Goa Board
Class X Mathematics
Term 1
Sample Paper – 1 Solution

Time: 3 hours **Total Marks: 90**

Section A

1. Correct answer: A

If the denominator of a rational number is of the form 2^n5^m, then it will terminate After n places if n>m or m places if m>n.

Now, $\frac{2^3}{2^2 5} = \frac{2}{5} = \frac{2}{2^{\circ}5}$ will ternminator after 1 decimal place.

2. Correct answer: B

Because -3 is the root of quadratic polynomial, we have:

$(k-1)(-3)^2 + 1 = 0$

$\Rightarrow 9(k-1) = -1$

$\Rightarrow k-1 = \frac{-1}{9}$

$\Rightarrow k = 1 - \frac{1}{9} = \frac{8}{9}$

3. Correct answer: C

We know:

Mean = $\frac{\text{Sum of observations}}{\text{Number of observations}}$

Mean of 6 numbers = 16

Sum of the 6 observations = 16 ×6 = 96

Mean of 5 observations = 17

Sum of the 5 observations = 17 5 = 85

$\therefore$ Number which is removed = 96 - 85 = 11

4. Correct answer: C

$\angle A = \angle R = 80^\circ$

$\angle B = \angle Q = 60^\circ$

Therefore, using the angle sum property, we have:

$\angle P = 180^\circ - (80^\circ + 60^\circ) = 40^\circ$

Section B

5. $870 = 225 \times 3 + 195$

$225 = 195 \times 1 + 30$

$195 = 30 \times 6 + 15$

$30 = 15 \times 2 + 0$

$\therefore$ HCF (870,225) = 15

6.

In ΔABC, $\angle 1 + \angle 3 = 90^0$

In ΔADE, $\angle 1 + \angle 2 = 90^0$

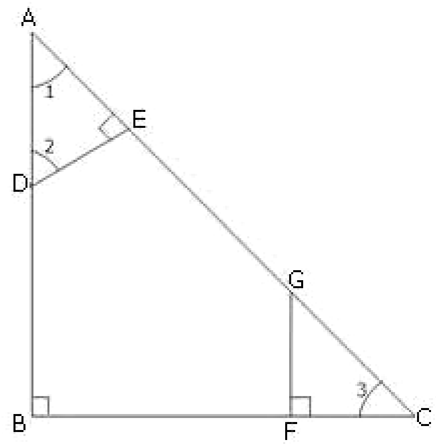
$\angle 1 + \angle 3 = \angle 1 + \angle 2 \Rightarrow \angle 3 = \angle 2$

In ΔADE and ΔGCF

$\angle E = \angle F = 90^0$

$\angle 2 = \angle 3$

$\therefore$ $\Delta ADE \sim \Delta GCF$ (By AA similarity criterion)



7. $\cot \theta = \frac{7}{8}$ (given)

$$\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{1-\sin^2\theta}{1-\cos^2\theta}$$

$$= \frac{\cos^2\theta}{\sin^2\theta}$$

$$= \cot^2\theta$$

$$= \frac{49}{64}$$

8. α, β are roots of $x^2 - (k + 6)x + 2(2k - 1)$

$\alpha + \beta = k + 6, \alpha\beta = 2(2k-1)$

Now, $\alpha + \beta = \frac{1}{2}\alpha\beta \Rightarrow k + 6 = \frac{1}{2} \times 2(2k-1)$

$\Rightarrow k + 6 = 2k - 1$

$\Rightarrow k = 7$

9. In ABC, we have

$AC^2 = BC^2 + AB^2$

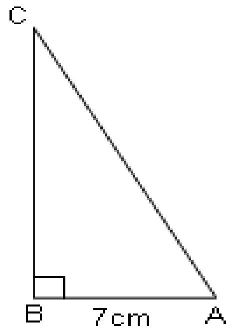
$(1 + BC)^2 = BC^2 + AB^2 (AC - BC = 1 \rightarrow AC = 1 + BC)$

$1 + BC^2 + 2BC = BC^2 + AB^2$

$1 + 2BC = 7^2$

BC = 24 cm and AC = 1 + BC = 25 cm

Hence, $\sin B = \frac{7}{25}$ and $\cos B = \frac{24}{25}$



10.

