Goa Board
Class IX Mathematics
Term II
Sample Paper – 9 Solution

(SECTION – A)

1. Correct Answer: D

Class mark of the class interval 30-38 is $\frac{30+38}{2}$ = 34

2. Correct Answer: A

Given r = 1.4 m and h = 6 m

Capacity of the tank = $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (1.4)^2 \times 6 = 12.32\ m^3$

3. Correct Answer: D

The equation of the x-axis is y = 0.

4. Correct Answer: A

Let base radius of the cylinder = r units

Then diameter of the sphere is = 2 × radius = 2r units

Height of the cylinder is equal to the diameter of sphere.

∴ Height of the cylinder = 2r units

Surface area of sphere = $4\pi r^2$ sq. units. ...(1)

Lateral surface area of cylinder = $2\pi r \cdot (2r) = 4\pi r^2$ ...(2)

From (1) and (2)

We get,

The relation between the surface area of the sphere and lateral surface area of the right circular cylinder which just encloses the sphere is surface area of the sphere is equal to the lateral surface area of the right circular cylinder.

5. Correct Answer: B

The mode is the value that occurs the most frequently in a data set. Hence mode of data is 2.

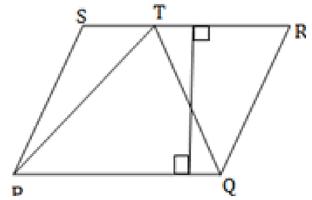
6. Correct Answer: A

Class size is the difference between two successive class marks.

i.e. 10 – 6 = 4

7. Correct Answer: D

In a rhombus and rectangle the diagonals are not equal. Only in a square, the diagonals are equal and bisect each other at right angles.



8. Correct Answer: B

ar(ΔPTQ) = $\frac{1}{2}$ ar(PQRS) = $\frac{1}{2}$ × 6 × 6 = 18 cm^2

(SECTION – B)

9.

P(E) = The probability of 2 students not having the same birthday = 0.0992

The probability of 2 student having the same birthday = $P(\bar{E})$

$P(\bar{E})$ = 1 – P(E) = 1 – 0.992 = 0.008.

10. Here 9 students have blood groups A, 6 have B, 3 have AB and 12 have O.
So, the table representing the data is as follows:

Blood group	**Number of students**
A	9
B	6
AB	3
O	12
Total	30

As 12 students have the blood group O and 3 have blood group AB, the most common blood group is O and the rarest blood group among these students is AB.

11. AD is a median of ΔABC. So, it will divide ΔABC into two triangles of equal areas.

∴ area(ΔABD) = area(ΔACD) ... (1)

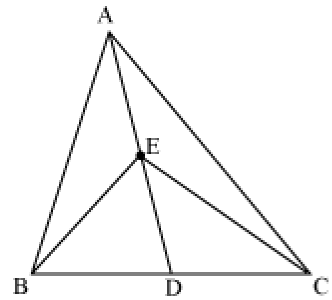
Now ED is median of ΔEBC.

∴ area(ΔEBD) = area(ΔECD) ... (2)

Subtracting equation (2) from equation (1), we have

area(ΔABD) – area(EBD) = area(ΔACD) – area(ΔECD)

area(ΔABE) = area(ΔACE)



12. The number of possible outcomes = 100 + 50 + 20 + 10 = 180

(i) Let E denote the event that the coin is of 50 p.

The number of outcomes favourable to the event E = 100

so, $P(E)=\frac{100}{180}=\frac{5}{9}$

The probability that the coin is 50 p is $\frac{5}{9}$

(*ii*) Let F denote the event the coin is not of Rs. 5.

The number of outcomes favourable to the event F = 170

So, $P(F)=\frac{170}{180}=\frac{17}{18}$

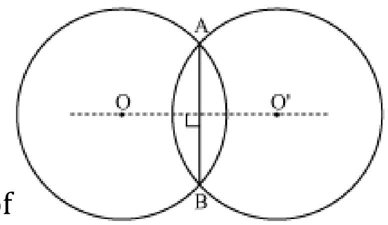
The probability that the coin is not of Rs. 5 coin is $\frac{17}{18}$

13. Consider two circles centered at point O and O', and intersect each other at point A and B respectively.

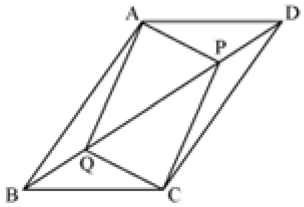
Join AB. AB is the chord for circle centered at O, so the perpendicular bisector of AB will pass through O.

