Goa Board
Class IX Mathematics
Term II
Sample Paper – 5 Solution

(SECTION – A)

1. Correct Answer: A

If we divide or multiply both sides of a linear equation with a non-zero number, then the solution of the linear equation remains the same as the graph of the equation remains the same in both cases.

2. Correct Answer: C

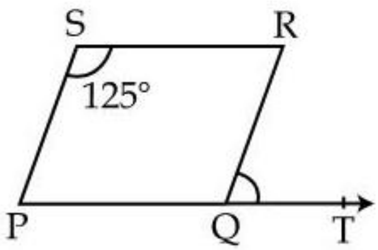
m∠PSR = m∠RQP = 125° (since PQRS is a parallelogram, opposite angles will be equal)

⇒∠PQT = 180° (PQT is a straight line)

⇒m∠PQR + m∠RQT = 180°

⇒125° + m∠RQT = 180°

⇒m∠RQT = 55°



3. Correct Answer: A

Class size is the difference between two successive class marks.

i.e. 10 – 6 = 4

4. Correct Answer: A

Surface area of sphere = $4\pi r^2$

We know that the right circular cylinder just encloses the sphere of radius r.

So height of cylinder = h = 2r.

Curved surface area of cylinder = $2\pi rh = 4\pi r^2$

Find the ratio of surface area of the sphere to the curved surface area of the cylinder = 1 : 1

5. Correct Answer: D

Standard form of the equation in two variables is ax + by + c = 0.

7x = 3

⇒7x + 0y – 3 = 0

6. Correct Answer: C

$$\text{Class mark} = \frac{\text{upper limit} + \text{lower limit}}{2}$$

7. Correct Answer: D

Let the drop in the oil level be h cm.

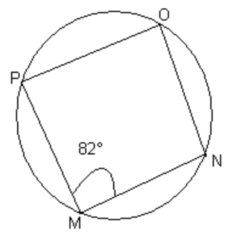
Volume of cylinder = $\pi r^2 h$

$$\Rightarrow 11000 = \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2} \times h$$

$$\Rightarrow h = \frac{11000 \times 7 \times 4}{22 \times 35 \times 35}$$

$$\Rightarrow h = 11\frac{3}{7} \text{ cm}$$

8. Correct Answer: B



The sum of opposite angles of a cyclic quadrilateral is 180°.

$m\angle M + m\angle O = 180°$

$m\angle O = 180° - 82° = 98°$

(SECTION – B)

9. Given,

T.S.A. of the cube = 294 cm^2

$6 \times l \times l = 294$ [$\because$ T.S.A. of cube = $6 \times l^2$]

$$\Rightarrow l^2 = \frac{294}{6}$$

$\Rightarrow l^2 = 49$

$\Rightarrow l = \sqrt{49} = 7$ cm

$\Rightarrow$ Side (l) = 7 cm.

Volume of cube = $l \times l \times l = 7 \times 7 \times 7 = 343$ cm^3

10. Number of students born in August = 6

Total number of students = 40

$$\text{Required probability} = \frac{\text{Number of students born in August}}{\text{Total number of students}} = \frac{6}{40} = \frac{3}{20}$$

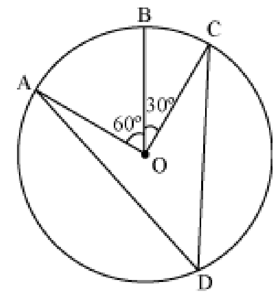
11. Making the class intervals continuous, we get

IQ	Number of children
54.5-64.5	0
64.5-74.5	120
74.5-84.5	140
84.5-94.5	134
94.5-104.5	90
104.5-114.5	108
114.5-124.5	38
124.5-134.5	15

12. We observe that

m∠AOC = m∠AOB + m∠BOC = 60° + 30° = 90°

We know that the angle subtended by an arc at its centre has twice the measure of any angle subtended by it any point on the remaining part of the circle.



$$m\angle ADC = \frac{1}{2} m\angle AOC = \frac{1}{2} \times 90° = 45°$$

13. Let the smallest value of data be x.

The largest value = 3x

We know that the range of a data = largest value – smallest value

⇒2x = 45

⇒x = 22.5

⇒ 3x = 67.5

The smallest value and the largest values of the data are 22.5 and 67.5 respectively.

