Goa Board
Class IX Mathematics
Term II
Sample Paper – 3 Solution

(SECTION – A)

1. Correct Answer: B
Arrange the data in ascending order and count the number of times the player scored more than 100 runs. Hence, the number of centuries scored by him is 3.

2. Correct Answer: D
Since the volume and surface area are the same,
$$\frac{4}{3}\pi r^3 = 4\pi r^2$$
$\Rightarrow r = 3$ units

3. Correct Answer: B
If for one of the solutions of the equation ax + by + c = 0, x is negative and y is positive, then a portion of the above line definitely lies in the II^{nd} Quadrant.

4. Correct Answer: A
We have, PQ ||RS and PS = QR = 7.
Therefore PQRS is an isosceles trapezium.
So, base angles ∠P and ∠Q are equal.
m∠Q = 70°
m∠Q + m∠R = 180°
⇒ m∠R = 110°
m∠P + m∠S = 180°
⇒ m∠S = 110°

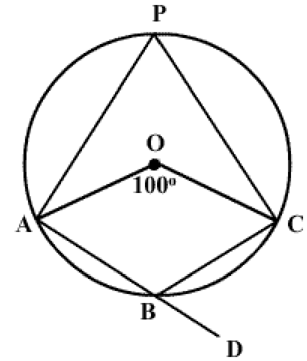
5. Correct Answer: B
Mode is the value that occurs most frequently in a data set. Hence mode of the given data is 2.

6. Correct Answer: B
The graph of the equation y = –4 is a line parallel to the x-axis, at a distance of 4 units from origin and below the x-axis.

7. Correct Answer: C
When two cubes of sides 5 cm are joined end to end then length = l = 5 + 5 = 10 cm
Volume (V) = l × b × h = 10 × 5 × 5 = 250 cm^3

8. Correct Answer: B

$\angle APC = \frac{1}{2}\angle AOC$



[∵ Central angle is twice the measure of the angles on the remaining part of the circumference.]

∴ $m\angle APC = 50°$

Now, $m\angle APC + m\angle ABC = 180°$

But $m\angle ABC = 130°$

$\Rightarrow m\angle ABC + m\angle CBD = 180°$ [linear pair]

$\Rightarrow m\angle CBD = 50°$

(SECTION – B)

9. In ΔABC

$m\angle BAC + m\angle ABC + m\angle ACB = 180°$ (Angle sum property of a triangle)

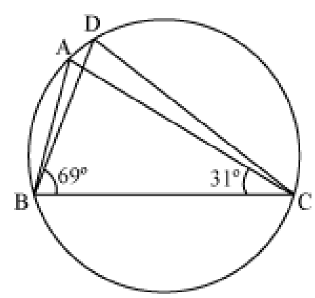
$\Rightarrow m\angle BAC + 69° + 31° = 180°$

$\Rightarrow m\angle BAC = 180° - 100°$

$\Rightarrow m\angle BAC = 80°$

$m\angle BDC = m\angle BAC = 80°$

(Angles in same segment of a circle are equal)



10. To construct a grouped frequency table of class size 0.5 and starting from class interval 2 – 2.5, class intervals will be 2 – 2.5, 2.5 – 3, 3 – 3.5, and so on. Hence the required grouped frequency distribution table is

Lives of batteries (in hours)	Number of batteries
2 – 2.5	2
2.5 – 3.0	6
3.0 – 3.5	14
3.5– 4.0	11
4.0 – 4.5	4
4.5 – 5.0	3
Total	40

11. Let ABCD be a rhombus in which the diagonals intersect at point O and a circle is drawn with side CD as the diameter.

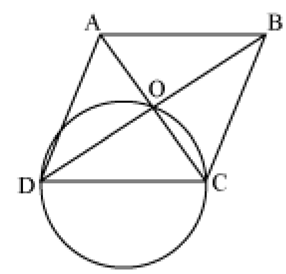
We know that the angle in a semicircle measures 90°.