C.I.	f	c.f.
135 - 140	4	4
140 - 145	7	11
145 - 150	11	22
150 - 155	6	28
155 - 160	7	35
160 - 165	5	40

Here, n = 40 $\Rightarrow \frac{n}{2} = 20$

Median class is 145 - 150

Also, since the highest frequency is 11, the modal class is 145 - 150.

Section C

11. $f(x) = x^2 - 2x + 1$

Zeroes of f(x) are α and β

Sum of zeroes = α + β = 2 and

Product of zeroes = α. β = 1

Now $\frac{2\alpha}{\beta}+\frac{2\beta}{\alpha}=2\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)$

$=2\left(\frac{\alpha^2+\beta^2}{\alpha\beta}\right)$

$=2\frac{\left((\alpha+\beta)^2-2\alpha\beta\right)}{\alpha\beta}=\frac{2\times 2}{1}=4$

Also, $\frac{2\alpha}{\beta}\times\frac{2\beta}{\alpha}=4$

Required polynomial = $k(x^2 - 4x + 4)$, where k is any integer.

12. Let the length and breadth of the rectangle be x and y respectively.

So the original area of the rectangle = xy

According to question,

(x + 2)(y - 2) = xy-28 or xy – 2x + 2y – 4 = xy – 28 or 2x – 2y = 24 .....(i)

Next, (x - 1) (y + 2) = xy + 33 or xy + 2x – y – 2 = xy + 33 or 2x – y = 35 ....(ii)

Now we need to solve (i) and (ii)

Subtracting (i) from (ii) we get, y = 11

Substituting this value in (ii) we get,

2x = 46

X = 23

So the length and breadth of the rectangle are 23 metres and 11 metres, respectively.

13. $\text{LHS} = \sqrt{\frac{\sec\theta - 1}{\sec\theta + 1}} + \sqrt{\frac{\sec\theta + 1}{\sec\theta - 1}}$

$$= \frac{\left(\sqrt{\sec\theta - 1}\right)^2 + \left(\sqrt{\sec\theta + 1}\right)^2}{\left(\sqrt{\sec\theta + 1}\right)\left(\sqrt{\sec\theta - 1}\right)}$$

$$= \frac{\sec\theta - 1 + \sec\theta + 1}{\sqrt{\sec^2\theta - 1}}$$

$$= \frac{2\sec\theta}{\sqrt{\tan^2\theta}}$$

$$= \frac{2\sec\theta}{\tan\theta} = 2 \times \frac{1}{\cos\theta} \times \frac{\cos\theta}{\sin\theta} = 2\operatorname{cosec}\theta = \text{RHS}$$

Hence, LHS = RHS.

14. The system has infinitely many solutions. Therefore,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{2}{a-b} = \frac{3}{a+b} = \frac{7}{3a+b-2}$$

Equating (1) and (2), we get:

2a + 2b = 3a - 3b

or, a = 5b ... (4)

Equating (2) and (3), we get:

9a + 3b - 6 = 7a + 7b

or, 2a - 4b = 6 ... (5)

On solving equations (4) and (5), we get,

10b - 4b = 6 or **b = 1**

Thus, from (4), we get, **a = 5**

15.

Age in yrs.	0-10	10-20	20-30	30-40	40-50	50-60	60-70
No. of persons (f_i)	10	15	25	22	13	10	5

Since, the maximum frequency is 25 and it lies in the class interval 20-30.

Therefore, modal class = 20 - 30

$l = 20, h = 10, f_0 = 15, f_1 = 25, f_2 = 22$

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$= 20 + \left(\frac{25 - 15}{2(25) - 15 - 22}\right) \times 10$$

$= 20 + 7.69 = 27.69$ years (approx.)

16. Given, XY || QR

By using the Basic Proportionality Theorem,

$$\frac{PX}{XQ} = \frac{PY}{YR}$$

$$\Rightarrow \frac{PX}{XQ} + 1 = \frac{PY}{YR} + 1$$

$$\Rightarrow \frac{PX+XQ}{XQ} = \frac{PY+YR}{YR}$$

$$\Rightarrow \frac{PQ}{XQ} = \frac{PR}{YR}$$

$$\Rightarrow \frac{7}{3} = \frac{6.3}{YR}$$

$$\Rightarrow YR = \frac{6.3 \times 3}{7} = 2.7 \text{ c.m}$$

17.

