Goa Board
Class IX Mathematics
Term II
Sample Paper - 7 Solution

(SECTION – A)

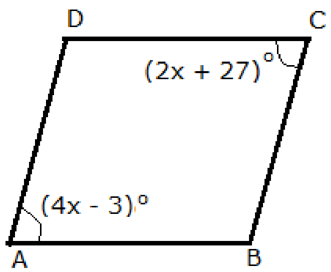
1. Correct Answer: D
The mode is the value which occurs most frequently in a data set. Here 5 appearing 5 times is the mode of the data.

2. Correct Answer: D
As it is dipped vertically to half its height, half the total surface area gets painted.

3. Correct Answer: A
There are six possible outcomes: {1, 2, 3, 4, 5, 6}
Only three outcomes produce even numbers: {2, 4, 6}
The probability of P(even) = $\frac{3}{6} = \frac{1}{2}$

4. Correct Answer: C
Surface area of sphere $= 4\pi r^2$
Curved surface area of cylinder $= 2\pi Rh$
$\Rightarrow 4\pi r^2 = 2\pi Rh$
$\Rightarrow r^2 = \frac{6 \times 12}{2}$
$\Rightarrow r^2 = 36$
$\Rightarrow r = 6$
The radius of the sphere is 6 cm.

5. Correct Answer: B
In a parallelogram, opposite angles are equal.
Thus, $(2x+27)^o = (4x-3)^o$
$\Rightarrow (27+3)^o = 4x - 2x$
$\Rightarrow 30^o = 2x$
$\Rightarrow x = \frac{30^o}{2}$
$\Rightarrow x = 15^o$

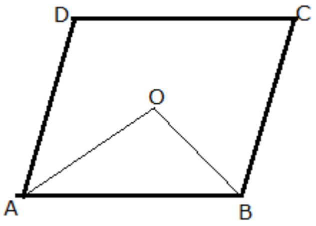


6. Correct Answer: C

In a parallelogram, the sum of consecutive angles is supplementary.

$\therefore m\angle A + m\angle B = 180^o$

$$\Rightarrow \frac{1}{2}m\angle A + \frac{1}{2}m\angle B = 90^o \ ....(1)$$



Consider the triangle AOB:

$$m\angle AOB + m\angle OBA + m\angle OAB = 180^o$$

$$\Rightarrow m\angle AOB + \frac{m\angle B}{2} + \frac{m\angle A}{2} = 180^o \ [\text{Given}]$$

$$\Rightarrow m\angle AOB + 90^o = 180^o \qquad [\text{from equation (1)}]$$

$$\Rightarrow m\angle AOB = 180^o - 90^o$$

$$\Rightarrow m\angle AOB = 90^o$$

7. Correct Answer: B

Given that the value of x is equal to two times the value of y.

x = 2y

8. Correct Answer: D

Since the volume and the surface area are same,

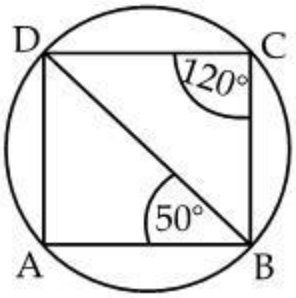
$$\frac{4}{3}\pi r^3 = 4\pi r^2$$

$$\Rightarrow r = 3 \text{ units}$$

(SECTION – B)

9. Area of cross section of pipe = 5 cm^2
Speed of water flowing out of the pipe = 30 cm/sec
Volume of water that flows out in 1 sec = 5 × 30 = 150 cm^3
Volume of water that flows out in 1 minute = 150 × 60 = 9000 cm^3 = 9 litres.

10. ABCD is a cyclic quadrilateral.



In a cyclic quadrilateral, opposite angles are supplementary.
$m\angle DAB + m\angle BCD = 180°$
$m\angle DAB + 120° = 180°$
$m\angle DAB = 60°$
In ΔDAB,
$m\angle DAB + m\angle ABD + m\angle ADB = 180°$
$60° + 50° + ADB = 180°$
$m\angle ADB = 180° - 110° = 70°$.

