \Rightarrow PQ = \frac{(H+60)}{\sqrt{3}}$ ....(ii)

From (i) and (ii),

$(H-60)\sqrt{3} = \frac{(H+60)}{\sqrt{3}}$

$\Rightarrow$ 3H − 180 = H + 60 $\quad\Rightarrow$ H = 120

Thus, the height of the cloud is 120 m.

31. Diameter of graphite = 1mm = 0.1cm

Therefore, radius of graphite = $\frac{0.1}{2}$ = 0.05 cm

Length of pencil = 10 cm

Volume of graphite = $\pi r^2 h = \frac{22}{7} \times (.05)^2 \times 10 = 0.0785\ cm^3$

Therefore, weight of graphite = volume × density

= 0.0785 × 2.3

= 0.180 gm

Diameter of the pencil = 0.7 cm

Therefore, radius of the pencil = 0.35 cm

Therefore, volume of the pencil = $\pi R^2 h = \frac{22}{7} \times (0.35)^2 \times 10$

Therefore, volume of wood = Volume of pencil – Volume of graphite

Therefore, Volume of wood = $\pi R^2 h - \pi r^2 h$

$= \pi h(R^2 - r^2)$

$= \frac{22}{7} \times 10[(0.35)^2 - (0.05)^2]$

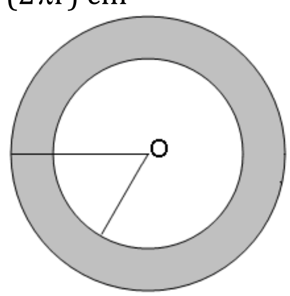
$= 3.771\ cm^3$

Weight of wood = Volume × density

= 3.771 × .6

= 2.2626 gm

32. Let the radius of inner circle be r cm. Then, its circumference = $(2\pi r)$ cm



$\therefore 2\pi r = 88 \Rightarrow 2 \times \frac{22}{7} \times r = 88$

$\therefore r = \frac{88 \times 7}{44} = 14$ cm

So, radius of the inner circle, i.e., $r_1 = 14$ cm

Let r_2 be the radius of the outer circle.

$\therefore$ Area of the ring = $\pi r_2^2 - \pi r_1^2 = \pi(r_2^2 - r_1^2)cm^2$

$$= \frac{22}{7}(r_2^2 - 14^2)cm^2 = \frac{22}{7}(r_2^2 - 196)\ cm^2$$

By the given condition,

$$\frac{22}{7} r_2^2 - \frac{22}{7} \times 196 = 346.5$$

$$\Rightarrow \quad \frac{22}{7} r_2^2 - 616 = 346.5 \Rightarrow \frac{22}{7} r_2^2 = 616 + 346.5 = 962.5$$

$$\Rightarrow \quad r_2^2 = \frac{962.5 \times 7}{22} = 306.25 \Rightarrow r_2 = \sqrt{306.25} = 17.5\ cm$$

Hence, the radius of the outer circle is 17.5 cm.

33. Let the number of blue, green, and white beads be b, g, and w respectively.

$\therefore\ b + g + w = 45$

Let the probability of selecting a blue, green, and white bead be P(B), P(G), and P(W) respectively.

$P(B) = \frac{1}{3}$ and $P(G) = \frac{4}{9}$

P(B) + P(G) + P(W) = 1

$\Rightarrow \frac{1}{3} + \frac{4}{9} + P(W) = 1$

$\Rightarrow \frac{7}{9} + P(W) = 1$

$\Rightarrow P(W) = 1 - \frac{7}{9} = \frac{2}{9}$

$\therefore \frac{w}{b+g+w} = \frac{2}{9}$

$\Rightarrow \frac{w}{45} = \frac{2}{9}$

$\therefore w = \frac{2 \times 45}{9} = 10$

Thus, there are ten white beads in the jar.

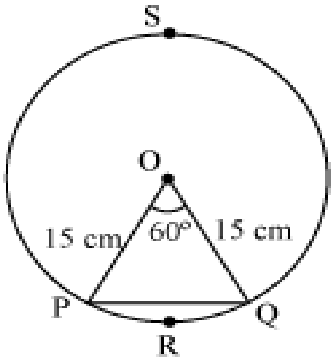
34. Radius (r) of circle = 15 cm

Area of sector OPRQ $= \frac{60°}{360°} \times \pi r^2$

$= \frac{1}{6} \times \frac{22}{7} \times (15)^2$

$= \frac{11 \times 75}{7} = \frac{825}{7}$

$= 117.85 \text{ cm}^2$



In ΔOPQ,

∠OPQ = ∠OQP (as OP = OQ)

∠OPQ + ∠OQP + ∠POQ = 180°

2 ∠OPQ = 120°

∠OPQ = 60°

ΔOPQ is an equilateral triangle.

$$\text{Area of } \Delta OPQ = \frac{\sqrt{3}}{4} \times (\text{side})^2$$
$$= \frac{\sqrt{3}}{4} \times (15)^2 = \frac{225\sqrt{3}}{4} \text{ cm}^2$$
$$= 56.25\sqrt{3}$$
$$= 97.425 \text{ cm}^2$$

Area of segment PRQ = Area of sector OPRQ – Area of ΔOPQ
= 117.85 – 97.425
= 20.425 cm^2

Area of major segment PSQ = Area of circle – Area of segment PRQ

$$= \pi(15)^2 - 20.425$$
$$= \frac{225 \times 22}{7} - 20.425$$
$$= \frac{4950}{7} - 20.425 = 707.14 - 20.425$$
$$= 686.715 \text{ cm}^2$$