Goa Board
Class IX Mathematics
Term II
Sample Paper – 4 Solution

(SECTION – A)

1. Correct Answer: C
An inconsistent system of two linear equations in two variables will have no solution.

2. Correct Answer: C
Range of data 70, 65, 75, 71, 36, 55, 61, 62, 41, 40, 39, 35 is
Range = Greatest value – Smallest value = 75 – 35 = 40

3. Correct Answer: D
The equation of x-axis is y = 0.

4. Correct Answer: C
Let their heights be h and 2h respectively and radii be r and R respectively.
$\pi r^2 h = \pi R^2 (2h)$

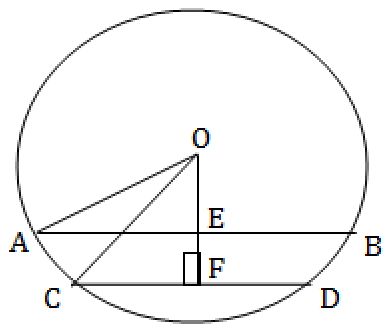
$$\Rightarrow \frac{r^2}{R^2} = \frac{2}{1}$$

$$\Rightarrow \frac{r}{R} = \frac{\sqrt{2}}{1}$$

$$\Rightarrow r : R = \sqrt{2} : 1$$

5. Correct Answer: B
Range = Maximum value – Minimum value = 61 – 9 = 52

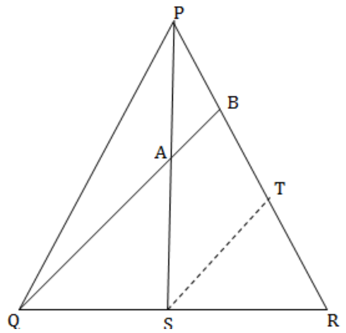
6. Correct Answer: C



AE = $\frac{1}{2}$AB
⇒ AE = 8 cm
CF = $\frac{1}{2}$CD
⇒ CF = 6 cm
Let EF = x.
We have,
Radius = OA = 10 cm
OE= $\sqrt{(10)^2-(8)^2}=\sqrt{100-64}=\sqrt{36}=6\text{cm}$
OF = $\sqrt{(10)^2-(6)^2}=\sqrt{100-36}=\sqrt{64}=8\text{cm}$
⇒ x = OF – OE = 8 – 6 = 2 cm

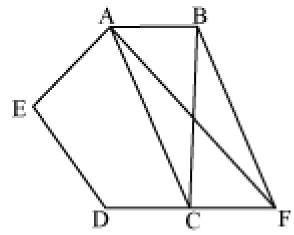
7. Correct Answer: B
Outside diameter = 16 cm
Outside radius (R) = 8 cm
Inside diameter = 12 cm
Inside radius (r) = 6 cm
Length of the cylindrical tube (h) = 7 m = 700 cm
Let V be the volume of metal in the tube.
$V = \pi (R^2 - r^2) h$
$\Rightarrow V = \frac{22}{7}(8^2 - 6^2) \times 700 \text{ cm}^2 = 61600 \text{ cm}^3$.
Therefore, the volume of metal in the tube is 61600 cm^3.

8. Correct Answer: D
Draw ST || BQ meeting PR at T
In ΔQBR,
ST || BQ
∴ TR = TB [i]
[Line drawn through the mid-point of a side of a triangle, parallel to the other bisects the third side]
Also in Δ PST,
AB || ST
∴ PB = BT [ii]
∴ TR = TB = PB [From (i) and (ii)]
∴ PB = $\frac{1}{3}$PR = $\frac{1}{3}$ × 9 = 3 cm



(SECTION – B)

9. ΔACB and ΔACF lie on the same base AC and are between the same parallel lines AC and BF.

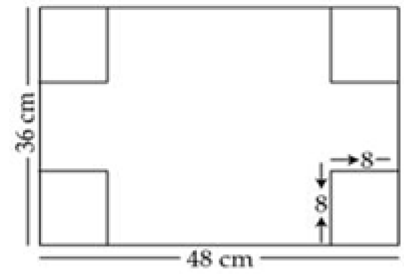


∴ area(ΔACB) = area(ΔACF)

10. Length of the box = l = 48 – 8 – 8 = 32 cm.

Breadth of the box = b = 36 – 8 – 8 = 20 cm.

Height = h = 8 cm



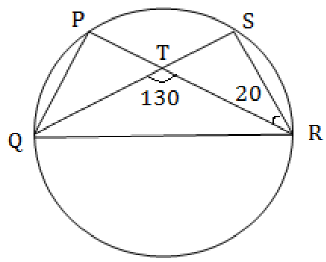
Volume of the box formed = l × b × h = 32 × 20 × 8 = 5120 cm^3.