11. Inner radius (r) of circular well $= \left(\frac{3.5}{2}\right)$ m $= 1.75$ m

Depth (h) of circular well = 10 m

i. Inner curved surface area = $2\pi rh = \left(2 \times \frac{22}{7} \times 1.75 \times 10\right) m^2 = (44 \times 0.25 \times 10)\ m^2$

Inner curved surface area = $2\pi rh = 110\ m^2$

ii. Cost of plastering 1 m^2 = Rs. 40

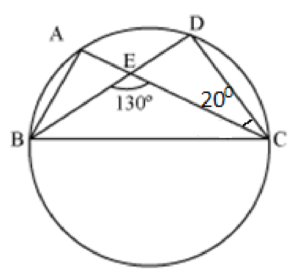
Cost of plastering 110 m^2 = Rs. (110 × 40) = Rs. 4400.

12. Lowest = 29, Highest = 95

Marks	Tally	Frequency
20-30	I	1
30-40	III	3
40-50	卌	5
50-60	卌 III	8
60-70	卌 III	8
70-80	卌 IIII	9
80-90	IIII	4
90-100	II	2

13. P(point up) = 0.73
P(point down) 1 − 0.73 = 0.27
(Since, the sum of probabilities is 1)

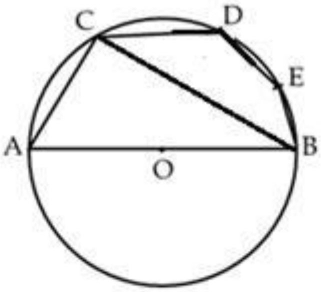
14. In ΔCDE
$\angle CDE + \angle DCE = \angle CEB$ (Exterior angle)
$\Rightarrow m\angle CDE + 20° = 130°$
$\Rightarrow m\angle CDE = 110°$
But $\angle BAC = \angle CDE$ (Angles in same segment of circle)
$\Rightarrow m\angle BAC = 110°$



15. Arranging the data in an ascending order
14, 14, 14, 14, 17, 18, 18, 18, 22, 23, 25, 28
Here 14 occurs 4 times and has the highest frequency. Hence, mode of the given data is 14.

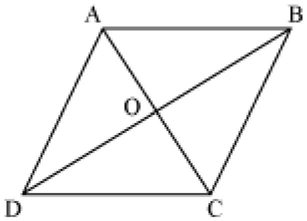
(SECTION – C)

16. Join BC.



Then m∠ACB = 90° [Angle in a semicircle]
Since DCBE is a cyclic quadrilateral
∴ m∠BCD + m∠BED = 180°
Adding ∠ACB on both sides
m∠BCD + m∠BED + m∠ACB = m∠ACB + 180°
(m∠BCD + m∠ACB) + m∠BED = 90° + 180°
m∠ACD + m∠BED = 270°

17. Let ABCD be a quadrilateral, whose diagonals AC and BD bisect each other at right angles.



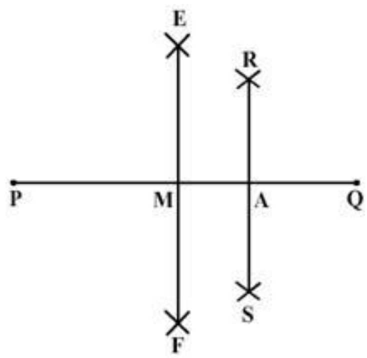
Hence, OA = OC, OB = OD and m∠AOB = m∠BOC = m∠COD = m∠AOD = 90°
To prove ABCD a rhombus, we need to prove ABCD is a parallelogram and all sides of ABCD are equal.
Now, in ΔAOD and ΔCOD

OA = OC	(Diagonal bisects each other)
∠AOD = ∠COD	(given)
OD = OD	(common)
∴ΔAOD ≅ ΔCOD	(by SAS congruence rule)
∴ AD = CD	(1)
Similarly we can prove that	
AD = AB and CD = BC	(2)

From equations (1) and (2), we can say that
AB = BC = CD = AD
Since opposite sides of quadrilateral ABCD are equal, so, we can say that ABCD is a parallelogram. Since all sides of a parallelogram ABCD are equal, we can say that ABCD is a rhombus.