14. AD is the median of ΔABC and hence will divide it into two triangles of equal areas.

∴ Area(ΔABD) = Area(ΔACD) ... (1)

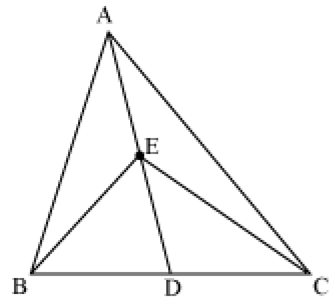
Now ED is the median of ΔEBC.

∴ Area(ΔEBD) = Area(ΔECD) ... (2)

Subtract equation (2) from equation (1), we have

Area(ΔABD) – Area(EBD) = Area(ΔACD) – Area(ΔECD)

Area(ΔABE) = Area(ΔACE)



(SECTION – C)

15. No. of white balls = x

Total no. of balls = 12

P(white ball) = $\frac{x}{12}$

If 6 white balls are added:

Total balls = 18

White balls = x + 6

∴ New probability of getting a white ball = p'(white ball) = $\frac{x+6}{18}$

According to the question:

$$\frac{x+6}{18} = \frac{2x}{12}$$

$$\Rightarrow \frac{x+6}{18} = \frac{x}{6}$$

$$\Rightarrow 6x + 36 = 18x$$

$$\Rightarrow x = 3$$

16. Length (l_1) of the storehouse = 40 m

Breadth (b_1) of the storehouse = 25 m

Height (h_1) of the storehouse = 10 m

Volume of storehouse = $l_1 \times b_1 \times h_1$ = $(40 \times 25 \times 10)$ m^3 = 10000 m^3

Length (l_2) of a wooden crate = 1.5 m

Breadth (b_2) of a wooden crate = 1.25 m

Height (h_2) of a wooden crate = 0.5 m

Volume of a wooden crate = $l_2 \times b_2 \times h_2$ = $(1.5 \times 1.25 \times 0.5)$ m^3 = 0.9375 m^3

Let n wooden crates be stored in the storehouse.

Volume of n wooden crates = volume of storehouse

$0.9375 \times n = 10000$

$$n = \frac{10000}{0.9375} = 10666.66$$

Thus, 10666 numbers of wooden crates can be stored in storehouse.

17. Inner radius of hemispherical bowl = 5 cm

Thickness of the bowl = 0.25 cm

∴ Outer radius (r) of hemispherical bowl = (5 + 0.25) cm = 5.25 cm

Outer C.S.A. of hemispherical bowl = $2\pi r^2 = 2 \times \frac{22}{7} \times (5.25\ \text{cm})^2 = 173.25\ \text{cm}^2$

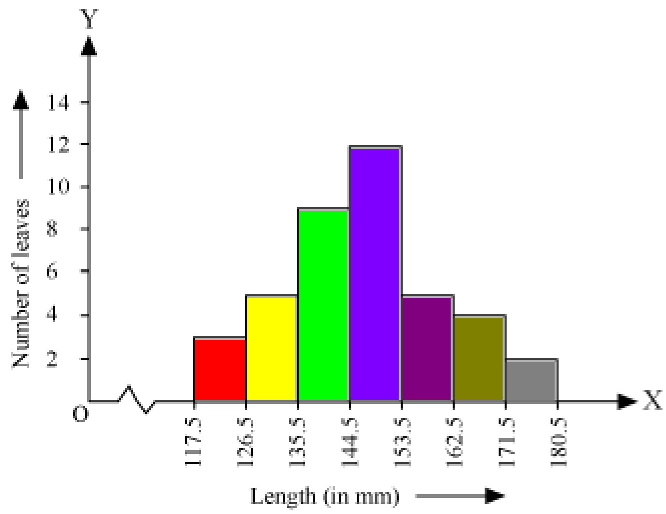
Thus, the outer curved surface area of the bowl is 173.25 cm^2.

18.

i. Length of leaves are represented in a discontinuous class intervals having a difference of 1mm in between them. So we have to add $\frac{1}{2} = 0.5\text{mm}$ to each upper class limit and also have to subtract 0.5mm from the lower class limits so as to make our class intervals continuous.

