Goa Board
Class X Mathematics
Term 1
Sample Paper – 4 Solution

Time: 3 hours **Total Marks: 90**

Section A

1. Correct answer: A

The graph of the polynomial touches the x-axis at only one point.
Thus, there is only one zero is the polynomial p(x).

2. Correct answer: B

Since DE || BC, corresponding angles are equal.

$\angle ADE = \angle ABC$

$\angle AED = \angle ACB$

$\therefore \Delta ADE \sim \Delta ABC$ (By AA similarity criterion)

We know that corresponding sides of similar triangles are proportional.

$\therefore \frac{AD}{AB} = \frac{AE}{AC}$ ...(1)

$\text{Given}: \frac{AD}{DB} = \frac{2}{3}$

$\Rightarrow \frac{AD}{DB + AD} = \frac{2}{3+2}$

$\Rightarrow \frac{AD}{AB} = \frac{2}{5}$

From (1):

$\frac{AD}{AB} = \frac{AE}{AC}$

$\Rightarrow \frac{2}{5} = \frac{AE}{AE + EC}$

$\Rightarrow \frac{2}{5} = \frac{AE}{AE + 4}$

$\Rightarrow 2AE + 8 = 5AE$

$\Rightarrow 3AE = 8 \text{ cm}$

$\Rightarrow AE = 2.6 \text{ cm}$

3. Correct answer: A

$\tan x = \sin 45^o \div \cos 45^o$

$\tan x = \frac{1}{\sqrt{2}} \div \frac{1}{\sqrt{2}} = 1$

We know that $\tan 45^o = 1$

Thus, $x = 45^o$

4. Correct answer: A
The highest frequency is corresponding to the class 30-40.
Thus, the modal class is 30-40.

Section B

5. **Sum of zeroes =** $3+\sqrt{2}+3-\sqrt{2}=6$

Product of zeroes = $\left(3+\sqrt{2}\right)\left(3-\sqrt{2}\right)=(3)^2-\left(\sqrt{2}\right)^2=9-4=5$

The required quadratic polynomial is

$x^2-(\text{Sum of zeroes})x+(\text{Product of zeroes})$

x^2-6x+5

6. Given, G is the mid-point of the side PQ of ΔPQR.

$\therefore$ PG = GQ or PQ = 2PG ... (1)

Now, since GH || QR. Using Basic Proportionality theorem, we get,

$\frac{PG}{PQ}=\frac{PH}{PR}$

$\Rightarrow \frac{PG}{2PG}=\frac{PH}{PR}$ [Using (1)]

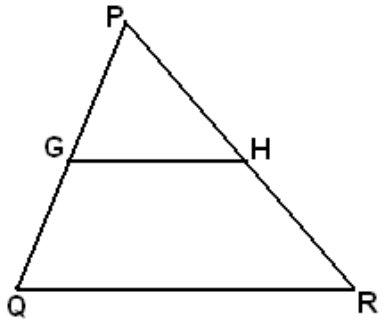
$\Rightarrow \frac{1}{2}=\frac{PH}{PR}$

$\Rightarrow PR=2PH$

$\Rightarrow PH+HR=2PH$

$\Rightarrow HR=PH$

Hence, H is the mid-point of PR.



7. 867 and 255

Since 867 > 255, apply Euclid's division lemma, to a =867 and b=255 to find q and r such that 867 = 255q+r, $0 \leq r < 255$

On dividing 867 by 255 we get quotient as 3 and remainder as 102

i.e. 867 = 255 × 3 + 102

Since remainder 102 ≠ 0, we apply the division lemma to a=255 and b= 102 to find whole numbers q and r such that

255 = 102q + r where $0 \leq r<102$

On dividing 255 by 102 we get quotient as 2 and remainder as 51

i.e. 255 = 102 × 2 + 51

Again remainder 51 is non-zero, so we apply the division lemma to a = 102 and b = 51 to find whole numbers q and r such that
$102 = 51q + r$ where $0 \leq r < 51$

On dividing 102 by 51 quotient is 2 and remainder is 0
i.e. $102 = 51 \times 2 + 0$
Since the remainder is zero, the divisor at this stage is the HCF
Since the divisor at this stage is 51, therefore, HCF of 867 and 255 is 51.