18. Steps of construction:

i. Draw a line segment PQ of length 10 cm.
ii. With P as centre and radius more than half of PQ, draw two arcs above and below PQ.
iii. With Q as centre and same radius, draw two arcs cutting the arcs drawn in step 2 at E and F respectively.
iv. Draw the line segment EF . Suppose it meets PQ at M. Then M bisects the line segment PQ.
v. With Q as centre and radius more than half of QM, draw two arcs above and below QM.
vi. With M as centre and the same radius as in step 5 draw arcs intersecting the previous arcs at R and S.
vii. Draw the line segment with R and S as end points. Suppose it meets QM at A.
viii. Then A bisects QM. AQ = AM = 2.5 cm.



19. Let the larger of the two supplementary angles be 'y' & smaller angle be 'x'.

Therefore, x + y = 180° --- (1)
(∵ Sum of supplementary angles is 180°)
Also, y = x + 18 (∵ Given) ---------- (2)
Substituting (2) in (1), we get,
x + x + 18 = 180
⇒ 2x + 18 = 180
⇒ 2x = 180 - 18
⇒ 2x = 162
⇒ x = 81 ----------- (3)
Substitute (3) in (2),
y = 81 + 18 = 99
Therefore, measure of the larger angle is 99° and measure of the smaller angle is 81°.

20.

i. Number of families earning Rs. 10,000 to Rs. 13,000 per month and owning exactly 2 television sets = 29.
ii. Number of families earning Rs. 16,000 or more per month and owning exactly 1 television set = 579.
iii. Number of families having not more than 1 television set and earning less than Rs. 7,000 = 10 + 160 = 170.

21. We have,

$$x-\frac{2}{3}y=\frac{8}{3},\frac{2x}{5}-y=\frac{7}{5}$$

$$x-\frac{2}{3}y=\frac{8}{3} \quad ....(1)$$

$$\frac{2x}{5}-y=\frac{7}{5} \quad ....(2)$$

From (1)

$$x=\frac{8}{3}+\frac{2}{3}y=\frac{8+2y}{3} \quad ......(3)$$

Substituting the value of x in (2),

$$\frac{2}{5}\left(\frac{8+2y}{3}\right)-y=\frac{7}{5}$$

$$\Rightarrow \frac{16+4y}{15}-y=\frac{7}{5}$$

$$\Rightarrow 16+4y-15y=21$$

$$\Rightarrow -11y=5$$

$$y=\frac{-5}{11}$$

Substituting the value of y in (3),

$$x = \frac{8+2\left(\frac{-5}{11}\right)}{3}=\frac{8-\frac{10}{11}}{3}$$

$$=\frac{88-10}{11\times 3}=\frac{78}{11\times 3}$$

$$x=\frac{26}{11}$$

$$x = \frac{26}{11}, y=\frac{-5}{11}$$

OR

i. Co-ordinates of points B and C are (3, 0) and (6, −3) respectively.
ii. (1, 2) is a solution of line passing through A and B.
iii. Equation of the x-axis is y = 0 and the y-axis is x = 0.

22. Given that we have to construct a grouped frequency distribution table of class size 5. So, the class intervals will be as 0 – 5, 5 – 10, 10 – 15, 15 –20, and so on.
Required grouped frequency distribution table is as follows:

Distance (in km)	Tally marks	Number of engineers
0 – 5	𝍸	5
5 – 10	𝍸 𝍸 𝍷	11
10 –15	𝍸 𝍸 𝍷	11
15 – 20	𝍸 𝍷𝍷𝍷𝍷	9
20 – 25	𝍷	1
25 – 30	𝍷	1
30 – 35	𝍷𝍷	2
Total		40

Only 4 engineers have homes at a distance of more than or equal to 20 km from their work place.
Most of the engineers have their workplace at a distance of upto 15 km from their homes.

23. Slant height (l) of conical tomb = 25 m

Base radius (r) of tomb = $\frac{14}{2}\text{m} = 7\text{ m}$

C.S.A. of conical tomb = $\pi rl = \left(\frac{22}{7}\times 7\times 25\right)\text{ m}^2 = 550\text{ m}^2$

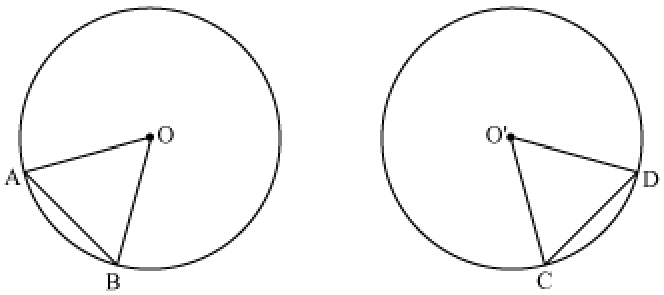
Cost of white-washing 100 m^2 = Rs. 210

Cost of white-washing 550 m^2 = Rs. $\left(\frac{210\times 550}{100}\right)$ = Rs. 1155

Thus, the cost of white washing the conical tomb is Rs. 1155.