Again AB is also chord of circle centered at O', so the perpendicular bisector of AB will also pass through O'.

Clearly centres of these circles lie on the perpendicular bisector of the common chord.



14.



i. In ΔAPD and ΔCQB

$\angle ADP = \angle CBQ$ (alternate interior angles for BC || AD)

AD = CB (opposite sides of parallelogram ABCD)

DP = BQ (given)

$\therefore \Delta APD \cong \Delta CQB$ (using SAS congruence rule)

ii. As we had observed that $\Delta APD \cong \Delta CQ$

$\therefore$ AP = CQ (CPCT)

(SECTION – C)

15. Let the number of pants purchased by Neha be 'x' & number of skirts purchased be 'y'.

Twice the number of pants = 2x.

From the given condition,

$y = 2x - 2$ (i)

i.e (Number of skirts are 2 less than twice the number of pants.)

$y = 4x - 4$ (ii)

i.e (Number of skirts are 4 less than four times the number of pants)

Subtracting (i) from (ii), we get

$2x = 2$

$\Rightarrow x = 1$

So, the number of pants purchased = 1 and number of skirts = 0.

16. Let the unit's digit be x and the ten's digit be y.

Sum of the digits of a two digit number is given to be 12.

$\therefore$ $x + y = 12$ ....(i)

$\therefore$ Original number = $10y + x$

Number obtained on reversing the digits = $10x + y$

According to the question:

$10y + x + 18 = 10x + y$

$\Rightarrow 9x - 9y = 18$

$\Rightarrow x - y = 2$ ....(ii)

(i) and (ii) are the required equations.

17. If a dice is rolled once, the total number of possible outcomes = 6.

(i) The number of multiples of 1 = 6

$\therefore$ The probability of a multiple of 1 = $\frac{6}{6} = 1$

(ii) The number of multiples of 7 = 0

$\therefore$ The probability of a multiple of 7 = $\frac{0}{6} = 0$

18. Let r be the common radius of a sphere, a cone and cylinder.

Height of sphere = its diameter = 2r

Then, the height of the cone = height of cylinder = height of sphere = 2r

let l be the slant height of cone=$\sqrt{r^2+h^2}=\sqrt{r^2+(2r)^2}=r\sqrt{5}$

$S_1=$ curved surface area of sphere = $4\pi r^2$

$S_2=$ curved surface area of cylinder = $2\pi rh$ = $2\pi r\times 2r$ = $4\pi r^2$

$S_3=$ curved surface area of cone=$\pi rl=\pi r\times r\sqrt{5}=\sqrt{5}\pi r^2$

Ratio of curved surface area as

$\therefore S_1:S_2:S_3=4\pi r^2:4\pi r^2:\sqrt{5}\pi r^2$ = 4 : 4 : $\sqrt{5}$

OR

If the radius of circular cone is given, (r) = 7 cm

Height(h) = 24 cm

$\therefore$ So, the slant height of the cone = $l^2=r^2+h^2$

l = $\sqrt{r^2+h^2}$ = $\sqrt{625}=25\ cm$

The surface area of right circular cone = πrl = $\frac{22}{7}\times 7\times 25$ = 550 cm^2

Area of sheet required to make 10 such caps = 10×550 = 5500 cm^2

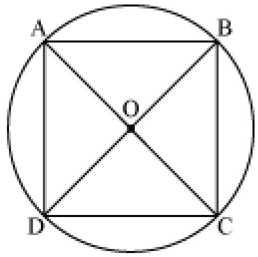
19. Let us find the class marks x_i of each class by taking the average of upper class limit and lower class limit and put them in a table.

Class interval	No. of students(f_i)	Class marks (x_i)	f_ix_i
10-14	15	12	180
15-19	110	17	1870
20-24	135	22	2970
25-29	115	27	3105
30-34	25	32	800
Total	$\sum f_i=400$		$\sum f_ix_i=8925$

$$\bar{x}=\frac{\sum f_ix_i}{\sum f_i}=\frac{8925}{400}=22.31$$

The mean number of oranges kept in basket is 22.31.We have used the 'Direct method'.