8. $p(x) = x^3 + 4x^2 - 3x - 18$
Now, 2 will be a zero of the polynomial if $p(2) = 0$.
We have:
$p(2) = (2)^3 + 4(2)^2 - 3(2) - 18$
$= 8 + 16 - 6 - 18$
$= 24 - 24$
$= 0$
Thus, 2 is a zero of the polynomial $x^3 + 4x^2 - 3x - 18$.

9.

Classes	Class Mark (x_i)	Frequency (f_i)	$f_i x_i$
	5		
0-10	15	7	35
10-20	25	3	45
20-30	35	15	375
30-40		5	175
		$\sum f_i = 30$	$\sum f_i x_i = 630$

Mean = $\frac{\sum f_i x_i}{\sum f_i} = \frac{630}{30} = 21$

10. Let if possible 6^n ends with digit 0.
$\Rightarrow 6^n = 10 \times q$
$(2 \times 3)^n = 2 \times 5 \times q$
$2^n \times 3^n = 2 \times 5 \times q$
5 is a prime factor of $2^n \times 3^n$
Which is not possible as $2^n \times 3^n$ can have only 2 and 3 as prime factors.
Hence, 6^n cannot end with the digit 5.

Section C

11. It is given that ABC is an isosceles triangle.

$\therefore$ AB = AC

$\Rightarrow \angle ABC = \angle ACB$ ... (1)

In ΔABD and ΔECF,

$\angle ADB = \angle EFC$ (Each 90°)

$\angle ABD = \angle ECF$ (Proved above)

$\therefore \Delta ABD \sim \Delta ECF$ (By using AA similarity criterion)

Hence, $\frac{AB}{EC} = \frac{AD}{EF}$

12. We know that:

$1 + \cot^2 A = \operatorname{cosec}^2 A$

$$\Rightarrow \frac{1}{1+\cot^2 A} = \sin^2 A$$

$$\Rightarrow \frac{1}{1+\cot^2 A} = \sin^2 A$$

$$\Rightarrow \sin A = \frac{1}{\sqrt{1+\cot^2 A}}$$

Also,

$1 + \tan^2 A = \sec^2 A$

$$\Rightarrow 1 + \frac{1}{\cot^2 A} = \sec^2 A$$

$$\Rightarrow \frac{1+\cot^2 A}{\cot^2 A} = \sec^2 A$$

$$\Rightarrow \sec A = \sqrt{\frac{1+\cot^2 A}{\cot^2 A}} = \frac{\sqrt{1+\cot^2 A}}{\cot A}$$

13. Let $f(x) = 2x^2 - 3x + p$

If $f(a) = 0$, then it is said that a is a zero of $f(x)$.

Given, 3 is a zero of $f(x)$.

$\therefore f(3) = 0$

$2(3)^2 - 3(3) + p = 0$

$18 - 9 + p = 0$

$p = -9$

$\therefore f(x) = 2x^2 - 3x - 9$

$= 2x^2 - 6x + 3x - 9$

$= 2x(x - 3) + 3(x - 3)$

$= (x - 3)(2x + 3)$

Thus, the other zero of $f(x)$ is $\frac{-3}{2}$.

14. Let the fixed charge for first three days and each day charge thereafter be Rs. *x* and Rs. *Y,* respectively.

According to the question,

$x + 4y = 27 \quad \ldots(1)$

$x + 2y = 21 \quad \ldots(2)$

Subtracting equation (2) from equation (1), we obtain:

$2y = 6$

$y = 3$

Substituting the value of *y* in equation (1), we obtain:

$x + 12 = 27$

$x = 15$

Hence, the fixed charge is Rs. 15 and the charge per day is Rs. 3.

Value reflected: Honesty, generosity and ethics is displayed through this activity.

15. From the data given as above we may observe that maximum class frequency is 12 belonging to class interval 65 – 75.
So, modal class = 65 – 75
Lower class limit (l) of modal class = 65
Frequency (f_1) of modal class = 12
Frequency (f_0) of class preceding the modal class = 11
Frequency (f_2) of class succeeding the modal class = 9
Class size (h) = 10

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$
$$= 65 + \left(\frac{12 - 11}{2(12) - 11 - 9}\right) \times 10$$
$$= 65 + \frac{1}{4} \times 10$$
$$= 65 + 2.5$$
$$= 67.5$$