Using sec(90°- θ) = cosec θ, tan(90° - θ) = cotθ

and cos(90° - θ) = sinθ

we have

$$\frac{\sec(90^0 - \theta).\text{cosec}\theta - \tan(90^0 - \theta)\cot\theta + \cos^2 25^\theta + \cos^2 65^0}{3\tan 27^0 \tan 63^0}$$

$$= \frac{\text{cos}ec\theta.\text{cos}ec\theta - \cot\theta.\cot\theta + \cos^2(90^0 - 65^0) + \cos^2 65^0}{3\tan(90^0 - 63^0)\tan 63^0}$$

$$= \frac{\text{cos}ec^2\theta - \cot^2\theta + \sin^2 65^0 + \cos^2 65^0}{3\cot 63^0 \tan 63^0}$$

$$= \frac{1+1}{3} = \frac{2}{3}$$ [Since, $\sin^2\theta + \cos^2\theta = 1$ and $\text{cosec}^2\theta - \cot^2\theta = 1$]

18. Assume the fixed charge = Rs. x

and the subsequent charge = Rs. y

According to the question, we have,

x + 4y = 27 ... (i)

and x + 2y = 21 ... (ii)

Subtracting (ii) from (i), we have,

2y = 6 or y = 3

So, from (i),

x = 27 - 12 = 15

Thus, the fixed charge is Rs. 15 and the charge for each extra day is Rs. 3.

19. In ΔABC, ∠B = 90°

We have:

$$\frac{AB}{AC} = \sin 30^0 = \frac{1}{2} \Rightarrow \frac{5}{AC} = \frac{1}{2} \Rightarrow AC = 10 \text{ cm}$$

And, $$\frac{BC}{AC} = \cos 30^0 = \frac{\sqrt{3}}{2} \Rightarrow \frac{BC}{10} = \frac{\sqrt{3}}{2} \Rightarrow BC = 5\sqrt{3} \text{ cm}$$

20.

CI	50-60	60-70	70-80	80-90	90-100	100-110	Total
f_i	5	3	4	p	2	13	27+p
x_i	55	65	75	85	95	105	
$f_i x_i$	275	195	300	85p	190	1365	2325+85p

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i}$$

Substituting the values,

$$86 = \frac{2325 + 85p}{27 + p}$$

$$86p + 2322 = 2325 + 85p$$

$$p = 3$$

Section D

21.

$$\text{LHS} = \frac{P^2 - 1}{P^2 + 1} = \frac{(\sec\theta + \tan\theta)^2 - 1}{(\sec\theta + \tan\theta)^2 + 1}$$

$$= \frac{\sec^2\theta + \tan^2\theta + 2\sec\theta \, . \, \tan\theta - 1}{\sec^2\theta + \tan^2\theta + 2\sec\theta \, . \, \tan\theta + 1}$$

$$= \frac{(\sec^2\theta - 1) + \tan^2\theta + 2\sec\theta\tan\theta}{\sec^2\theta + (1 + \tan^2\theta) + 2\sec\theta\tan\theta}$$

$$= \frac{\tan^2\theta + \tan^2\theta + 2\sec\theta\tan\theta}{\sec^2\theta + \sec^2\theta + 2\sec\theta\tan\theta}$$

$$= \frac{2\tan^2\theta + 2\sec\theta\tan\theta}{2\sec^2\theta + 2\sec\theta\tan\theta}$$

$$= \frac{2\tan\theta(\tan\theta + \sec\theta)}{2\sec\theta(\tan\theta + \sec\theta)} = \frac{\tan\theta}{\sec\theta}$$

$$= \sin\theta = \text{RHS}$$

22. If the number 15^n, where $n \in N$, was to end with a zero, then its prime factorisation must have 2 and 5 as its factors.

$15 = 5 \times 3$

$15^n = (5 \times 3)^n = 5^n \times 3^n$

So, the prime factors of 15^n include only 5 but not 2.

Also, from the fundamental theorem of Arithmetic, the prime factorisation of a number is unique.

Hence, a number of the form 15^n, where $n \in N$, will never end with a zero.