24. Let the common ratio between the angles be x. So, the measures of the angles will be 3x, 5x, 9x and 13x respectively.
Since the sum of all interior angles of a quadrilateral is 360°.
$\therefore$ 3x + 5x + 9x + 13x = 360°
30x = 360°
x = 12°
Hence, the angles are
3x = 3 × 12 = 36°
5x = 5 × 12 = 60°
9x = 9 × 12 = 108°
13x = 13 × 12 = 156°

25. A circle is a collection of points which are equidistant from a fixed point. This fixed point is called the centre of the circle and this equal distance is called the radius of the circle. Thus the shape of a circle depends on its radius.
So, if we try to superimpose two circles of equal radius on each other, both circles will cover each other.
Hence two circles are congruent if they have an equal radius.
Now consider two congruent circles having centres O and O' and two chords AB and CD of equal lengths



Now in ΔAOB and ΔCO'D

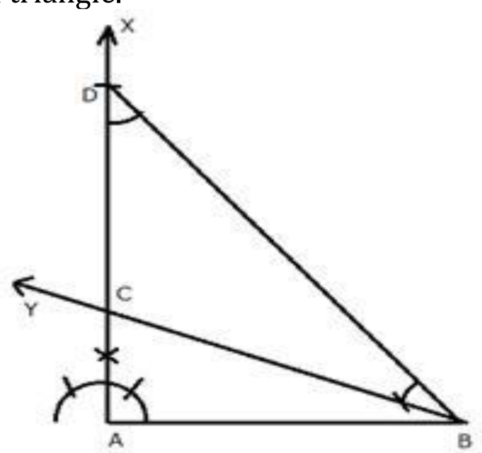
AB = CD	(chords of same length)
OA = O'C	(radii of congruent circles)
OB = O'D	(radii of congruent circles)
∴ ΔAOB ≅ ΔCO'D	(SSS congruence rule)
⇒ ∠AOB = ∠CO'D	(by CPCT)

Hence equal chords of congruent circles subtend equal angles at their centers.

(SECTION – D)

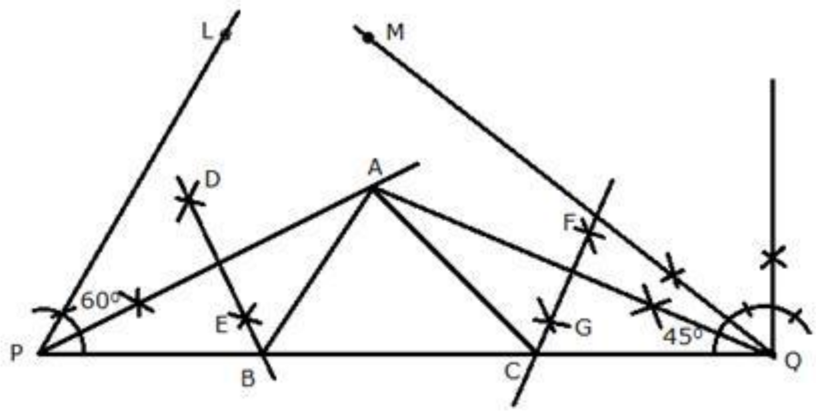
26. Steps of construction:
1. Draw a line segment AB = 12 cm.
2. Draw a ray AX making right angle with AB.
3. Cut a line segment AD of length 18 cm from ray AX.
4. Join BD and make ∠DBY equal to ∠ADB.
5. Let BY intersect AX at C. Join AC and BC.

ΔABC is the required triangle.



OR

Steps of Construction:
1. Draw a line segment PQ = 11 cm (= AB + BC + CA).
2. At P construct an angle of 60° and at Q an angle of 45°.
3. Bisect these angles. Let the bisectors of these angles intersect at point A.
4. Draw perpendicular bisectors DE of AP to intersect PQ at B and FG of AQ to intersect PQ at C.
5. Join AB and AC
ΔABC is the required triangle.