∴ $m\angle COD = 90°$

Also in rhombus the diagonals intersect each other at 90°.

$m\angle AOB = m\angle BOC = m\angle COD = m\angle DOA = 90°$

So, point O has to lie on the circle.



12. Height (h) of cylindrical tank is 1 m

Base radius (r) of cylindrical tank = $\left(\frac{140}{2}\right)\text{cm} = 70\text{ cm}$

Base radius (r) of cylindrical tank = 0.7 m

Surface area of cylinder = $2\pi r[h+r]$

$$= 2 \times \frac{22}{7} \times 0.7[1+0.7]$$

$$= 7.48\text{m}^2$$

Therefore, it will require 7.48 m^2 of sheet.

13. Let B denote the event that the batsman did not hit a boundary.

Total number of trials = 45

Number of trials in which the event B happened = 45 – 9 = 36

∴ The probability that he didn't hit a boundary = $P(B) = \frac{36}{45} = \frac{4}{5} = 0.8$

14. Mean height = $\frac{151+158+155+144+152}{5} = \frac{760}{5} = 152\text{ cm}$

(SECTION – C)

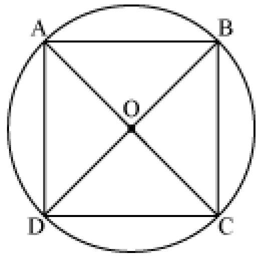
15.

i. (+, +) are the signs of co-ordinates of points in the Ist quadrant.
∴ A(2, 2) lies in the Ist quadrant.

ii. (–, –) are the signs of the co- ordinates of points in the IIIrd quadrant.
∴ B(–3, –5) lies in the IIIrd quadrant.

iii. (+, –) are the signs of the co-ordinates of points in the IVth quadrant.
∴ C(2, –3) lies in the IVth quadrant.

16. Let ABCD be a cyclic quadrilateral having diagonals as BD and AC intersecting each other at point O.



$m\angle BAD = \frac{1}{2} m\angle BOD = \frac{180^\circ}{2} = 90^\circ$ (Consider BD as a chord)

$m\angle BCD + m\angle BAD = 180^\circ$ (Cyclic quadrilateral)

$m\angle BCD = 180^\circ - 90^\circ = 90^\circ$

$m\angle ADC = \frac{1}{2} m\angle AOC = \frac{1}{2}(180^\circ) = 90^\circ$ (Considering AC as a chord)

$m\angle ADC + m\angle ABC = 180^\circ$ (Cyclic quadrilateral)

$90^\circ + m\angle ABC = 180^\circ$

$m\angle ABC = 90^\circ$

Here, each interior angle of cyclic quadrilateral is of 90°. Hence it is a rectangle.

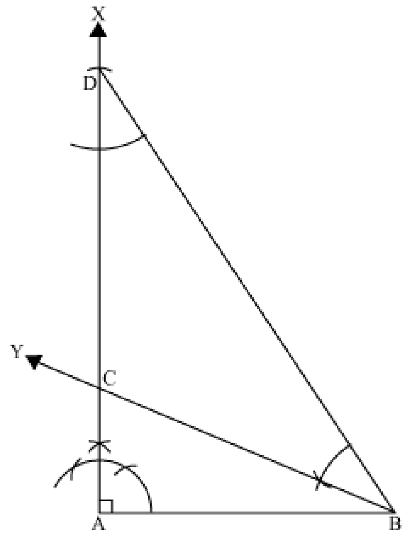
17. The steps of construction for the required triangles are as follows:

Step I: Draw line segment AB of 12 cm. Draw a ray AX making 90° with AB.

Step II: Cut a line segment AD of 18 cm. (As sum of other two sides is 18) from ray AX.

Step III: Join DB such that ∠DBY is equal to ∠ADB.

Step IV: Let BY intersect AX at C. Join AC, BC. ΔABC is the required triangle.