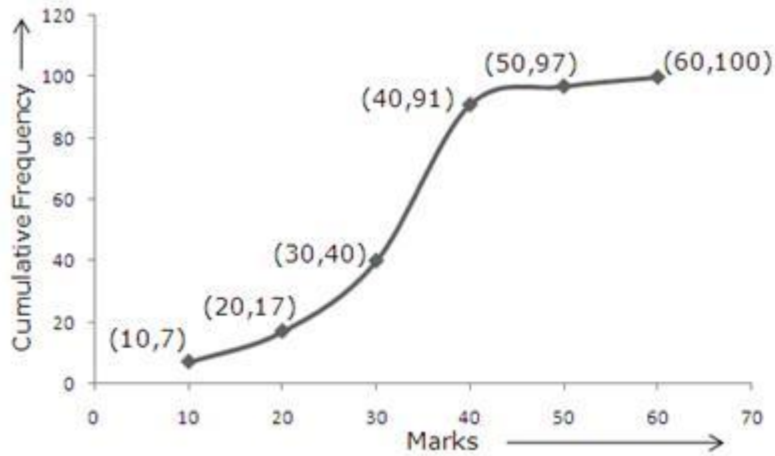
23. We first prepare the cumulative frequency distribution table as given below:

Marks	No. of students	Marks less than	Cumulative frequency
0-10	7	10	7
10-20	10	20	17
20-30	23	30	40
30-40	51	40	91
40-50	6	50	97
50-60	3	60	100

Now, we mark the upper class limits along x-axis by taking a suitable scale and the cumulative frequencies along the y-axis by taking a suitable scale.

Thus, we plot the points (10,7), (20,17), (30,40), (40,91), (50,97) and (60,100).

Join the plotted points by a free hand to obtain the required ogive.



24. To solve the equations, make the table corresponding to each equation.

$2x - y + 6 = 0$

$\Rightarrow y = 2x + 6$ $2x - y + 6 = 0$

$y = 2x+6$

x	-1	-2	-3
y	4	2	0

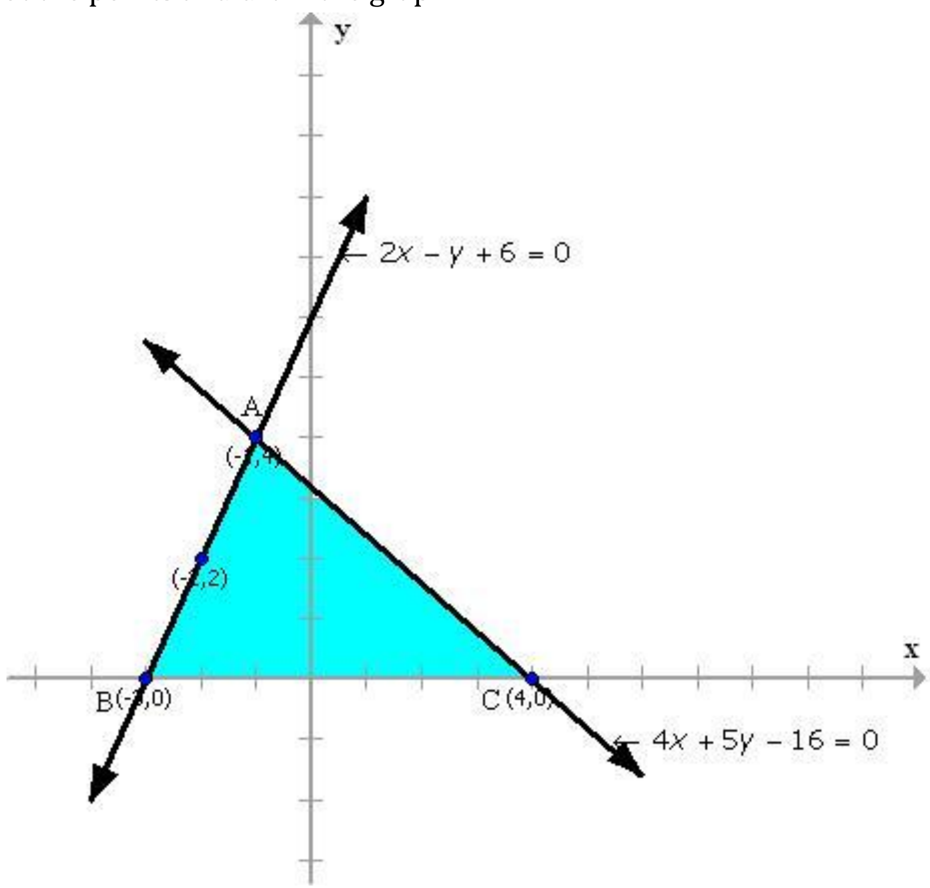
$4x + 5y - 16 = 0$

$\Rightarrow y = \frac{16 - 4x}{5}$ $4x + 5y - 16=0$

$y=\frac{16 - 4x}{5}$

x	4	-1
y	0	4

Now plot the points and draw the graph.