27. Radius of circular end of cylindrical penholder = 3 cm
Height of penholder = 10.5 cm
Surface area of 1 penholder = C.S.A. of penholder + Area of base of penholder
Surface area of 1 penholder = $2\pi rh + \pi r^2$

$$= \left[2 \times \frac{22}{7} \times 3 \times 10.5 + \frac{22}{7} \times (3)^2\right] \text{cm}^2$$

$$= \left(132 \times 1.5 + \frac{198}{7}\right) \text{cm}^2$$

$$= \left(198 + \frac{198}{7}\right) \text{cm}^2$$

$$= \frac{1584}{7} \text{cm}^2$$

Area of cardboard sheet used by 1 competitor $= \frac{1584}{7} \text{cm}^2$

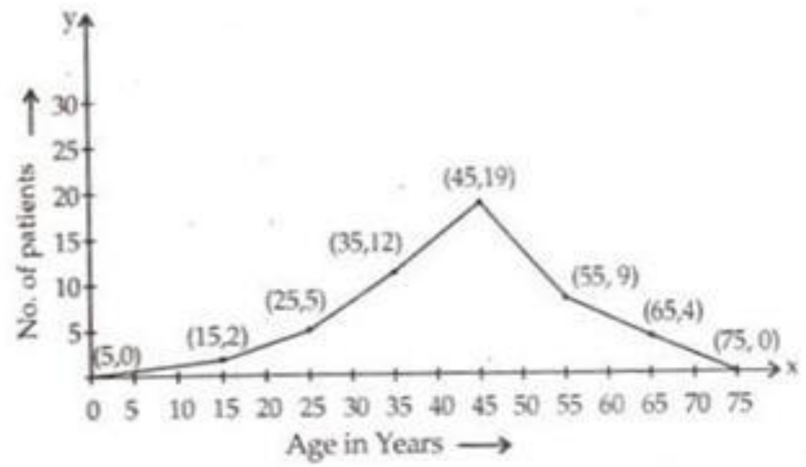
Area of cardboard sheet used by 35 competitors $= \left(\frac{1584}{7} \times 35\right) \text{cm}^2 = 7920 \text{ cm}^2$

Thus, 7920 cm² cardboard sheet will be bought for the competition.

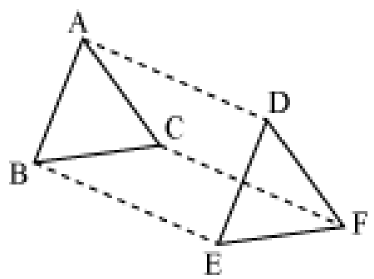
28. The frequency distribution table with class marks is given below:

Class-Intervals	Class marks	Frequency
0-10	5	0
10-20	15	2
20-30	25	5
30-40	35	12
40-50	45	19
50-60	55	9
60-70	65	4
70-80	75	0

Plot points (5, 0), (15, 2)………(65, 4) and (75, 0) to get the frequency polygon as follows:

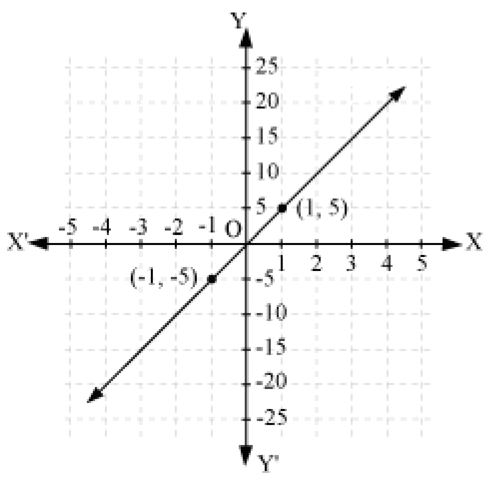


29. In ∆ABC and ∆DEF, AB = DE, AB || DE, BC = EF and BC || EF. Vertices A, B and C are joined to vertices D, E and F respectively.



i. Here AB = DE and AB || DE. Now, if two opposite sides of a quadrilateral are equal and parallel to each other, it will be a parallelogram.
Therefore, quadrilateral ABED is a parallelogram.
ii. Again BC = EF and BC || EF. Therefore, quadrilateral BCEF is a parallelogram.
iii. Here ABED and BEFC are parallelograms.
AD = BE, and AD || BE
(Opposite sides of parallelogram are equal and parallel)
And BE = CF, and BE || CF
(Opposite sides of parallelogram are equal and parallel)
∴ AD = CF, and AD || CF