11. First seven natural numbers are 1, 2, 3, 4 5, 6, and 7.

$\overline{X}$ = arithmetic mean.

$$\therefore \overline{X} = \frac{1+2+3+4+5+6+7}{7} = \frac{28}{7} = 4$$

12. Given, P, Q, R and S are four points on a circle. PR and QS intersect at a point T such that m∠QTR = 130° and m∠TRS = 20°



m∠PQT = m∠SRT = 20°
m∠PTQ = 180° – 130° = 50°
m∠QPR = 180° – (20° + 50°) = 110°

13. The event that at most one head occurred = 550

$\therefore$ Probability of at most one head occuring = $\dfrac{550}{1000} = \dfrac{11}{20}$

14.

$$\text{Mean} = \frac{\text{Sum of all observations}}{\text{Total number of observations}}$$

$$\Rightarrow 15 = \frac{10+12+18+11+p+19}{6}$$

$$\Rightarrow 90 = 70 + p$$

$$\Rightarrow p = 90 - 70$$

$$\Rightarrow p = 20$$

(SECTION – C)

15. $2x + 3y = 9$

$\Rightarrow 2x = 9 - 3y$

$\Rightarrow x = \dfrac{9-3y}{2}$

(Expressing one variable in terms of the other)

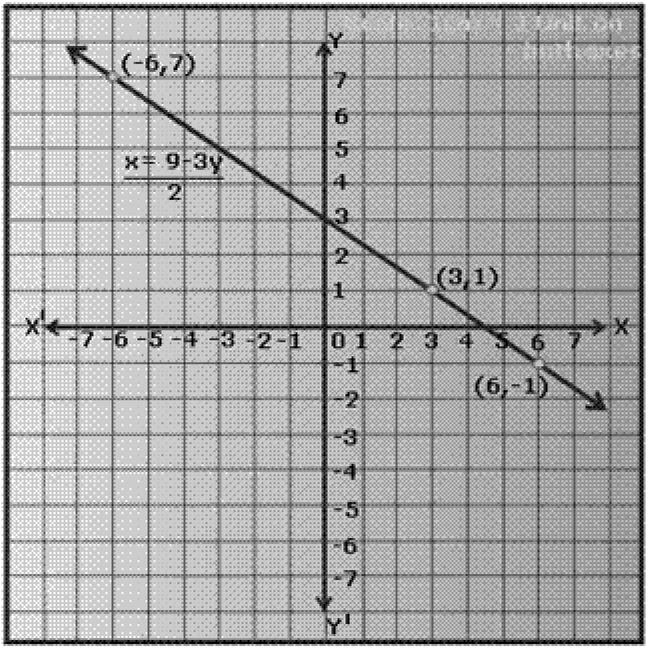
Put y = 1, then x = 3

Put y = −1, then x = 6

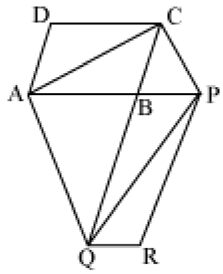
Put y = 7, then x = −6

x	3	6	-6
y	1	−1	7

On plotting the graph, we get



16. Construction: Let us join AC and PQ.



ΔACQ and ΔAQP are on same base AQ and between same parallel lines AQ and CP.

$\therefore$ area(ΔACQ) = area(ΔAPQ)

$\Rightarrow$ area(ΔACQ) – area(ΔABQ) = area(ΔAPQ) – area(ΔABQ)

$\Rightarrow$ area(ΔABC) = area(ΔQBP) ... (1)

Since AC and PQ are diagonals of parallelograms ABCD and PBQR respectively

$\therefore$ area(ΔABC) = $\frac{1}{2}$area(ABCD) ... (2)

$\therefore$ area(ΔQBP) = $\frac{1}{2}$area(PBQR) ... (3)

From equations (1), (2) and (3), we have

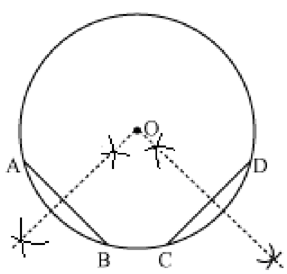
$\frac{1}{2}$area(ABCD) = $\frac{1}{2}$area(PBQR)

$\therefore$ area(ABCD) = area(PBQR)

17. Following are the steps of construction:

Step 1: On the given circle, take any two different chords AB and CD and draw the perpendicular bisectors of these chords.

Step 2**:** Let these perpendicular bisectors meet at point O. O is the centre of the given circle.



18. The roller is cylindrical in shape.

Height (h) of cylindrical roller = Length of roller = 120 cm

Radius (r) of the circular end of roller = $\left(\frac{84}{2}\right)$ cm $= 42$ cm

C.S.A. of roller = $2\pi rh = 2\times\frac{22}{7}\times 42\times 120 = 31680\ \text{cm}^2$

Area of field = 500 × C.S.A. of roller = (500 × 31680) cm^2 = 15840000 cm^2

Area of field = 1584 m^2.

OR

Height of the cylinder = 14 cm

Let diameter of the cylinder be ‘d’ and the radius of its base be ‘r’.

Curved surface area of cylinder = 88 cm^2

$\Rightarrow 2\pi rh = 88$

$\Rightarrow \pi dh = 88$

$\Rightarrow d = 88 \times \frac{7}{22} \times \frac{1}{14}$

$\Rightarrow d = 2$

The diameter of the base of the cylinder is 2 cm.

19. Total observations in the given data set are 10 (even number). So median of this data set will be mean of the $\frac{10}{2}$ i.e. 5^{th} and $\frac{10}{2}+1$ i.e. the 6^{th} observations.