18. Radius of circular end of cylindrical penholder = 3 cm

Height of penholder = 10.5 cm

Surface area of 1 penholder = C.S.A. of penholder + Area of base of penholder

Surface area of 1 penholder = $2\pi rh + \pi r^2$

$$= \left[2 \times \frac{22}{7} \times 3 \times 10.5 + \frac{22}{7} \times (3)^2\right] \text{cm}^2$$

$$= \left(132 \times 1.5 + \frac{198}{7}\right) \text{cm}^2$$

$$= \left(198 + \frac{198}{7}\right) \text{cm}^2$$

$$= \frac{1584}{7} \text{cm}^2$$

Area of cardboard sheet used by 1 participant $= \frac{1584}{7} \text{cm}^2$

Area of cardboard sheet used by 35 participant = $\left(\frac{1584}{7} \times 35\right) \text{cm}^2$ = 7920 cm^2

Thus, 7920 cm^2 cardboard sheet will be bought by the organizers for the competition.

OR

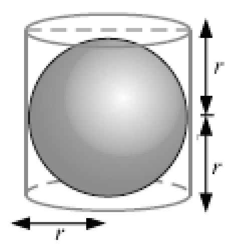
i. Surface area of sphere = $4\pi r^2$

ii. Height of cylinder = r + r = 2r

Radius of cylinder = r

C.S.A. of cylinder = $2\pi rh = 2\pi r(2r) = 4\pi r^2$

iii. Required ratio $= \frac{\text{Surface area of sphere}}{\text{CSA of cylinder}} = \frac{4\pi r^2}{4\pi r^2} = \frac{1}{1}$



19. Number of goals scored by the team:

2, 3, 4, 5, 0, 1, 3, 3, 4, 3

$$\text{Mean of data} = \frac{\text{Sum of all observations}}{\text{Total number of observations}}$$

$$\text{Mean score} = \frac{2+3+4+5+0+1+3+3+4+3}{10}$$

$$= \frac{28}{10} = 2.8$$

$$= 2.8 \text{ goals}$$

Arranging the number of goals in ascending order

0, 1, 2, 3, 3, 3, 3, 4, 4, 5

As the number of observations is 10. 10 is an even number. So, median score will be

$$\text{Median score} = \frac{5^{th}\text{ observation} + 6^{th}\text{ observation}}{2}$$
$$= \frac{3+3}{2}$$
$$= \frac{6}{2}$$
$$= 3$$

Mode of data is the observation with the maximum frequency.
As 3 has the maximum frequency (occurs 4 times), it is the mode of the given data set.

20. 3 – (x – 5) = y + 2

⇒ 3 – x + 5 = y + 2

⇒ 8 – x = y + 2

⇒ x + y = 8 – 2

⇒ x + y = 6 ...[1]

2 (x + y) = 4 – 3y

⇒ 2x + 2y = 4 – 3y

⇒2x + 5y = 4

⇒2x + 5y = 4 ...[2]

Form (1) $x = 6 - y$ .....(3)
Substituting the value of x in (2),
$2(6 - y) + 5y = 4$
$12 - 2y + 5y = 4$
$3y = 4 - 12 = -8$
$$y = \frac{-8}{3}$$
Substituting the value of y in (3),
$$x = 6 - \left(-\frac{8}{3}\right) = 6 + \frac{8}{3} = \frac{18+8}{3} = \frac{26}{3}$$
$$x = \frac{26}{3}, y = \frac{-8}{3}$$

OR

Total distance covered = x km.