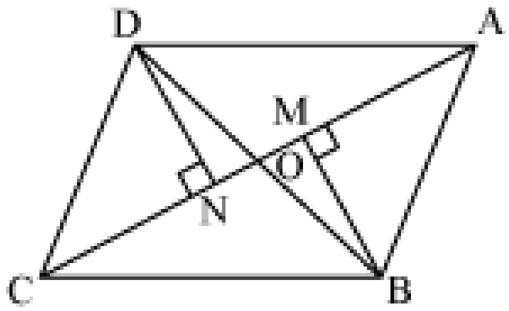
30. Let the distance travelled by the body and work done by the body be d and *w* respectively.
Work done $\propto$ distance traveled
$w \propto d$
w = kd
Where *k* is a constant
If constant force is 5 units, work done w = 5d
Now we may observe that point (1, 5) and (−1, −5) satisfy the above equation.
So (1, 5) and (−1, −5) are solutions of this equation.
The graph can be drawn as follows:



i. From the graphs we observe that the value of y corresponding to x = 2 is 10. Thus the work done by the body is 10 units when the distance traveled by it is 2 units.
ii. From the graphs we may observe that the value of y corresponding to x = 0 is 0. Thus the work done by the body is 0 units when the distance traveled by it is 0 units.

31. In the given figure, diagonals AC and BD of quadrilateral ABCD intersect at O such that OB = OD.

Let us draw DN ⊥ AC and BM ⊥ AC.



In ΔDON and ΔBOM

∠DNO = ∠BMO (By construction)

∠DON = ∠BOM (Vertically opposite angles)

OD = OB (Given)

By A–A–S congruence rule

ΔDON ≅ ΔBOM

∴ DN = BM ... (1)

We know that congruent triangles have equal areas.

∴ Area(ΔDON) = Area(ΔBOM) ... (2)

In ΔDNC and ΔBMA

∠DNC = ∠BMA (By construction)

CD = AB (Given)

DN = BM [Using equation (1)]

∴ ΔDNC ≅ ΔBMA (RHS congruence rule)

⇒ area(ΔDNC) = area(ΔBMA) ... (3)

On adding equation (2) and (3), we have

area(ΔDON) + area(ΔDNC) = area(ΔBOM) + area(ΔBMA)

So, area(ΔDOC) = area(ΔAOB)

32. Radius (r) of a wooden sphere = $\left(\frac{21}{2}\right)$ cm = 10.5 cm

Surface area of a wooden sphere = $4\pi r^2 = \left[4\times\frac{22}{7}\times(10.5)^2\right]$ cm^2 = 1386 cm^2

Radius (r_1) of cylindrical support = 1.5 cm

Height (h) of cylindrical support = 7 cm

C.S.A. of cylindrical support = $2\pi r^2 h = \left[2\times\frac{22}{7}\times(1.5)^2\times 7\right]$ cm^2 = 99 cm^2

Area of circular end of cylindrical support = $\pi r^2 = \left[\frac{22}{7}\times(1.5)^2\right]$ cm^2 = 7.07 cm^2

Area to be painted silver = [8 × (1386 – 7.07)] cm^2

Area to be painted silver = (8 × 1378.93) cm^2 = 11031.44 cm^2

Cost occurred in painting silver colour = Rs. (11031.44 × 0.25) = Rs. 2757.86

Area to painted black = (8 × 99) cm^2 = 792 cm^2

Cost occurred in painting black colour = Rs. (792 × 0.05) = Rs. 39.60

∴ Total cost occurred in painting = Rs. (2757.86 + 39.60) = Rs. 2797.46

OR

Let r be the radius of the base and h be the height of the cylinder.

Total surface area = 462 cm^2

Curved surface area = $\frac{1}{3}\times 462 = 154$ cm^2

$2\pi rh + 2\pi r^2 = 462$

$154 + 2\pi r^2 = 462$

$\Rightarrow 2\pi r^2 = 462 - 154 = 308$

$\Rightarrow r^2 = \frac{308\times 7}{2\times 22} = 49$

$\Rightarrow$ r = 7 cm.