So, median of data $= \frac{5^{th} \text{ observation} + 6^{th} \text{ observation}}{2}$

$\Rightarrow 63 = \frac{x + x + 2}{2}$

$\Rightarrow 63 = \frac{2x + 2}{2}$

$\Rightarrow 63 = x + 1$

$\Rightarrow x = 62$

20.

$x - \frac{2}{3}y = \frac{8}{3}$ ....(1)

$\frac{2x}{5} - y = \frac{7}{5}$ .... (2)

From (1)

$x = \frac{8}{3} + \frac{2}{3}y = \frac{8+2y}{3}$ ...... (3)

Substituting the value of y in (2),

$\frac{2}{5}\left(\frac{8+2y}{3}\right) - y = \frac{7}{5}$

$\Rightarrow \frac{16+4y}{15} - y = \frac{7}{5}$

$\Rightarrow 16 + 4y - 15y = 21$

$\Rightarrow -11y = 5$

$y = \frac{-5}{11}$

Substituting the value of y in (3),

$x = \frac{8 + 2\left(\frac{-5}{11}\right)}{3} = \frac{8 - \frac{10}{11}}{3}$

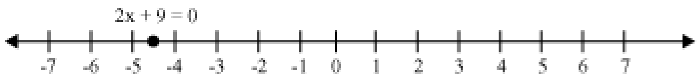
$= \frac{88-10}{11 \times 3} = \frac{78}{11 \times 3}$

$x = \frac{26}{11}$

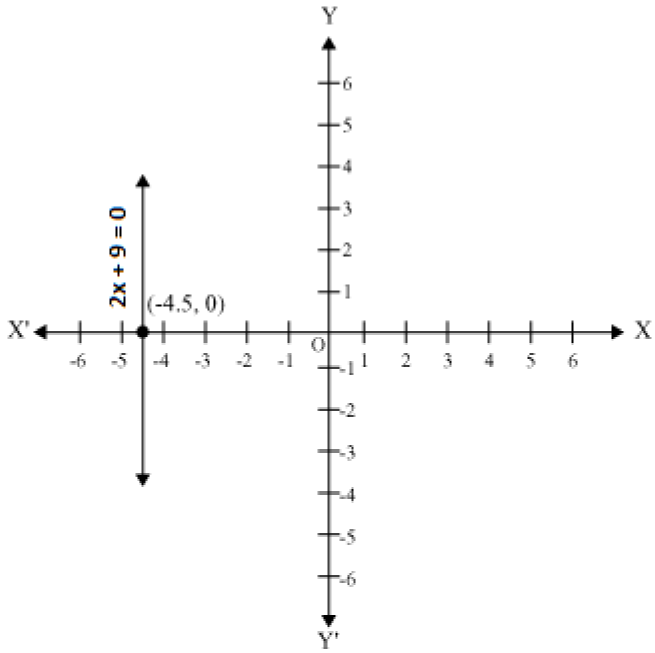
$x = \frac{26}{11}, y = \frac{-5}{11}$

OR

i. In one variable 2x + 9 = 0 represents a point $x = \frac{-9}{2} = -4.5$ as shown in following figure.



ii. In two variables 2x + 9 = 0 represents a straight line passing through point (–4.5, 0) and parallel to the y-axis.
As it is a collection of all points of plane, having their x-coordinate as 4.5.



21. Side (a) of the cube = 12 cm

Volume of the cube = $a^3 = (12\text{ cm})^3 = 1728\text{ cm}^3$

Let the side of each smaller cube be l.

Volume of each smaller cube $=\left(\frac{1728}{8}\right)\text{cm}^3 = 216\text{ cm}^3$

$l^3 = 216\text{ cm}^3$

$\Rightarrow l = 6\text{ cm}$

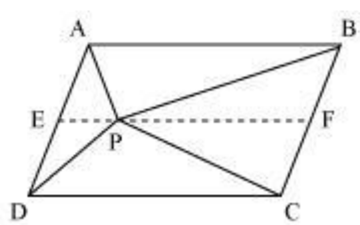
Thus, the side of each smaller cube is 6 cm.

Ratio between surface areas of the cubes $=\frac{\text{Surface area of the bigger cube}}{\text{Surface area of the smaller cube}}$

$$=\frac{6a^2}{6l^2}=\frac{6(12)^2}{6(6)^2}=\frac{4}{1}$$

So, the required ratio between surface areas of the cubes is 4: 1.

22.



Let us draw a line segment EF, passing through the point P and parallel to line segment AB.