Length (in mm)	Number of leaves
117.5 – 126.5	3
126.5 – 135.5	5
135.5 – 144.5	9
144.5 – 153.5	12
153.5 – 162.5	5
162.5 – 171.5	4
171.5 – 180.5	2

Now taking length of leaves on the x-axis and number of leaves on the y-axis we can draw the histogram of this information as below:



Here 1 unit on the y-axis represents 2 leaves.

ii. Other suitable graphical representation of this data could be frequency polygon.

iii. No as maximum numbers of leaves (i.e. 12) have their length in between of 144.5 mm and 153.5 mm. It is not necessary that all have their lengths as 153 mm.

19. Let the digit in the units place be x and the digit in the tens place be y.

Then x = 3y and the number = 10y + x

The number obtained by reversing the digits is 10x + y.

If 36 is added to the number, the digits interchange their places.

So we have 10y + x + 36 = 10x + y

$\Rightarrow 9y - 9x + 36 = 0$

$\Rightarrow 9x - 9y = 36$

$\Rightarrow 9(x - y) = 36$

$\Rightarrow x - y = 4$ .... (i)

Substituting the value of x = 3y in equation (i), we get

$3y - y = 4$

$\Rightarrow 2y = 4$

$\Rightarrow y = 2$

$\Rightarrow y = 2$

Substituting the value of y = 2 in equation (i),we get

$x - 2 = 4$

$\Rightarrow x = 4 + 2$

$\Rightarrow x = 6$

Therefore, the number becomes 26.

OR

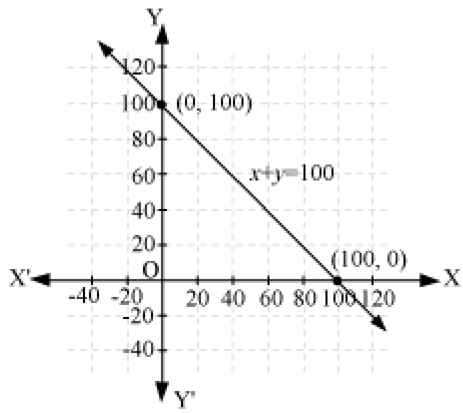
Let Yamini and Fatima contributed x and y respectively towards the Prime Minister's Relief fund.

Amount contributed by Yamini + amount contributed by Fatima = 100

$x + y = 100$

Now we observe that (100, 0) and (0, 100) satisfy the above equation.

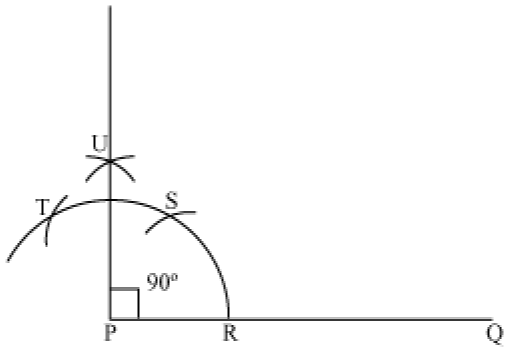
So, (100, 0) and (0, 100) are solutions of above equation.
The graph of equation $x + y = 100$ can be drawn as follows:



Here we may find that variable x and y are representing the amount contributed by Yamini and Fatima respectively and these quantities may not be negative. Hence we will consider only those values of x and y which are lying in 1st quadrant.

20. Following are the steps of construction:

i. Take the given ray PQ. Draw an arc of some radius taking point P as its centre, which intersects PQ at R.
ii. Taking R as the centre and with the same radius as before, draw an arc intersecting the previously drawn arc at S.
iii. Taking S as centre and with the same radius as before, drawn an arc intersecting the arc at T (see figure)
iv. Taking S and T as the centres, draw arcs of the same radius which intersect each other at U.
v. Join PU, which is the required ray making 90° with the given ray PQ.



Justification of construction:

We can justify the construction, if we can prove m∠UPQ = 90°.