20. Let ABCD be a cyclic quadrilateral having diagonals BD and AC, intersecting each other at point O.



$m\angle BAD = \frac{1}{2} m\angle BOD = \frac{180°}{2} = 90°$ (Consider BD as a chord)

$m\angle BCD + m\angle BAD = 180°$ (Cyclic quadrilateral)

$m\angle BCD = 180° - 90° = 90°$

$m\angle ADC = \frac{1}{2} m\angle AOC = \frac{1}{2}(180°) = 90°$ (Considering AC as a chord)

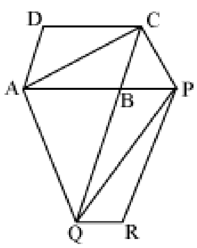
$m\angle ADC + m\angle ABC = 180°$ (Cyclic quadrilateral)

$90° + m\angle ABC = 180°$

$m\angle ABC = 90°$

Here, each interior angle of cyclic quadrilateral is of 90°. Hence it is a rectangle.

21. Construction: Let us join AC and PQ.



ΔACQ and ΔAQP are on same base AQ and between same parallels AQ and CP.

$\therefore$ area (ΔACQ) = area (ΔAPQ)

$\Rightarrow$ area (ΔACQ) – area (ΔABQ) = area (ΔAPQ) – area (ΔABQ)

$\Rightarrow$ area (ΔABC) = area (ΔQBP) ... (1)

Since AC and PQ are diagonals of parallelograms ABCD and PBQR respectively

$\therefore$ area$(\Delta ABC) = \frac{1}{2}$area(ABCD) ... (2)

$\therefore$ area$(\Delta QBP) = \frac{1}{2}$area(PBQR) ... (3)

From equations (1), (2) and (3), we have

$\frac{1}{2}$area(ABCD) = $\frac{1}{2}$area(PBQR)

Therefore, area(ABCD) = area(PBQR)

22. The given tank is cuboidal in shape having its length (l) as 20 m, breadth (b) as 15 m, and height (h) as 6 m.

Capacity of tank = $l \times b \times h$ = $(20 \times 15 \times 6)$ m^3 = 1800 m^3 = 1800000 litres

Water consumed by the people of the village in 1 day = (4000 × 150) litres = 600000 litres

Let water in this tank last for n days.

Water consumed by all people of village in n days = Capacity of tank

$n \times 600000 = 1800000$

$n = 3$

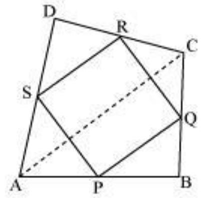
Therefore, the water of this tank will last for 3 days.

OR

We can find the required values from the graph.
The line corresponding to the equation given passes through (6,0).
Therefore, when y = 0 , then x = 6
Similarly, line corresponding to the equation given passes through (0,2)
Therefore, when x = 0, then y = 2

23.

i. SR || AC and SR = ½ AC
ii. PQ = SR
iii. PQRS is a parallelogram.



In ΔADC, S and R are the mid-points of sides AD and CD respectively.
In a triangle, the line segment joining the mid-points of any two sides of the triangle is parallel to the third side and is half of it. (by mid-point theorem)
∴ SR || AC and SR = ½ AC ... (1)

ii. In ΔABC, P and Q are mid-points of sides AB and BC respectively. Therefore, by using the mid-point theorem,
PQ || AC and PQ = ½ AC ... (2)
Using equations (1) and (2), we obtain
PQ || SR and PQ = SR ... (3)
⇒ PQ = SR

iii. From equation (3), we obtained
PQ || SR and PQ = SR
Clearly, one pair of opposite sides of quadrilateral PQRS is parallel and equal.
Hence, PQRS is a parallelogram.