In parallelogram ABCD we find that

AB || EF (By construction) ...(1)

ABCD is a parallelogram

$\therefore$ AD || BC (Opposite sides of a parallelogram)

$\Rightarrow$ AE || BF ....(2)

From equations (1) and (2), we have

AB || EF and AE || BF

So, quadrilateral ABFE is a parallelogram

Now, we may observe that ΔAPB and parallelogram ABFE are lying on the same base AB and between the same parallel lines AB and EF.

$$\text{area}(\Delta APB) = \frac{1}{2}\text{area}(ABFE) \quad ...(3)$$

Similarly, for ΔPCD and parallelogram EFCD

$$\text{area}(\Delta PCD) = \frac{1}{2}\text{area}(EFCD) \quad ...(4)$$

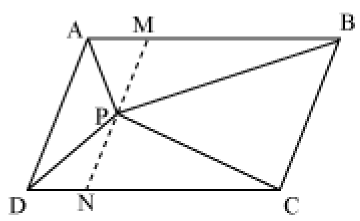
Adding equations (3) and (4), we have

$$\text{area}(\Delta APB) + \text{area}(\Delta PCD) = \frac{1}{2}\left[\text{area}(ABFE) + \text{area}(EFCD)\right]$$

$$\text{area}(\Delta APB) + \text{area}(\Delta PCD) = \frac{1}{2}\text{area}(ABCD) \quad ...(5)$$

Draw a line segment MN, passing through point P and parallel to line segment AD.

In parallelogram ABCD we may observe that,



MN || AD (By construction) ... (6)

ABCD is a parallelogram

$\therefore$ AB || DC (Opposite sides of a parallelogram)

$\Rightarrow$ AM || DN ... (7)

From equations (6) and (7), we have

MN || AD and AM || DN

So, quadrilateral AMND is a parallelogram

Now, ΔAPD and parallelogram AMND are lying on the same base AD and between the same parallel lines AD and MN.

$$\therefore \text{area}(\Delta APD) = \frac{1}{2}\text{area}(AMND) \quad ...(8)$$

Similarly, for ΔPCB and parallelogram MNCB

$$\text{area}(\Delta PCB) = \frac{1}{2}\text{area}(MNCB) \quad ...(9)$$

Adding equations (8) and (9), we have

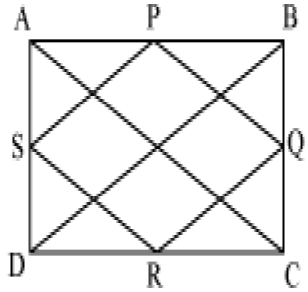
$$\text{Area}(\Delta\text{APD}) + \text{area}(\Delta\text{PCB}) = \frac{1}{2}[\text{area}(\text{AMND}) + \text{area}(\text{MNCB})]$$

$$\text{Area}(\Delta\text{APD}) + \text{area}(\Delta\text{PCB}) = \frac{1}{2}\text{area}(\text{ABCD}) \quad ...(10)$$

On comparing equations (5) and (10), we have

Area(ΔAPD) + area(ΔPBC) = area(ΔAPB) + area(ΔPCD)

23. Let us join AC and BD



In ΔABC,

P and Q are the mid-points of AB and BC respectively

$\therefore$ PQ || AC and PQ = $\frac{1}{2}$AC (Mid-point theorem) ... (1)

Similarly in ΔADC

SR || AC and SR = $\frac{1}{2}$ AC (Mid-point theorem). .. (2)

Clearly, PQ || SR and PQ = SR

As in quadrilateral PQRS, one pair of opposite sides is equal and parallel to each other, so, it is a parallelogram.

$\therefore$ PS || QR and PS = QR (opposite sides of parallelogram)... (3)

Now, in ΔBCD, Q and R are the mid-points of sides BC and CD respectively.

$\therefore$ QR || BD and QR = $\frac{1}{2}$BD (Mid-point theorem) ... (4)

But diagonals of a rectangle are equal

$\therefore$ AC = BD (5)

Now, by using equation (1), (2), (3), (4), (5) we can say that

PQ = QR = SR = PS

So, PQRS is a rhombus.

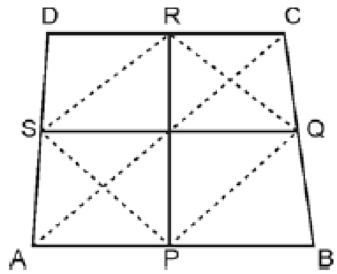
24. Number of times 2 heads come up = 72

Total number of times the coins were tossed = 200

$$P(2\text{ heads will come up}) = \frac{\text{Number of times 2 heads come up}}{\text{Total number of times the coins were tossed}} = \frac{72}{200} = \frac{9}{25}$$

(SECTION – D)

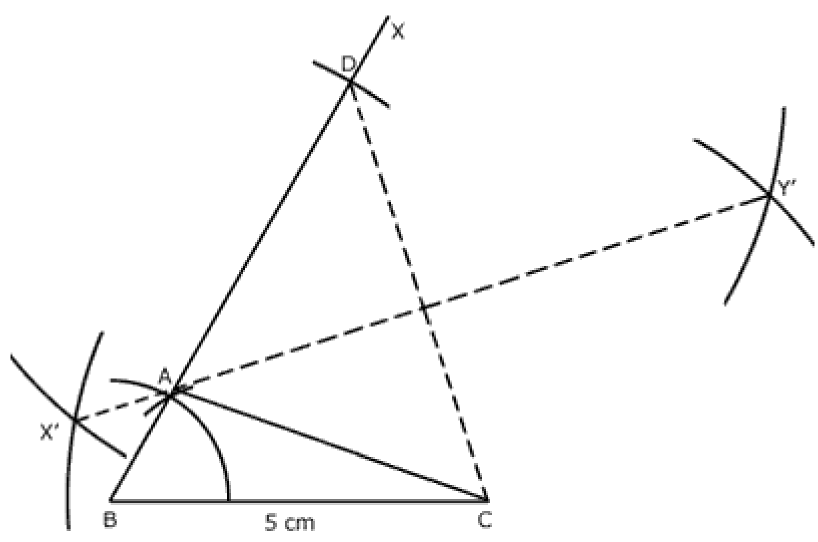
25. Let ABCD be a quadrilateral. P, Q, R, and S are mid points of AB, BC, CD and DA respectively.