For this let us join PS and PT

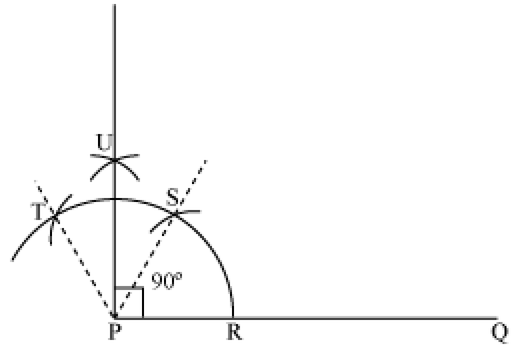
We have m∠SPQ = m∠TPS = 60°. In (iii) and (iv) steps of this construction, we have drawn PU as the bisector of ∠TPS.

$\therefore$ m∠UPS = $\frac{1}{2}$ m∠TPS $= \frac{1}{2} \times 60° = 30°$

Now, ∠UPQ = ∠SPQ + ∠UPS

= 60° + 30°

= 90°



21. Side (a) of the cube = 12 cm

Volume of the cube = a^3 = $(12\text{ cm})^3$ = 1728 cm^3

Let the side of each smaller cube be l.

Volume of each smaller cube $= \left(\frac{1728}{8}\right)\text{cm}^3 = 216\text{ cm}^3$

$l^3 = 216\text{ cm}^3$

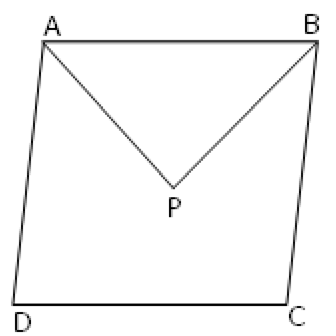
$\Rightarrow$ l = 6 cm

Thus, the side of each smaller cube is 6 cm.

Ratio between surface areas of the cubes $= \frac{\text{Surface area of the bigger cube}}{\text{Surface area of the smaller cube}}$

$= \frac{6a^2}{6l^2} = \frac{6(12)^2}{6(6)^2} = \frac{4}{1}$

So, the required ratio between the surface areas of the cubes is 4 : 1.



22. Given: ABCD is a parallelogram such that angle bisector of adjacent angles A and B intersect at point P.

To Prove that m∠ APB = 90°.

$\Rightarrow$ AD||BC

$\Rightarrow$ m∠A + m∠B = 180° [∠A and ∠B are consecutive interior angles.]

$\Rightarrow \frac{1}{2}$ m∠A + $\frac{1}{2}$ m∠B = 90°

But,

$\frac{1}{2}$ m∠A+ $\frac{1}{2}$ m∠B + m∠APB = 180° [Sum of angles of a triangle is 180°.]

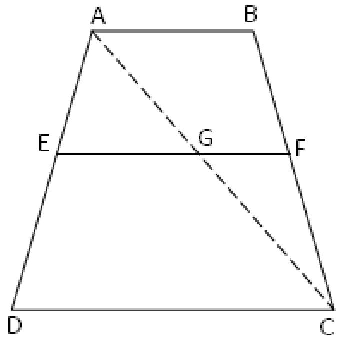
$\Rightarrow$ 90° + m∠APB = 180°

$\Rightarrow$ m∠APB = 90°

23. Given: ABCD is a trapezium. E is the mid-point of AD and AB || CD, EF || AB.

To prove: F is the mid-point of BC

Construction: Join AC to intersect EF at point G.



Proof: EF || DC [Given]

⇒ EG || DC

Since E is mid-point of AD.

∴G is the mid-point of AC. [By converse of midpoint theorem]

In ΔABC,

FG || AB

G is the mid-point of AC

∴ F is the mid-point of BC.

24. Total number of families = 475 + 814 + 211 = 1500

i. Number of families with 2 girls = 475

$$\text{Required probability} = \frac{\text{Number of families with 2 girls}}{\text{Total number of families}}$$

$$= \frac{475}{1500} = \frac{19}{60}$$

ii. Number of families with 1 girl = 814

$$\text{Required probability} = \frac{\text{Number of families with 1 girl}}{\text{Total number of families}}$$

$$= \frac{814}{1500} = \frac{407}{750}$$

iii. Number of families with no girls = 211

$$\text{Required probability} = \frac{\text{Number of families with no girls}}{\text{Total number of families}}$$

$$= \frac{211}{1500}$$

$$\text{Sum of all these probabilities} = \frac{19}{60} + \frac{407}{750} + \frac{211}{1500}$$

$$= \frac{475+814+211}{1500}$$

$$= \frac{1500}{1500} = 1$$

Thus, the sum of all these probabilities is 1.