Because the lines intersect at the point (-1, 4), x = -1 and y = 4 is the solution. Also, by observation, vertices of triangle formed by lines and x-axis are A (-1, 4), B (-3, 0) and C (4, 0).

25.

$$\text{LHS} = \sqrt{\frac{1+\sin A}{1-\sin A}} = \sqrt{\frac{1+\sin A}{1-\sin A} \times \frac{1+\sin A}{1+\sin A}}$$

$$= \sqrt{\frac{(1+\sin A)^2}{(1-\sin A)(1+\sin A)}}$$

$$= \sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}}$$

$$= \frac{1+\sin A}{\sqrt{\cos^2 A}}$$

$$= \frac{1+\sin A}{\cos A}$$

$$= \frac{1}{\cos A} + \frac{\sin A}{\cos A}$$

$$= \sec A + \tan A = \text{RHS}$$

26. Let $p(x) = x^3 + 2x^2 + kx + 3$

Using the Remainder theorem, we have:

$p(3) = 3^3 + 2 \times 3^2 + 3k + 3 = 21$

$\Rightarrow k = -9$

Thus, $p(x) = x^3 + 2x^2 - 9x + 3$

$$x^2 + 5x + 6$$

$$x-3 \overline{)\, x^3 + 2x^2 - 9x + 5}$$

$$x^3 - 3x^2$$

$$- \quad +$$

$$5x^2 - 9x + 3$$

$$5x^2 - 15x$$

$$- \quad +$$

$$6x + 3$$

$$6x - 18$$

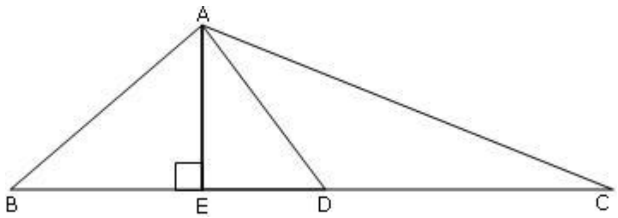
$$- \quad +$$

$$21$$

When p(x) is divided by (x - 3), the quotient is $x^2 + 5x + 6$.

Value indicated: Promotion of cooperative learning among students and helping each other in the study and removal of gender bias.

27. AD is the median of triangle ABC since D is the mid-point of BC.



$\Rightarrow BD = DC = \frac{BC}{2}$ ....(i)

In right triangle AEB,

$AB^2 = AE^2 + BE^2$ (Pythagoras theorem)

$AB^2 = (AD^2 - DE^2) + (BD - DE)^2$

(By using Pythagoras theorem for right triangle AED and BE=BD-DE)

$AB^2 = AD^2 - DE^2 + \left(\frac{BC}{2} - DE\right)^2$ ....from (i)

$AB^2 = AD^2 - DE^2 + \frac{BC^2}{4} + DE^2 - 2\left(\frac{BC \times DE}{2}\right)$

$\Rightarrow AB^2 = AD^2 - BC \times DE + \frac{BC^2}{4}$

Hence proved.

28. To prove: $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec}A + \cot A$

We will make use of the identity: $\operatorname{cosec}^2 A = 1 + \cot^2 A$

LHS

$$= \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$$

$$= \frac{\frac{\cos A}{\sin A} - \frac{\sin A}{\sin A} + \frac{1}{\sin A}}{\frac{\cos A}{\sin A} + \frac{\sin A}{\sin A} - \frac{1}{\sin A}}$$

$$= \frac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A}$$

$$= \frac{\{(\cot A) - (1 - \operatorname{cosec} A)\}\{(\cot A) - (1 - \operatorname{cosec} A)\}}{\{(\cot A) + (1 - \operatorname{cosec} A)\}\{(\cot A) - (1 - \operatorname{cosec} A)\}}$$

$$= \frac{(\cot A - 1 + \operatorname{cosec} A)^2}{(\cot A)^2 - (1 - \operatorname{cosec} A)^2}$$