Join PQ, QR, RS and SP.
Join AC.
In ΔDAC, SR || AC
And SR = $\frac{1}{2}$AC (Mid-point theorem)
In ΔBAC, PQ || AC and PQ = $\frac{1}{2}$AC

Clearly, PQ || SR and PQ = SR
As in quadrilateral PQRS, one pair of opposite sides is equal and parallel to each other, so, it is a parallelogram.
⇒ PQRS is a parallelogram
∴ PR and SQ are diagonals of PQRS, therefore PR & SQ bisect each other.

26. Construction:
1. Draw BC = 5 cm
2. Draw m∠CBX = 60° and cut off BD = 7.7 cm.
3. Join CD and draw its perpendicular bisector meeting BD at A.
4. Join AC. ΔABC is the required triangle.

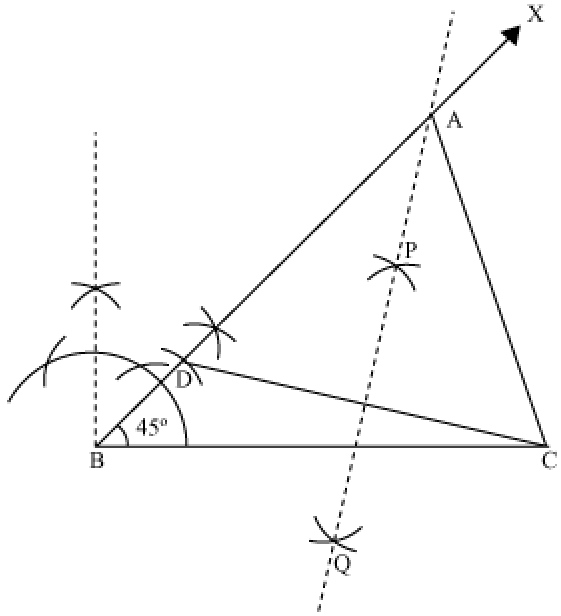


OR

The steps of construction for the required triangle are as follows:
Step 1: Draw the line segment BC = 8 cm and at point B make an angle of 45° say ∠XBC.
Step 2: Cut the line segment BD = 3.5 cm (equal to AB – AC) on ray BX.

Step 3: Join DC and draw the perpendicular bisector PQ of DC.

Step 4: Let it intersect BX at point A. Join AC. ΔABC is the required triangle.



27. According to given condition,

$x + y = 100$ ...(i)

Now, put the value $x = 0$ in equation (i).

$0 + y = 100 \Rightarrow y = 100$.

The solution is (0, 100)

Putting the value $x = 50$ in equation (i)

We get,

$50 + y = 100 \Rightarrow y = 100 - 50 \Rightarrow y = 50$.

The solution is (50, 50).

Put the value $x = 100$ in equation (i).