(SECTION D)

25. Length of the bigger box = 25 cm

Breadth of the bigger box = 20 cm

Height of the bigger box = 5 cm

Total surface area of the bigger box = 2(lb + lh + bh)

= [2(25 × 20 + 25 × 5 + 20 × 5)] cm^2

= [2(500 + 125 + 100)] cm^2 = 1450 cm^2

Extra area required for overlapping $= \left(\frac{1450 \times 5}{100}\right) \text{cm}^2$ = 72.5 cm^2

Considering all overlaps, total surface area of 1 bigger box = (1450 + 72.5) cm^2

∴ Considering all overlaps, total surface area of 1 bigger box = 1522.5 cm^2

Area of cardboard sheet required for 250 bigger boxes = (1522.5 × 250) cm^2

∴ Area of cardboard sheet required for 250 such bigger boxes = 380625 cm^2

Total surface area of smaller box = [2(15 ×12 + 15 × 5 + 12 × 5] cm^2

= [2(180 + 75 + 60)] cm^2

= (2 × 315) cm^2

= 630 cm^2

Extra area required for overlapping $= \left(\frac{630 \times 5}{100}\right) \text{cm}^2 = 31.5\, \text{cm}^2$

Considering all overlaps, total surface area of 1 smaller box = (630 + 31.5) cm^2

∴ considering all overlaps, total surface area of 1 smaller box = 661.5 cm^2

Area of cardboard sheet required for 250 smaller boxes = (250 × 661.5) cm^2

∴ Area of cardboard sheet required for 250 smaller boxes = 165375 cm^2

Total cardboard sheet required = (380625 + 165375) cm^2 = 546000 cm^2

Cost of 1000 cm^2 cardboard sheet = Rs. 4

∴ Cost of 546000 cm^2 cardboard sheet $= \text{Rs.} \left(\frac{546000 \times 4}{1000}\right) = \text{Rs. } 2184$

So, cost of cardboard sheet required for 250 boxes of each kind will be Rs. 2184.

26.

i. Edge of cube = 10 cm

Length (l) = 12.5 cm, Breadth (b) = 10 cm, Height (h) = 8 cm

Lateral surface area of cubical box = 4(edge)^2 = 4(10 cm)^2 = 400 cm^2

Lateral surface area of cuboidal box = 2[lh + bh]

= [2(12.5 × 8 + 10 × 8)] cm^2

= (2 × 180) cm^2

= 360 cm^2

Clearly, the lateral surface area of the cubical box is greater than the lateral surface area of the cuboidal box.

Lateral surface area of cubical box − Lateral surface area of cuboidal box = 400 cm^2 − 360 cm^2 = 40 cm^2

Therefore, the lateral surface area of the cubical box is greater than the lateral surface area of the cuboidal box by 40 cm^2.

ii. Total surface area of cubical box = 6(edge)^2 = 6(10 cm)^2 = 600 cm^2

Total surface area of cuboidal box = 2[lh + bh + lb]

= [2(12.5 × 8 + 10 × 8 + 12.5 × 100] cm^2

= 610 cm^2

Clearly, the total surface area of the cubical box is smaller than that of the cuboidal box.

Total surface area of cuboidal box − Total surface area of cubical box = 610 cm^2 − 600 cm^2 = 10 cm^2.

Therefore, the total surface area of the cubical box is smaller than that of the cuboidal box by 10 cm^2.