$$= \frac{\cot^2 A + 1 + \operatorname{cosec}^2 A - 2\cot A - 2\operatorname{cosec} A + 2\cot A \operatorname{cosec} A}{\cot^2 A - (1 + \operatorname{cosec}^2 A - 2\operatorname{cosec} A)}$$

$$= \frac{2\operatorname{cosec}^2 A + 2\cot A \operatorname{cosec} A - 2\cot A - 2\operatorname{cosec} A}{\cot^2 A - 1 - \operatorname{cosec}^2 A + 2\operatorname{cosec} A}$$

$$= \frac{2\operatorname{cosec} A(\operatorname{cosec} A + \cot A) - 2(\cot A + \operatorname{cosec} A)}{\cot^2 A - \operatorname{cosec}^2 A - 1 + 2\operatorname{cosec} A}$$

$$= \frac{(\operatorname{cosec} A + \cot A)(2\operatorname{cosec} A - 2)}{-1 - 1 + 2\operatorname{cosec} A}$$

$$= \frac{(\operatorname{cosec} A + \cot A)(2\operatorname{cosec} A - 2)}{(2\operatorname{cosec} A - 2)}$$

= cosecA + cotA

= RHS

29.

Class Interval	0-6	6-12	12-18	18-24	24-30
Frequency	4	x	5	y	1
Cumulative frequency	4	4+x	9+x	9+x+y	10+x+y

It is given that total frequency N is 20

So, 10 + x + y = 20, i.e. x + y = 10 ....(i)

Given 50% of the observations are greater than 14.4.
So median = 14.4, which lies in the class interval 12-18.
l = 12, cf = 4 + x, h = 6, f = 5, N = 20

$$\text{Median} = \ell + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h$$

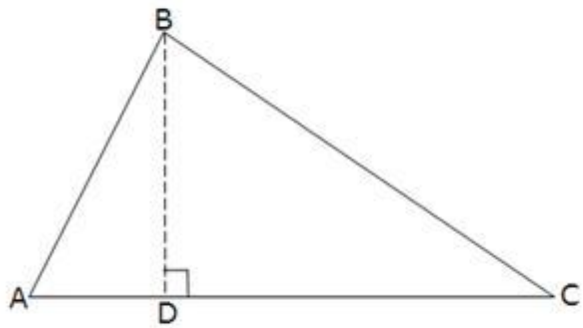
$$14.4 = 12 + \left(\frac{10 - (4 + x)}{5}\right) \times 6$$

$$14.4 - 12 = \frac{(6 - x)}{5} \times 6$$

$$\frac{2.4 \times 5}{6} = 6 - x$$

x = 4
Now using the equation 10 + x + y = 20, we get y = 6.
Hence x = 4 and y = 6.

30. Given: A right triangle ABC right angled at B.
To prove: $AC^2 = AB^2 + BC^2$
Construction: Draw BD $\perp$ AC



Proof:
We know that if a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse, then triangles on both sides of the perpendicular are similar to the whole triangle and to each other.
$\Delta ADB \sim \Delta ABC$
So, $\frac{AD}{AB} = \frac{AB}{AC}$ (Sides are proportional)
Or, $AD.AC = AB^2$ ... (1)
Also, $\Delta BDC \sim \Delta ABC$

So, $\frac{CD}{BC} = \frac{BC}{AC}$

Or CD.AC = BC^2 ... (2)

Adding (1) and (2),

AD.AC + CD.AC = $AB^2 + BC^2$

AC (AD + CD) = $AB^2 + BC^2$

AC.AC = $AB^2 + BC^2$

$AC^2 = AB^2 + BC^2$

Hence proved.

31. Let us assume to the contrary, that $\sqrt{n-1} + \sqrt{n-1}$ is a rational number.

$\Rightarrow \left(\sqrt{n-1} + \sqrt{n-1}\right)^2$ is rational.

$\Rightarrow$ (n-1) + (n+1) + 2$\left(\sqrt{n-1} + \sqrt{n-1}\right)$ is rational

$\Rightarrow$ 2n+2 $\sqrt{n^2 - 1}$ is rational

However, we know that $\sqrt{n^2 - 1}$ is an irrational number.

So 2n+2 $\sqrt{n^2 - 1}$ is also an irrational number

So our basic assumption that the given number is rational is wrong.

Hence, $\sqrt{n-1} + \sqrt{n-1}$ is an irrational number.