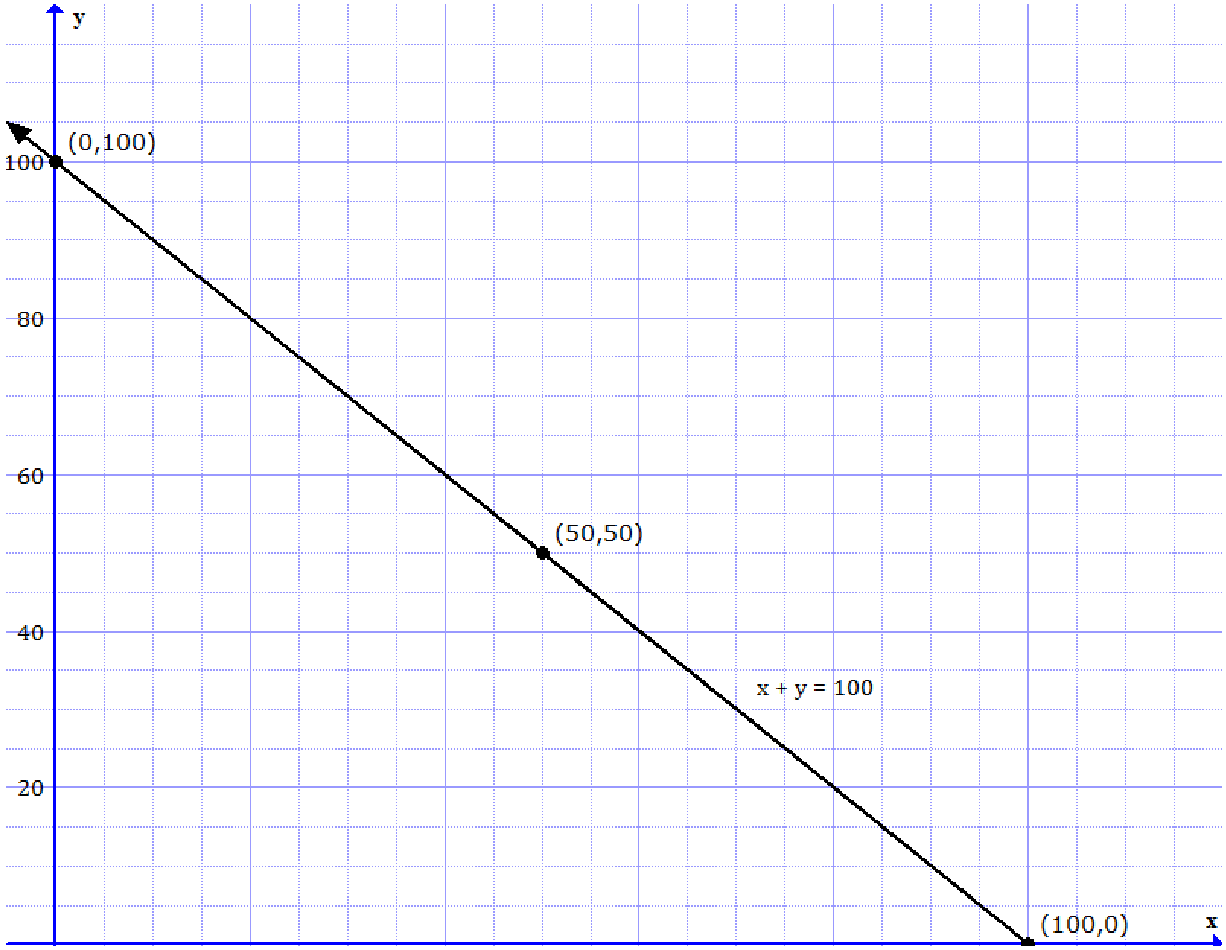
$100 + y = 100$,

$y = 100 - 100 \Rightarrow y = 0$.

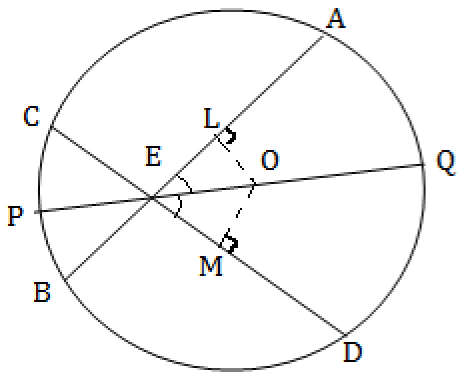
The solution is (100, 0).

X	0	50	100
y	100	50	0

Now, plot the points (0, 100), (50, 50), (100, 0) and draw lines passing through the points.



28. Given that AB and CD are two chords of a circle with centre O intersecting at a point E. PQ is diameter through E, such that ∠AEQ = ∠DEQ.



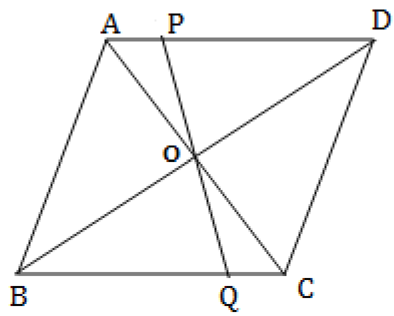
To prove that AB = CD.
Draw perpendicular OL and OM on chords AB and CD respectively.
Now,
m∠LOE = 180° – 90° – m∠LEO = 90° – m∠LEO [Angle sum property of a triangle]
⇒ m∠LOE = 90° – m∠AEQ
⇒ m∠LOE = 90° – m∠ DEQ

⇒ m∠LOE = 90° – m∠ MEQ
⇒ ∠LOE = ∠ MOE
In Δ OLE and Δ OME,
∠LEO = ∠ MEO
∠LOE = ∠ MOE
EO = EO
Δ OLE ≅ Δ OME
OL = OM
AB = CD

29. i. Cost of white washing the dome from inside = Rs 498.96
Cost of white washing 1 m^2 area = Rs. 2
∴ C.S.A. of the inner side of dome = $\left(\frac{498.96}{2}\right) m^2 = 249.48\ m^2$

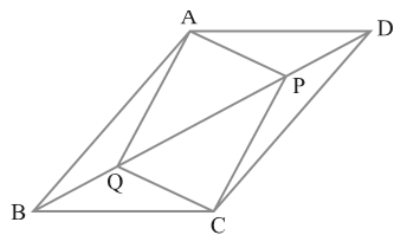
ii. Let inner radius of hemispherical dome be r.
C.S.A of the inner side of the dome = 249.48 m^2
$2\pi r^2 = 249.48\ m^2$
$\Rightarrow 2\times\frac{22}{7}\times r^2 = 249.48\ m^2$
$\Rightarrow r^2 = \left(\frac{249.48\times 7}{2\times 22}\right) m^2 = 39.69\ m^2$
$\Rightarrow r = 6.3$ m
Volume of air inside the dome = Volume of the hemispherical dome
$= \frac{2}{3}\pi r^3$
$= \left[\frac{2}{3}\times\frac{22}{7}\times(6.3)^3\right] m^3$
= 523.908 m^3
Thus, the volume of air inside the dome is approximately 523.9 m^3.