Curved surface area = 154 cm^2

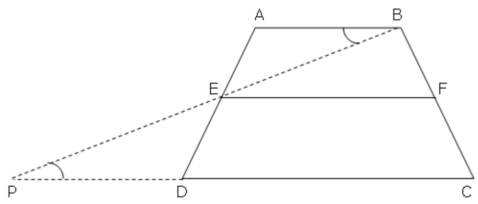
$2\pi rh = 154$

$\Rightarrow 2\times\frac{22}{7}\times 7\times h = 154$

$\Rightarrow h = \frac{7}{2}$ cm.

$\therefore$ Volume of the cylinder = $\pi r^2 h = \frac{22}{7}\times 7\times 7\times\frac{7}{2} = 539$ cm^3

33. Join BE and extend it to meet CD produced at point P.
In ΔAEB and DEP, we have
∠ABE = ∠EPD [Alternate angles]
∴ AB || PC and transversal BP meets them at B, P.
∠AEB = ∠PED (vertically opposite angles)
AE = ED [∵ E is the mid – point of AD]
∴ By AAS criterion of congruence ΔAEB ≅ ΔDEP
⇒ BE = PE ...(1) (cpct)
And AB = DP ...(2) (cpct)



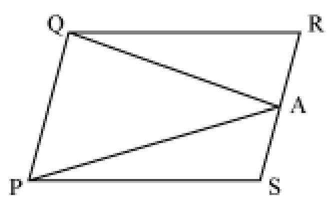
Now in ΔBPC, E is a mid-point of BP [proved in (1) and F is the mid-point of BC (given)]

∴ EF || PC and EF = $\frac{1}{2}$ PC

i.e., EF || AB and EF = $\frac{1}{2}$ (PD + DC) = $\frac{1}{2}$ (AB + DC)

Hence, EF || AB and EF = $\frac{1}{2}$ (AB + DC)

34. From the figure it is clear that point A divides the field into three parts. These parts are triangular in shape – ΔPSA, ΔPAQ and ΔQRA.



Area of ΔPSA + Area of ΔPAQ + Area of ΔQRA = area of parallelogram PQRS ... (1)
We know that if a parallelogram and triangle are on the same base and between the same parallel lines, the area of triangle is half the area of the parallelogram.

∴ area (ΔPAQ) = $\frac{1}{2}$ area (PQRS) ... (2)

From equations (1) and (2), we have

area (ΔPSA) + area (ΔQRA) = $\frac{1}{2}$ area (PQRS) ... (3)

Clearly, the farmer must sow wheat in the triangular part PAQ and pulses in the other two triangular parts PSA and QRA or wheat in the triangular part PSA and QRA and pulses in the triangular parts PAQ.

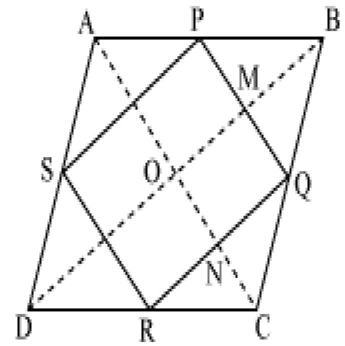
35. In ΔABC, P and Q are mid-points of sides AB and BC respectively.

$\therefore$ PQ || AC and PQ = $\frac{1}{2}$AC (using mid-point theorem) ... (1)

In ΔADC

R and S are the mid-points of CD and AD respectively

$\therefore$ RS || AC and RS = $\frac{1}{2}$AC (using mid-point theorem) ... (2)



From equations (1) and (2), we have

PQ || RS and PQ = RS

As in quadrilateral PQRS one pair of opposite sides are equal and parallel to each other, so, it is a parallelogram.

Let diagonals of rhombus ABCD intersect each other at point O.

Now in quadrilateral OMQN

MQ || ON ($\because$ PQ || AC)

QN || OM ($\because$ QR || BD)

So, OMQN is parallelogram

$\Rightarrow \angle MQN = \angle NOM$

$\Rightarrow \angle PQR = \angle NOM$

But, $m\angle NOM = 90°$ (diagonals of a rhombus are perpendicular to each other)

$\therefore m\angle PQR = 90°$

Clearly PQRS is a parallelogram having one of its interior angle as 90°.

Hence, PQRS is a rectangle.