24. The number of outcomes = 90

i. Favourable outcomes for two-digit number
E=10, 11, 12, 13, 14, ...90
No. of favourable outcomes n(E) = 81

$$P(E) = \frac{81}{90} = \frac{9}{10}$$

ii. Favourable outcomes for perfect square
S = 1, 4, 9, 16, 25, 36, 49, 64, 81
No. of favourable outcomes n(S) = 9

$$P(S) = \frac{9}{90} = \frac{1}{10}$$

iii. Favourable outcomes for number divisible by 5

F = 5, 10, 15, 20, 25, 30, 35, ...90.

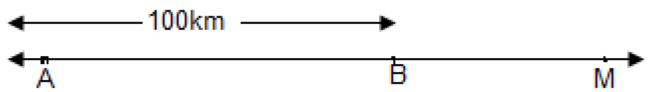
No. of favourable outcomes n(F) = 18

P(F) = $\frac{18}{90} = \frac{1}{5}$

(SECTION – D)

25. Let P and Q be the cars starting from A and B respectively. Let their speeds be x km/hr and y km/hr respectively.

Case – I: When the cars P and Q move in the same direction,



Distance covered by the car P in 5 hours = 5x km

Distance covered by the car Q in 5 hours = 5y km

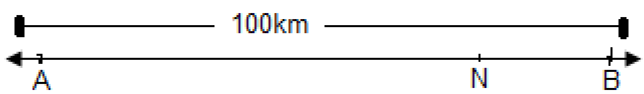
∴ AM = 5x km and BM = 5y km

∴ AM – BM = AB ⇒ 5x – 5y = 100

⇒ 5(x − y) = 100

⇒ x – y = 20 --- (1)

Case – II: When the car P and Q move towards each other,



Distance covered by P in 1 hour = x km

Distance covered by Q in 1 hour = y km

∴ AN = x km and BN = y km

∴ AN + BN = AB

⇒ x + y = 100 ---- (2)

Adding (1) and (2) we get,

2x = 120

⇒ x = 60

Subs. x = 60 in (1) we get y = 40.

Therefore, the speeds of the cars are 60km/hr and 40km/hr respectively.

26. Let the amount to be invested at 10% be x and amount to be invested at 15% be y.

Total amount to be invested is Rs. 12000.

Therefore, x + y = 12000 ... (I)

S.I. on Rs. X at 10% p.a. for 1 year = $\frac{x \times 10 \times 1}{100} = \frac{x}{10}$

S.I. on Rs. X at 15% p.a. for 1 year = $\frac{y \times 15 \times 1}{100} = \frac{3y}{20}$

Now, total amount is to be invested at 12% = $\frac{12000 \times 12 \times 1}{100} = 1440$

$$\frac{x}{10} + \frac{3y}{20} = 1440$$

$2x + 3y = 28800$ --- (2)

Multiplying eq. (1) by 2 and subtracting it from eq. (2) we get,

$$\begin{array}{l} 2x + 3y = 28800 \\ 2x + 2y = 24000 \\ \underline{- \quad - \qquad -} \\ \qquad y = 4800 \end{array}$$

Subs. $y = 4800$ in (1) we get,

$x + 4800 = 12000$

$x = 7200$

Hence the amount to be invested at 10% is Rs. 7200 and the amount to be invested at 15% is Rs. 4800.

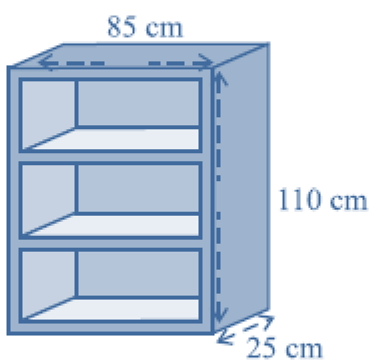
27. External length (l) of bookshelf = 85 cm

External breadth (b) of bookshelf = 25 cm

External height (h) of bookshelf = 110 cm

External surface area of shelf while leaving front face of shelf = lh + 2 (lb + bh)

External surface area of shelf while leaving front face of shelf = $[85 \times 110 + 2(85 \times 25 + 25 \times 110)]$ cm^2 = 19100 cm^2



Area of front face = $[85 \times 110 - 75 \times 100 + 2(75 \times 5)]$ cm^2 = 1850 + 750 cm^2

Area of front face = 2600 cm^2

Area to be polished = (19100 + 2600) cm^2 = 21700 cm^2

Cost of polishing 1 cm^2 area = Rs. 0.20

Cost of polishing 21700 cm^2 area = Rs. (21700×0.20) = Rs. 4340

Now, length (l), breadth (b) height (h) of each row of bookshelf is 75 cm, 20 cm, and 30 cm $\left(= \frac{110-20}{3}\right)$ respectively.