30. We have to prove that
area(APQB) = area(PQCD) = $\frac{1}{2}$ (ABCD)



Since diagonals of a parallelogram divide it into two triangles of equal area.
Therefore, area(Δ ABC) = area(Δ ACD)
$\Rightarrow$ area(ABQO) + area(Δ COQ) = area(CDPO) +area (Δ AOP) ...(i)
Consider Δ AOP and Δ COQ.
In these two triangles, we have:
$\angle$AOP = $\angle$COQ [Vertically opposite angles]
OA = OC [Diagonals of a ||gm bisect each other]
$\angle$OAP = $\angle$OCQ [Alternate angles]
$\Rightarrow$ Δ AOP $\cong$ Δ COQ
$\Rightarrow$ area(Δ AOP) = area(Δ COQ) ...(ii)
From (i) and (ii),
area(ABQO) + area(Δ AOP) = area(CDPO) + area(Δ COQ)
$\Rightarrow$ area(ABQP) = area(CDPQ)
$\Rightarrow$ area(APQB) = area(PQCD)

OR



i. In ΔAPD and ΔCQB,
$\angle$ ADP = $\angle$ CBQ (Alternate interior angles for BC || AD)
AD = CB (Opposite sides of parallelogram ABCD)
DP = BQ (Given)
$\therefore$ Δ APD $\cong$ Δ CQB (Using SAS congruence rule)
As we had observed that ΔAPD ≅ ΔCQB,
$\therefore$ AP = CQ (CPCT)

ii. In ΔAQB and ΔCPD,
$\angle$ ABQ = $\angle$ CDP (Alternate interior angles for AB || CD)
AB = CD (Opposite sides of parallelogram ABCD)
BQ = DP (Given)
$\therefore$ Δ AQB ≅ ΔCPD (Using SAS congruence rule)
Δ AQB $\cong$ Δ CPD,
$\therefore$ AQ = CP (CPCT)

iii. From the result obtained above,
AQ = CP and AP = CQ
Since opposite sides in quadrilateral APCQ are equal to each other, APCQ is a parallelogram.

31. The equation of the line given to us is x – 2y = 4.

x – 2y = 4

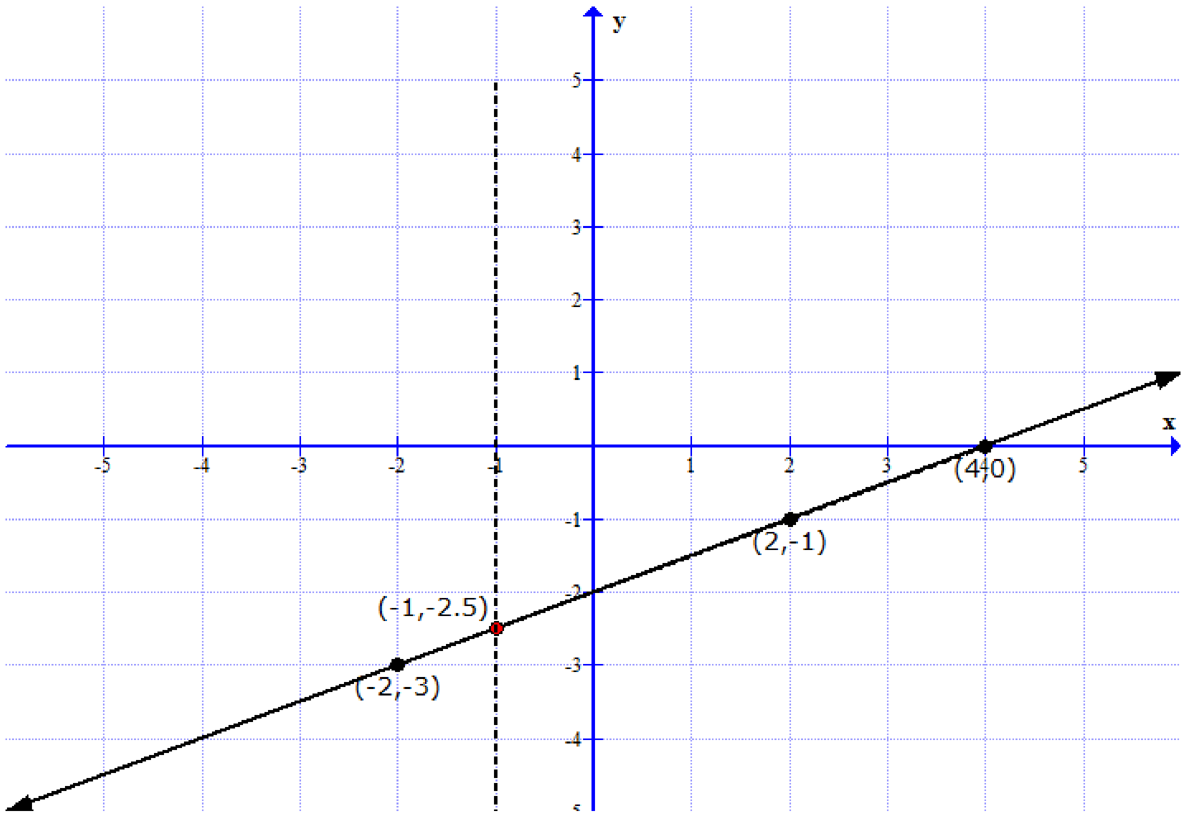
$\Rightarrow y = \frac{x-4}{2}$

When x = 0, y = –2; When x = 2, y = –1; When x = –2, y = –3; and so on.