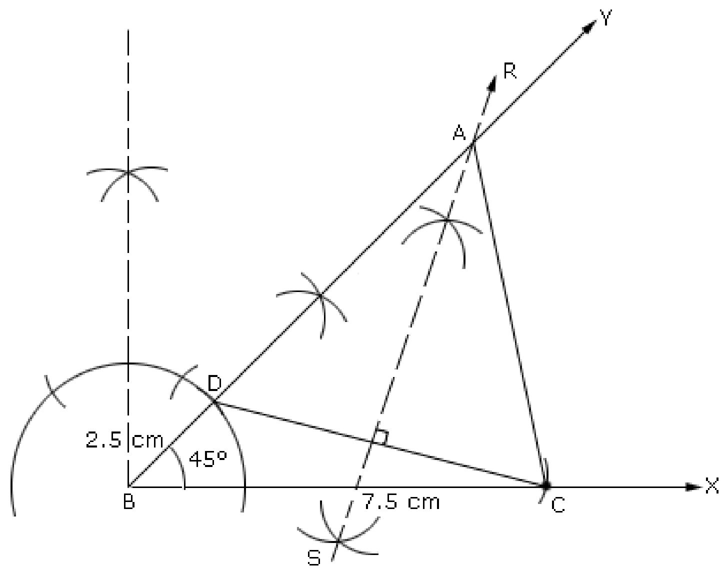
27. Given: Base BC = 7.5 cm, the difference between the other two sides AB − AC or AC − AB = 2.5 cm and one base angle = 45^o.

Let AB > AC

AB − AC = 2.5 cm

Steps of Construction:

1. Draw a ray BX and cut off a line segment BC = 7.5 cm from it.
2. Construct m∠YBC = 45^o
3. Cut off a line segment BD = 2.5 cm from BY.
4. Join CD.
5. Draw perpendicular bisector RS of CD intersecting BY at a point A.
6. Join AC. ΔABC is the required triangle.



Justification:

RS is the perpendicular bisector of DC.

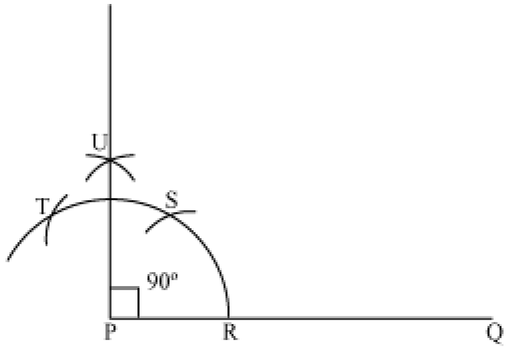
So, AD = AC

BD = AB - AD = AB – AC

OR

Following are the steps of construction:

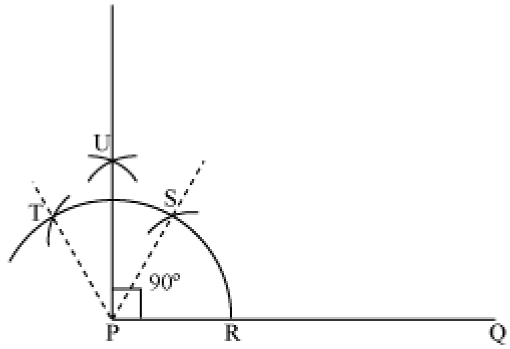
i. Take the given ray PQ. Draw an arc of some radius taking point P as its centre, which intersects PQ at R.
ii. Taking R as the centre and with the same radius as before, draw an arc intersecting the previously drawn arc at S.
iii. Taking S as centre and with the same radius as before, draw an arc intersecting the arc at T (see figure)
iv. Taking S and T as centre, draw arc of same radius to intersect each other at U.
v. Join PU, which is the required ray making 90° with given ray PQ.



Justification of Construction:

We can justify the construction, if we can prove $\angle UPQ = 90°$.

For this let us join PS and PT



We have $\angle SPQ = \angle TPS = 60°$. In (iii) and (iv) steps of this construction, we have drawn PU as the bisector of $\angle TPS$.