Value depicted: dedication towards growth of society, responsible

16.
$$\frac{\cos 37^\circ . \operatorname{cosec} 53^\circ}{\tan 5^\circ . \tan 25^\circ . \tan 45^\circ . \tan 65^\circ . \tan 85^\circ}$$
$$= \frac{\cos 37^\circ . \operatorname{cosec}(90^\circ - 37^\circ)}{\tan 5^\circ . \tan 25^\circ . \tan 45^\circ . \tan(90^\circ - 25^\circ) . \tan(90^\circ - 5^\circ)}$$
$$= \frac{\cos 37^\circ . \sec 37^\circ}{\tan 5^\circ . \tan 25^\circ . \tan 45^\circ . \cot 25^\circ . \cot 5^\circ} \qquad \left[\because \operatorname{cosec}(90^\circ - \theta) = \sec\theta,\ \tan(90^\circ - \theta) = \cot\theta\right]$$
$$= \frac{1}{\tan 5^\circ . \cot 5^\circ . \tan 25^\circ . \cot 25^\circ . 1} \qquad \left[\because \sec\theta = \frac{1}{\cos\theta},\ \tan 45^\circ = 1\right]$$
$$= \frac{1}{1} = 1 \qquad \left[\because \cot\theta = \frac{1}{\tan\theta}\right]$$

17. Let us assume, on the contrary that $\sqrt{5}$ is a rational number.

Therefore, we can find two integers a, b ($b \neq 0$) such that $\sqrt{5} = \frac{a}{b}$

Where a and b are co-prime integers.

$\sqrt{5} = \frac{a}{b}$

$\Rightarrow a = \sqrt{5}b$

$\Rightarrow a^2 = 5b^2$

Therefore, a^2 is divisible by 5 then a is also divisible by 5.

So a = 5k, for some integer k.

$\text{Now}, a^2 = (5k)^2 = 5(5k^2) = 5b^2$

$\Rightarrow b^2 = 5k^2$

This means that b^2 is divisible by 5 and hence, b is divisible by 5.

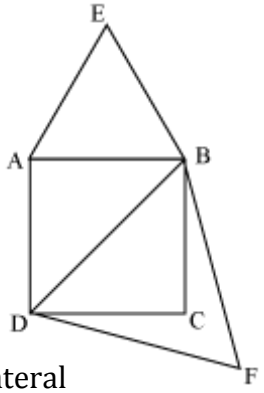
This implies that a and b have 5 as a common factor.

And this is a contradiction to the fact that a and b are co-prime.

So our assumption that $\sqrt{5}$ is rational is wrong.

Hence, $\sqrt{5}$ cannot be a rational number. Therefore, $\sqrt{5}$ is irrational.

18.



Let ABCD be a square of side a.

Therefore, its diagonal $= \sqrt{2}a$

Two desired equilateral triangles are formed as ΔABE and ΔDBF

Length of one side of ΔABE = a

Length of one side of $\Delta DBF = \sqrt{2}a$

We know that equilateral triangles have all angles as 60°. So, all equilateral triangles are similar to each other.

So, the ratio between areas of these triangles will be equal to the square of the ratio of their corresponding sides of these triangles.

$\therefore \frac{\text{area of } \Delta ABE}{\text{area of } \Delta DBF} = \left(\frac{a}{\sqrt{2}a}\right)^2 = \frac{1}{2}$

Hence, proved.

19.

Class	Class Marks (x)	Frequency (f)	Fx
0-10	5	8	40
10-20	15	p	15p
20-30	25	12	300
30-40	35	13	455
40-50	45	10	450
		$\sum f_i = 43 + p$	$\sum f_i x_i = 1245 + 15p$

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i}$$

$$27 = \frac{1245 + 15p}{43 + p}$$

$$1161 + 27p = 1245 + 15p$$

$$27p - 15p = 1245 - 1161$$

$$12p = 84$$

$$p = 7$$

20.