We can plot a table of value of x and y as:

x	0	2	–2	4
y	–2	–1	–3	0

Now plot the points from the table on a graph paper and join them to get a straight line:



From the graph we can see that when x = –1, y = –2.5.

32. Edge of the cubical tank = 1.5 m = 150 cm

Surface area of the tank = 5 × 150 × 150 cm^2

Area of each square tile = side × side = 25 × 25 cm^2

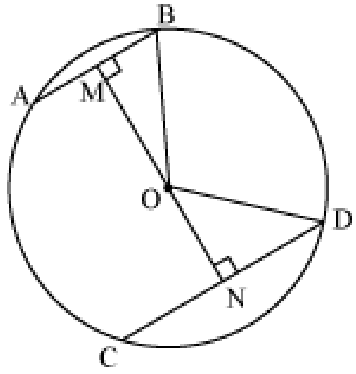
The number of tiles required = $\frac{\text{Suirface area of the tank}}{\text{area of each tile}} = \frac{5 \times 150 \times 150}{25 \times 25} = 180$

Cost of 1 dozen tiles, i.e. cost of 12 tiles = Rs. 360

Cost of one tile = Rs. $\frac{360}{12}$ = Rs. 30

The cost of 180 tiles = 180 × 30 = Rs. 5400

33. Construction: Draw OM ⊥ AB and ON ⊥ CD. Join OB and OD.



$BM = \frac{AB}{2} = \frac{5}{2}$ (Perpendicular from centre bisects the chord)

$ND = \frac{CD}{2} = \frac{11}{2}$

Let ON be x, so OM will be 6 – x

In ΔMOB

$OM^2 + MB^2 = OB^2$

$(6-x)^2 + \left(\frac{5}{2}\right)^2 = OB^2$

$36 + x^2 - 12x + \frac{25}{4} = OB^2$ ... (1)

In ΔNOD

$ON^2 + ND^2 = OD^2$

$x^2 + \left(\frac{11}{2}\right)^2 = OD^2$

$x^2 + \frac{121}{4} = OD^2$ ... (2)

We have OB = OD (radii of same circle)

So, from equation (1) and (2)

$36 + x^2 - 12x + \frac{25}{4} = x^2 + \frac{121}{4}$

$\Rightarrow 12x = 36 + \frac{25}{4} - \frac{121}{4} = \frac{144+25-121}{4} = \frac{48}{4} = 12$

$\Rightarrow 12x = 12$

$\Rightarrow x = 1$

From equation (2)

$$\Rightarrow (1)^2 + \left(\frac{121}{4}\right) = OD^2$$

$$\Rightarrow OD^2 = 1 + \frac{121}{4} = \frac{125}{4}$$

$$\Rightarrow OD = \frac{5}{2}\sqrt{5}$$

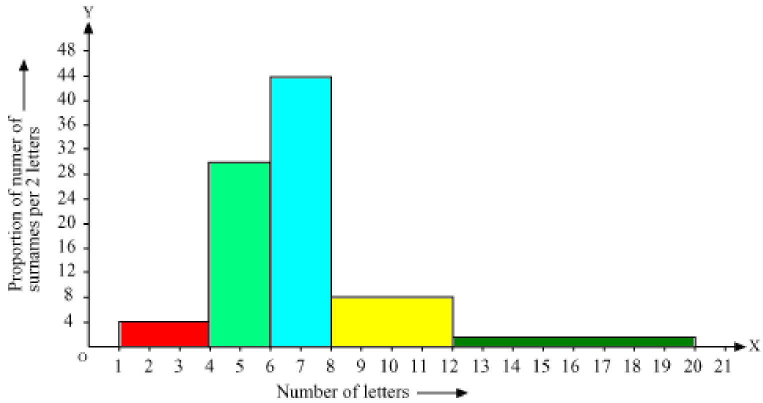
Hence, radius of the circle is $\frac{5}{2}\sqrt{5}$ cm.

34.

i. Given data has class interval of varying width. We need to compute the adjusted Frequency

Number of letters	Frequency (Number of surnames)	Width of class	Length of rectangle
1 – 4	6	3	$\frac{6\times2}{3}=4$
4 – 6	30	2	$\frac{30\times2}{2}=30$
6 – 8	44	2	$\frac{44\times2}{2}=44$
8 – 12	16	4	$\frac{16\times2}{4}=8$
12 – 20	4	8	$\frac{4\times2}{8}=1$

Now by taking number of letters on the x-axis and proportion of number of surnames per 2 letters interval on the y-axis and choosing an appropriate scale (1 unit = 4 students for y-axis) we will construct the histogram as below.



ii. The class interval in which maximum number of surnames lies is (6 – 8) with 44 surnames.