Area to be painted in 1 row = 2 (l + h) b + lh = $[2(75 + 30) \times 20 + 75 \times 30]$ cm^2

Area to be painted in 1 row = (4200 + 2250) cm^2

Area to be painted in 1 row = 6450 cm^2

Area to be painted in 3 rows = (3×6450) cm^2 = 19350 cm^2

Cost of painting 1 cm^2 area = Rs. 0.10

Cost of painting 19350 cm^2 area = Rs. (19350×0.10) = Rs. 1935

Total expense required for polishing and painting the surface of the bookshelf = Rs. (4340 + 1935) = Rs. 6275

28. Cost of white washing 1 m^2 area = Rs. 2

$\therefore$ C.S.A. of the inner side of the dome = $\left(\frac{498.96}{2}\right) m^2$ = 249.48 m^2

Let inner radius of the hemispherical dome be r.

i. C.S.A. of the inner side of the dome = 249.48 m^2

$2\pi r^2 = 249.48\ m^2$

$\Rightarrow 2 \times \frac{22}{7} \times r^2 = 249.48\ m^2$

$\Rightarrow r^2 = \left(\frac{249.48 \times 7}{2 \times 22}\right) m^2 = 39.69\ m^2$

$\Rightarrow$ r = 6.3 m

ii. Volume of air inside the dome = Volume of the hemispherical dome $= \frac{2}{3}\pi r^3$

Volume of air inside the dome $= \left[\frac{2}{3} \times \frac{22}{7} \times (6.3)^3\right] m^3$

Volume of air inside the dome = 523.908 m^3

Thus, the volume of air inside the dome is approximately 523.9 m^3.

29.

i. There are 3 aces in the remaining pack

The number of possible outcomes = 39

Let E be the event 'the card is an ace'.

The number of outcomes favourable to E = 3

Therefore, $P(E) = \frac{3}{39} = 0.0769$

ii. There are 13 diamonds in a pack

The number of possible outcomes = 39

Let E be the event 'the card is a diamond'.

The number of outcomes favourable to E = 13

Therefore, $P(E) = \frac{13}{39} = 0.333$

iii. A card that is not a heart is 39

The number of possible outcomes = 39

Let E be the event 'the card is not an heart'.

The number of outcomes favourable to E = 39

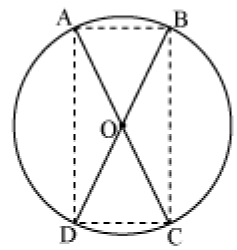
Therefore, $P(E) = \frac{39}{39} = 1$

iv. There are no aces of heart in the remaining pack

The number of possible outcomes = 39

$P(E)=\frac{0}{39}=0$

30. Let two chords AB and CD intersect each other at point O.



In ΔAOB and ΔCOD

OA = OC (given)

OB = OD (given)

$\angle AOB = \angle COD$ (vertically opposite angles)

$\Delta AOB \cong \Delta COD$ (SAS congruence rule)

AB = CD (by CPCT)

Similarly, we can prove $\Delta AOD \cong \Delta COB$

$\therefore$ AD = CB (by CPCT)

Since in quadrilateral ACBD opposite sides are equal in length.

Hence, ACBD is a parallelogram.

We know that opposite angles of a parallelogram are equal

$\therefore \angle A = \angle C$

But $m\angle A + m\angle C = 180°$ (ABCD is a cyclic quadrilateral)

$\Rightarrow m\angle A + m\angle A = 180°$

$\Rightarrow 2m\angle A = 180°$

$\Rightarrow m\angle A = 90°$

As ACBD is a parallelogram and one of its interior angles is 90°, so it is a rectangle.

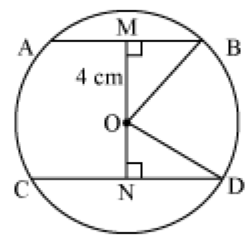
$\angle A$ is the angle subtended by chord BD. And as $m\angle A = 90°$, so BD should be the diameter of the circle. Similarly AC is diameter of the circle.