$$= \frac{\frac{\cos A}{\sin A} - \frac{\sin A}{\sin A} + \frac{1}{\sin A}}{\frac{\cos A}{\sin A} + \frac{\sin A}{\sin A} - \frac{1}{\sin A}}$$

$$= \frac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A}$$

$$= \frac{\{(\cot A) - (1 - \operatorname{cosec} A)\}\{(\cot A) - (1 - \operatorname{cosec} A)\}}{\{(\cot A) + (1 - \operatorname{cosec} A)\}\{(\cot A) - (1 - \operatorname{cosec} A)\}}$$

$$= \frac{(\cot A - 1 + \operatorname{cosec} A)^2}{(\cot A)^2 - (1 - \operatorname{cosec} A)^2}$$

$$= \frac{\cot^2 A + 1 + \operatorname{cosec}^2 A - 2\cot A - 2\operatorname{cosec} A + 2\cot A \operatorname{cosec} A}{\cot^2 A - (1 + \operatorname{cosec}^2 A - 2\operatorname{cosec} A)}$$

$$= \frac{2\operatorname{cosec}^2 A + 2\cot A \operatorname{cosec} A - 2\cot A - 2\operatorname{cosec} A}{\cot^2 A - 1 - \operatorname{cosec}^2 A + 2\operatorname{cosec} A}$$

$$= \frac{2\operatorname{cosec} A(\operatorname{cosec} A + \cot A) - 2(\cot A + \operatorname{cosec} A)}{\cot^2 A - \operatorname{cosec}^2 A - 1 + 2\operatorname{cosec} A}$$

$$= \frac{(\operatorname{cosec} A + \cot A)(2\operatorname{cosec} A - 2)}{-1 - 1 + 2\operatorname{cosec} A}$$

$$= \frac{(\operatorname{cosec} A + \cot A)(2\operatorname{cosec} A - 2)}{(2\operatorname{cosec} A - 2)}$$

$$= \operatorname{cosec} A + \cot A$$

$$= \text{R.H.S.}$$

Section D

21. Let *a* be any odd positive integer we need to prove that a is of the form 6q + 1, or $6q+3$, or $6q+5$, where q is some integer.

Since a is an integer. Consider b = 6 as another integer.
Applying Euclid's division lemma, we get,
a = 6q + r, for some integer $q \geq 0$, and $r = 0, 1, 2, 3, 4, 5$ since $0 \leq r < 6$.

Therefore, $a = 6q$ or $6q + 1$ or $6q + 2$ or $6q + 3$ or $6q + 4$ or $6q + 5$
However since a is odd so a cannot take the values 6q, 6q+2 and 6q+4
(since all these are divisible by 2)

Also, $6q + 1 = 2 \times 3q + 1 = 2k_1 + 1$, where k_1 is a positive integer
$6q + 3 = (6q + 2) + 1 = 2(3q + 1) + 1 = 2k_2 + 1$, where k_2 is an integer
$6q + 5 = (6q + 4) + 1 = 2(3q + 2) + 1 = 2k_3 + 1$, where k_3 is an integer
Clearly, 6q + 1, 6q + 3, 6q + 5 are of the form 2k + 1, where k is an integer. So, 6q + 1, 6q + 3, 6q + 5 are odd numbers.

Thus, any odd integer can be expressed in of the form $6q + 1$, or $6q + 3$, or $6q + 5$, where q is some integer.

22.

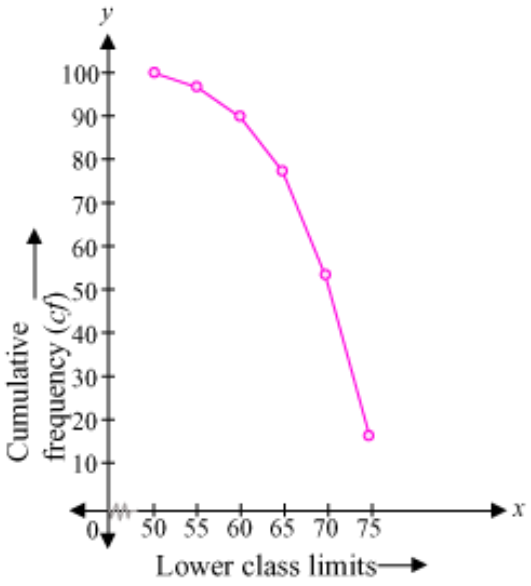
$$2x+3y-7=0$$
$$(a-b)x+(a+b)y-(3a+b-2)=0$$
Here, $a_1=2,\ b_1=3,\ c_1=-7$
$$a_2=(a-b),\ b_2=(a+b),\ c_2=-(3a+b-2)$$
$$\frac{a_1}{a_2}=\frac{2}{a-b},\quad \frac{b_1}{b_2}=\frac{3}{a+b},\quad \frac{c_1}{c_2}=\frac{-7}{-(3a+b-2)}=\frac{7}{(3a+b-2)}$$
For the equations to have infinitely many solutions, we have:
$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$
$$\frac{2}{a-b}=\frac{7}{3a+b-2}$$
$$6a+2b-4=7a-7b$$
$$a-9b=-4 \qquad \ldots(1)$$
$$\frac{2}{a-b}=\frac{3}{a+b}$$
$$2a+2b=3a-3b$$
$$a-5b=0 \qquad \ldots(2)$$