$\therefore \angle UPS = \frac{1}{2}\angle TPS = \frac{1}{2} \times 60° = 30°$

Now, $\angle UPQ = \angle SPQ + \angle UPS = 60° + 30° = 90°$

28. Length (l) of the greenhouse = 30 cm

Breadth (b) of the greenhouse = 25 cm

Height (h) of the greenhouse = 25 cm

Total surface area of the greenhouse= 2[lb + lh + bh]

$= [2(30 \times 25 + 30 \times 25 + 25 \times 25)]\ \text{cm}^2$

$= [2(750 + 750 + 625)]\ \text{cm}^2$

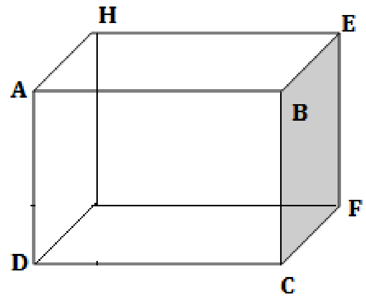
$= (2 \times 2125)\ \text{cm}^2 = 4250\ \text{cm}^2$

Therefore, area of glass is 4250 cm^2.

(ii)Total length of tape = 4(l + b + h)

= [4(30 + 25 + 25)] cm = 320 cm

Therefore, 320 cm tape is required for all the 12 edges.



29. Height (h) of the cylindrical tank = 4.5 m

Radius (r) of circular end of the cylindrical tank =

$\left(\frac{4.2}{2}\right) \text{m} = 2.1 \text{ m}$

i. Lateral or curved surface area of tank = 2πrh $= \left(2 \times \frac{22}{7} \times 2.1 \times 4.5\right) \text{m}^2$ = 59.4 m^2

ii. Total surface area of tank = 2πr (r + h) $= \left[2 \times \frac{22}{7} \times 2.1 \times (2.1 + 4.5)\right] \text{m}^2$ = 87.12 m^2

Let A m^2 of steel sheet be actually used to make the tank.

$\therefore A\left(1 - \frac{1}{12}\right) = 87.12\ \text{m}^2$

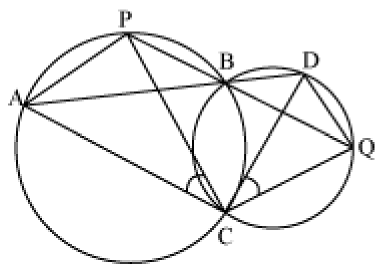
$\Rightarrow A = \left(\frac{12}{11} \times 87.12\right) \text{m}^2$

$\Rightarrow A = 95.04\ \text{m}^2$

Thus, 95.04 m^2 steel was used in actually making the tank.

30. Given: Two circles intersect at two points B and C.

Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively.



To Prove: ∠ACP = ∠QCD.

Construction: Join chords AP and DQ

Consider chord AP,

∠PBA = ∠ACP (Angles in the same segment) ... (1)

Consider chord DQ,

∠DBQ =∠QCD (Angles in the same segment) ... (2)

ABD and PBQ are line segments intersecting at B.

∴ ∠PBA = ∠DBQ (Vertically opposite angles) ... (3)

From equations (1), (2), and (3), we obtain

∠ACP = ∠QCD

31. Consider the triangle BFC:

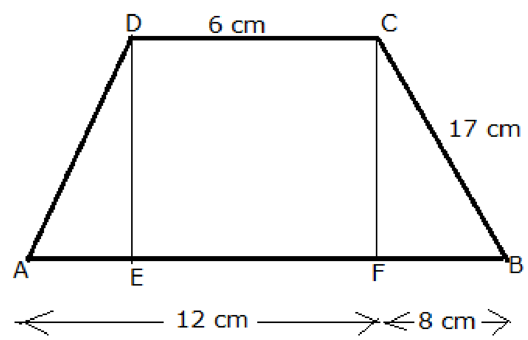
Using Pythagoras Theorem, we have,

$CF^2 + BF^2 = BC^2$

$\Rightarrow CF^2 + 8^2 = 17^2$

$\Rightarrow CF^2 = 225$

$\Rightarrow CF = 15$



CF and DE are the perpendiculars from C and D to AB. Also AB is parallel to CD.