31. Let AB and CD be two parallel chords in a circle with centre O. Join OB and OD

Distance of smaller chord AB from centre of circle = 4 cm.

OM = 4 cm

$MB = \frac{AB}{2} = \frac{6}{2} = 3cm$



In ΔOMB

$OM^2 + MB^2 = OB^2$

$(4)^2 + (3)^2 = OB^2$

$16 + 9 = OB^2$

$OB = \sqrt{25}$

$OB = 5cm$

In ΔOND

$OD = OB = 5cm \qquad \text{(radii of same circle)}$

$ND = \frac{CD}{2} = \frac{8}{2} = 4cm$

$ON^2 + ND^2 = OD^2$

$ON^2 + (4)^2 = (5)^2$

$ON^2 = 25 - 16 = 9$

$ON = 3$

So, distance of bigger chord from the centre is 3 cm.

32. To Construct: ΔABC

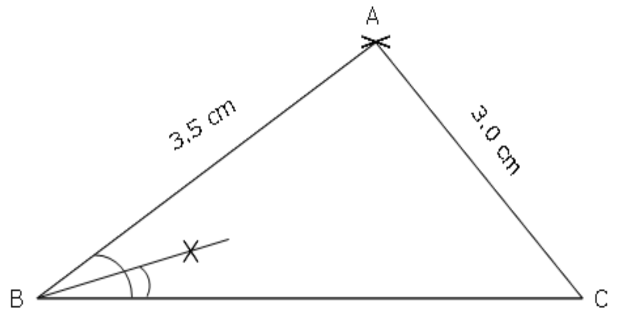
Steps of Construction

i. Draw base BC = 4.8 cm
ii. With B as centre and radius = 3.5 cm, draw an arc.
iii. With C as centre and radius = 3 cm, draw another arc to meet the previous arc at point A.
iv. Join A and B, A and C. Then ΔABC is the required triangle
v. Construction is possible using SSS criterion of congruence.

Since side AC = 3.0 cm is the smallest side.

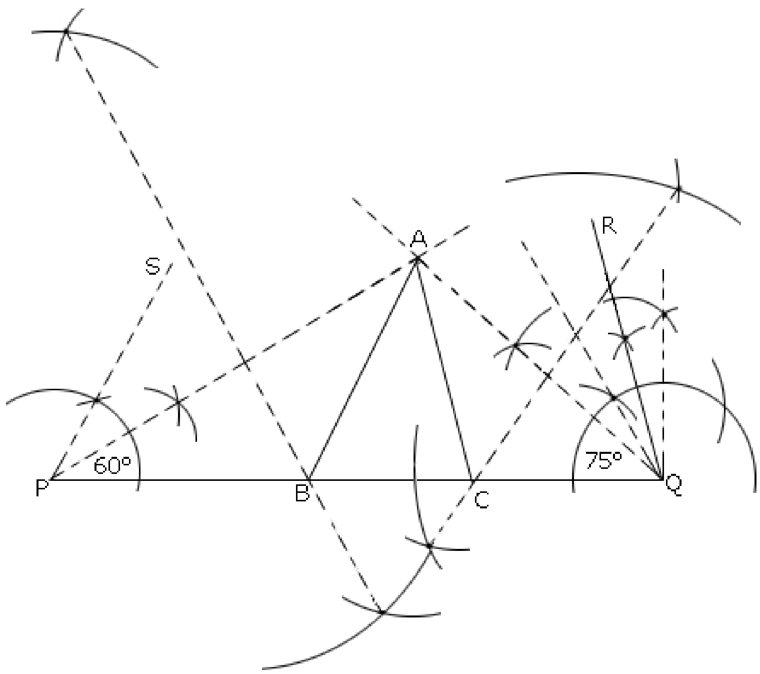
Therefore, ∠ABC is the smallest angle.

i. Bisect ∠ABC.



Measure the equal parts of the angle m∠B = 25°

OR



$$m\angle A = \frac{3}{12} \times 180° = 45°$$

$$m\angle B = \frac{4}{12} \times 180° = 60°$$

$$m\angle B = \frac{5}{12} \times 180° = 75°$$

Steps of construction:

(a) Draw a line PQ = 12.5 cm.