Subtracting (1) from (2), we obtain:
4b = 4
b = 1
Substituting the value of *b* in equation (2), we obtain:
a - 5 x 1 = 0
a = 5
Thus, the values of *a* and *b* are 5 and 1 respectively.

23. We can obtain cumulative frequency distribution of more than type as following –

Over (lower class limits)	**Cumulative frequency**
More than or equal to 50	100
More than or equal to 55	100 – 2 = 98
More than or equal to 60	98 – 8 = 90
More than or equal to 65	90 – 12 = 78
More than or equal to 70	78 – 24 = 54
More than or equal to 75	54 – 38 = 16

Now taking lower class limits on *x*-axis and their respective cumulative frequencies on *y*-axis, we can obtain its ogive as follows.



24.

$$\text{L.H.S} = (\sin\theta + \text{cosec}\,\theta)^2 + (\cos\theta + \sec\theta)^2$$

$$= \sin^2\theta + \text{cosec}^2\theta + 2\sin\theta\,\text{cosec}\,\theta + \cos^2\theta + \sec^2\theta + 2\cos\theta\sec\theta$$

$$= \sin^2\theta + \cos^2\theta + \text{cosec}^2\theta + \sec^2\theta + 2\sin\theta\left(\frac{1}{\sin\theta}\right) + 2\cos\theta\left(\frac{1}{\cos\theta}\right)$$

$$= 1 + 1 + \cot^2\theta + 1 + \tan^2\theta + 2 + 2$$

$$= 7 + \tan^2\theta + \cot^2\theta$$

$$= \text{R.H.S}$$

25. Since triangle ABC and PQR are similar ,

So $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = k(\text{say})$

$\Rightarrow \frac{AB + BC + AC}{PQ + QR + PR} = k$

So according to question $k = \frac{4}{5}$

Let a side of triangle ABC be 4y and that of PQR is 5y.We know that for two similar triangles the ratio of areas is equal to the ratio of squares of their corresponding sides.

Let the area of triangle ABC is S and that of triangle PQR is S'.

So $\frac{s}{s'} = \frac{16y^2}{25y^2}$.but s + s' = 164 cm^2(given)

So $16y^2 + 25y^2 = 164\ cm^2$

$41y^2 = 164\ cm^2$

$y^2 = 4$

So the area of triangle ABC is 64 cm^2.

Also, the area of triangle PQR is 100 cm^2.

26.

Let the price of Apple per Kg = Rs. x

And the price of Grapes per Kg = Rs. y

According to the given situation, the equations are:-

$2x + y = 160$ (1)

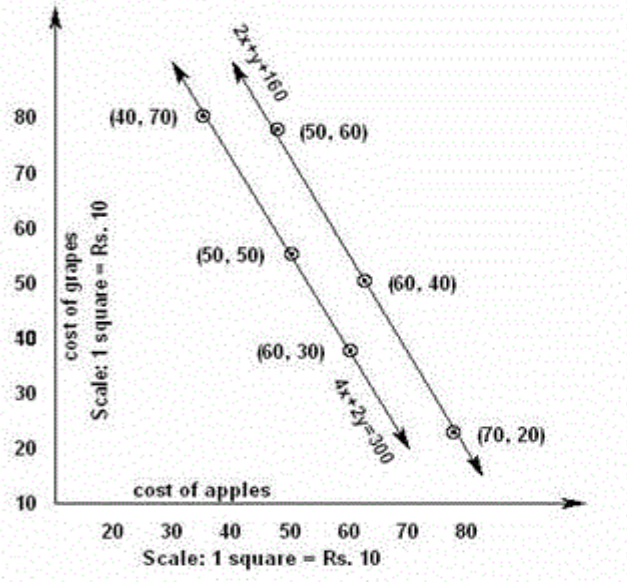
$4x + 2y = 300$ (2)