Thus, we have, CD = EF and CF = DE

Since AF = 12 cm,

AE = AF – EF

$\Rightarrow$AE = AF – CD

$\Rightarrow$AE = 12 – 6

$\Rightarrow$AE = 6 cm

Thus, area of ΔADE = $= \frac{1}{2} \times AE \times DE = \frac{1}{2} \times 6 \times 15 = 45\ cm^2$

Area of rectangle CDEF = 6×15 = 90 cm^2

Area of ΔBCF = $= \frac{1}{2} \times BF \times CF = \frac{1}{2} \times 8 \times 15 = 60\ cm^2$

Area of quadrilateral ABCD = Area of ΔADE + Area of rectangle CDEF + Area of ΔBCF

Thus,

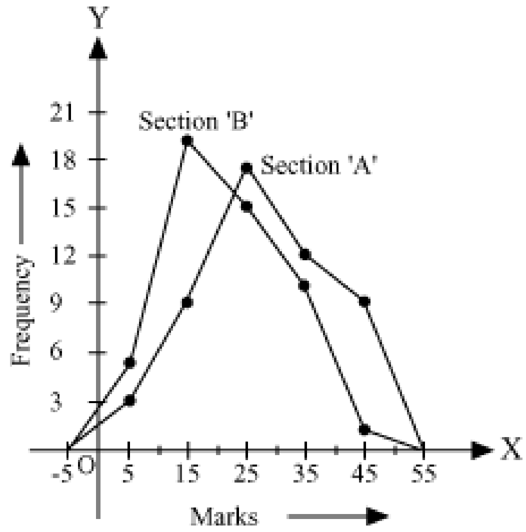
Area of quadrilateral ABCD = 45 + 90 + 60 = 195 cm^2.

32. We can find class marks of given class intervals by using the formula –

$$\text{Class mark } = \frac{\text{upper class limit } + \text{ lower class limit}}{2}$$

Section A			Section B		
Marks	Class marks	Frequency	Marks	Class marks	Frequency
0 – 10	5	3	0 – 10	5	5
10 – 20	15	9	10 – 20	15	19
20 – 30	25	17	20 – 30	25	15
30 – 40	35	12	30 – 40	35	10
40 – 50	45	9	40 – 50	45	1

Now taking class marks on the x-axis and frequency on the y-axis and choosing an appropriate scale (1 unit = 3 for the y-axis) we can draw a frequency polygon as below –



From the graph we can see performance of students of section 'A' is better than the students of section 'B' as for good marks.

33. Diameter= 24 m ⇒ radius =12 m

Radius of the conical part = Radius of the cylindrical part (r) = 12 m
Height of cylindrical part (h) = 11 m, height of the cone (h) = 5 m
For the conical part of the circus tent,

$l^2 = r^2 + h^2$

$\Rightarrow l = \sqrt{r^2 + h^2}$

$\Rightarrow l = \sqrt{12^2 + 5^2} = 13\ \text{m}$

Thus, l = 13 m

Surface area of the tent = Curved surface area of the conical part + curved surface area of the cylindrical part

Surface area of the tent = $\pi rl + 2\pi rh = \pi r(l + 2h) = \frac{22}{7} \times 12\ (13 + 22) = \frac{22}{7} \times 12 \times 35$

Surface area of the tent = 1320 m^2

Breadth of canvas (B) = 5 m, Let length of canvas = L

Area of canvas required = surface area of the tent

L × B = 1320

$\Rightarrow L = \frac{1320}{5} = 264\,\text{m}$

Thus 264 m long canvas is required.

34. Let us assume that Laxmi purchased x bananas and y oranges.
Since each banana costs Rs. 2, x bananas cost Rs. $2 \times x$ = Rs. 2x
Similarly, each orange costs Rs. 3.
Thus, y oranges cost Rs.$3 \times y$ = Rs. 3y
Thus, the total amount paid by Laxmi is Rs. (2x + 3y), which equals Rs. 30
Thus, we can express the given information in the form of a linear equation as
$2x + 3y = 30$
Now, we know that Laxmi purchased 6 oranges, i.e., the value of y is 6.
Substitute this value of y in the equation $2x + 3y = 30$, thereby reducing it to a linear equation in one variable.
We can then solve the equation to obtain the value of x.
$2x + 3 \times 6 = 30$
$\Rightarrow 2x + 18 = 30$
This is a linear equation in one variable.
$\Rightarrow 2x = 30 - 18$
$\Rightarrow 2x = 12$
$\Rightarrow x = 6$
Thus, we see that the value of x is 6, i.e., Laxmi purchased 6 bananas.