(b) At P, construct m∠SPQ = 60° and at Q, construct m∠RQP = 75°.

(c) Draw the bisectors of ∠SPQ and ∠RQP, intersecting at A.

(d) Draw perpendicular bisectors of AP and AQ and cause them to intersect PQ at B and C respectively.

(e) Join A to B and A to C.

ΔABC is the required triangle.

33. a) Let *S* be the sample space and *A* be the event of a class having 2 left-handed students.

n(*S*) = 30

n(*A*) = 5

$$P(A) = \frac{5}{30} = \frac{1}{6}$$

b) Let *B* be the event of a class having at least 3 left-handed students.

n(*B*) = 12 + 8 + 2 = 22

$$P(B) = \frac{22}{30} = \frac{11}{15}$$

c) First find the total number of left-handed students:

No. of left-handed students, x	0	1	2	3	4	5
Frequency, f (no. of classes)	1	2	5	12	8	2
fx	0	2	10	36	32	10

Total number of left-handed students = 2 + 10 + 36 + 32 + 10 = 90

Here, the sample space is the total number of students in the 30 classes, which was given as 960.

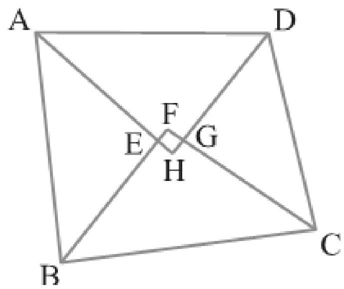
Let T be the sample space and C be the event that a student is left-handed.

n(T) = 960

n(C) = 90

$$P(C)=\frac{90}{960}=\frac{3}{32}$$

34. Given: In quadrilateral ABCD, AH, BF, CF, DH are the angle bisectors of angles of ∠A, ∠B, ∠C and ∠D respectively.



To prove the quadrilateral EFGH is cyclic.

Proof: ABCD is a quadrilateral in which the angle bisectors AH, BF, CF and DH of internal angles ∠A, ∠B, ∠C and ∠D respectively form a quadrilateral EFGH.

Now, ∠FEH = ∠ AEB (vertically opposite angles)

= 180° – m∠EAB – m∠EBA (ASP of a triangle)

m∠FEH = 180° – $\frac{1}{2}$(m∠A + m∠B)   (1)

And m∠FGH = m∠CGD = 180° – m∠GCD – m∠GDC (ASP of a triangle)

m∠FGH = 180° – $\frac{1}{2}$ (m∠C + m∠ D) (2)

Therefore, m∠FEH + m∠ FGH = 180° – $\frac{1}{2}$ (m∠ A + m∠ B) + 180° – $\frac{1}{2}$ (m∠C + m∠D)

From (1) and (2),

m∠FEH +m∠ FGH = 360° – $\frac{1}{2}$ (m∠A + m∠ B + m∠C + m∠ D) = 360° – $\frac{1}{2}$ × 360°

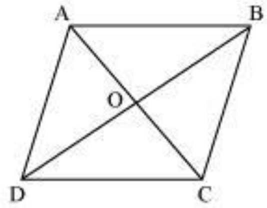
m∠FEH + m∠ FGH = 360° – 180° = 180°

We know that if the sum of a pair of opposite angles of a quadrilateral is 180°, the quadrilateral is cyclic.

Therefore, the quadrilateral EFGH is cyclic.

OR

We know that the diagonals of a parallelogram bisect each other.
Therefore, O is the mid-point of AC and BD.
BO is the median in ΔABC. Therefore, it will divide it into two triangles of equal areas.



∴ area(ΔAOB) = area(ΔBOC) ... (1)
In ΔBCD, CO is the median.
∴ area(ΔBOC) = area(ΔCOD) ... (2)
Similarly, area(ΔCOD) = area(ΔAOD) ... (3)
From equations (1), (2), and (3), we obtain
area(ΔAOB) = area(ΔBOC) = area(ΔCOD) = area(ΔAOD)
Therefore, it is evident that the diagonals of a parallelogram divide it into four triangles of equal area.