Table for (1) $y = 160 - 2x$

Table for (2) $y = \frac{300 - 4x}{2}$

x	50	60	70
y	60	40	20

x	40	50	60
y	70	50	30



In this graph, clearly we can see that the graphical representation of the two lines is parallel, which show that the system has no unique solution. (1)

This shows that price is different in the two markets

27.

$$\frac{\sin 30^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ}$$

$$= \frac{\frac{1}{2} - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2}}$$

$$= \frac{\frac{\sqrt{3} - 4}{2\sqrt{3}}}{\frac{4 + \sqrt{3}}{2\sqrt{3}}}$$

$$= \frac{\sqrt{3} - 4}{\sqrt{3} + 4}$$

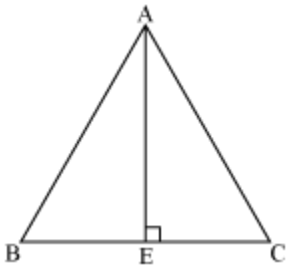
$$= \frac{\sqrt{3} - 4}{\sqrt{3} + 4} \times \frac{\sqrt{3} - 4}{\sqrt{3} - 4}$$

$$= \frac{3 + 16 - 8\sqrt{3}}{3 - 16}$$

$$= \frac{19 - 8\sqrt{3}}{-13}$$

$$= \frac{8\sqrt{3} - 19}{13}$$

28.



Let side of equilateral triangle be a. Also, let AE be the altitude of ΔABC

So, $BE = EC = \frac{BC}{2} = \frac{a}{2}$

By applying the Pythagoras theorem in ΔABE, we have:

$AB^2 = AE^2 + BE^2$

$a^2 = AE^2 + \left(\frac{a}{2}\right)^2$

$AE^2 = a^2 - \frac{a^2}{4}$

$AE^2 = \frac{3a^2}{4}$

$4AE^2 = 3a^2$.

Or, $4AE^2 = 3 \times (\text{square of one side})^2$

Hence, proved.

29. $\tan(A + B) = \sqrt{3}$

$\Rightarrow \tan(A + B) = \tan 60^\circ$

$\Rightarrow A + B = 60^\circ$ ... (i)

$\tan(A - B) = \frac{1}{\sqrt{3}}$

$\Rightarrow \tan (A - B) = \tan 30^\circ$

$\Rightarrow A - B = 30^\circ$ ... (ii)

Adding equations (i) and (ii), we get:

$2A = 90°$

$A = 45°$

From equation (i),

$45° + B = 60°$

$B = 15°$

Thus, $\angle A = 45°$ and $\angle B = 15°$

30. We can find cumulative frequencies with their respective class intervals as shown below:

No. of hours per week	Number of students (f_i)	Cumulative frequency
1 – 4	6	6
4 – 7	30	30 + 6 = 36
7 – 10	40	36 + 40 = 76
10 – 13	16	76 + 16 = 92
13 – 16	4	92 + 4 = 96
16 – 19	4	96 + 4 = 100
	Total (n) = 100	

Now we may observe that cumulative frequency just greater than $\frac{n}{2}\left(\text{i.e. } \frac{100}{2} = 50\right)$ is 76 belonging to class interval 7 – 10.

Median class = 7 – 10

Lower limit (l) of median class = 7

Cumulative frequency (cf) of class preceding median class = 36

Frequency (f) of median class = 40

Class size (h) = 3

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$= 7 + \left(\frac{50 - 36}{40}\right) \times 3$$

$$= 7 + \frac{14 \times 3}{40}$$

$$= 8.05$$

Value depicted: Team spirit, cooperation

31. Let $f(x) = -x^3 + 3x^2 - 3x + 5$ and $g(x) = -x^2 + x - 1$

$$
\begin{array}{r}
x - 2 \\
-x^2 + x - 1 \overline{)\, -x^3 + 3x^2 - 3x + 5} \\
\underline{-x^3 + x^2 - x} \\
2x^2 - 2x + 5 \\
\underline{2x^2 - 2x + 2} \\
3
\end{array}
$$

We have:

Quotient, $q(x) = x - 2$ and remainder, $r(x) = 3$

Now,

Quotient $\times$ Divisor + Remainder

$= (x - 2)(-x^2 + x - 1) + 3$

$= -x^3 + x^2 - x + 2x^2 - 2x + 2 + 3$

$= -x^3 + 3x^2 - 3x + 5$ = Dividend

Hence, the division algorithm is verified.