Fare for 1^{st} kilometer = Rs. 8

Fare for the remaining distance = (x – 1) 5

Total fare = 8 + (x – 1) 5

y = 8 + 5x –5

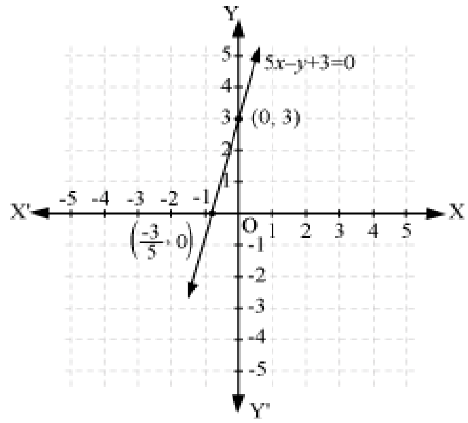
y = 5x + 3

5x – y + 3 = 0

We may observe that points (0, 3) and $\left(-\frac{3}{5}, 0\right)$ satisfy the above equation. So these are solutions of this equation.

x	0	$\frac{-3}{5}$
y	3	0

Now we may draw the graph of this equation as below



Here we may find that variable x and y are representing the distance covered and fare paid for that distance respectively and these quantities may not be negative. Hence we will consider only those values of x and y which are lying in 1^{st} quadrant.

21. Length (l_1) of the storehouse = 40 m

Breadth (b_1) of the storehouse = 25 m

Height (h_1) of the storehouse = 10 m

Volume of storehouse = $l_1 \times b_1 \times h_1$ = (40 × 25 × 10) m^3 = 10000 m^3

Length (l_2) of a wooden crate = 1.5 m

Breadth (b_2) of a wooden crate = 1.25 m

Height (h_2) of a wooden crate = 0.5 m

Volume of a wooden crate = $l_2 \times b_2 \times h_2$ = (1.5 × 1.25 × 0.5) m^3 = 0.9375 m^3

Let n wooden crates be stored in the storehouse.

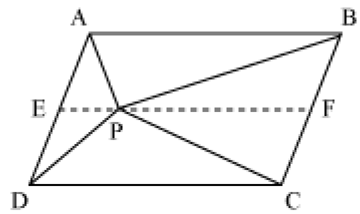
Volume of n wooden crates = volume of storehouse

0.9375 × n = 10000

$$n = \frac{10000}{0.9375} = 10666.66$$

Thus, 10666 numbers of wooden crates can be stored in the storehouse.

22. Let us draw a line segment EF, passing through point P and parallel to line segment AB.



In parallelogram ABCD we find that

AB || EF (By construction) ... (1)

ABCD is a parallelogram

∴ AD || BC (Opposite sides of a parallelogram)

⇒ AE || BF ... (2)

From equations (1) and (2), we have

AB || EF and AE || BF

So, quadrilateral ABFE is a parallelogram

Now, we may observe that ΔAPB and parallelogram ABFE are lying on the same base AB and between the same parallel lines AB and EF.

∴ Area (ΔAPB) = $\frac{1}{2}$area (ABFE) ... (3)

Similarly, for ΔPCD and parallelogram EFCD

area (ΔPCD) = $\frac{1}{2}$area (EFCD) ... (4)

Adding equations (3) and (4), we have

$$\text{Area }(\Delta APB)+\text{area }(\Delta PCD)=\frac{1}{2}\left[\text{area }(ABFE)+\text{area }(EFCD)\right]$$

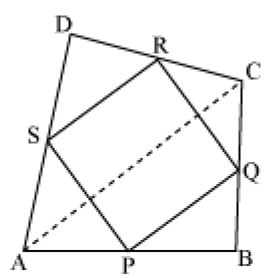
$$\text{Area }(\Delta APB)+\text{area }(\Delta PCD)=\frac{1}{2}\text{area }(ABCD) \quad ...(5)$$

23.

i. In ΔADC, S and R are the mid points of sides AD and CD respectively.
In a triangle the line segment joining the mid points of any two sides of the triangle is parallel to the third side and is half of it.

∴ SR || AC and SR = $\frac{1}{2}$AC ... (1)

ii. In ΔABC, P and Q are mid points of sides AB and BC respectively. So, by using mid-point theorem, we have



PQ || AC and PQ = $\frac{1}{2}$AC ... (2)

Now using equations (1) and (2), we have

PQ || SR and PQ = $\frac{1}{2}$AC = SR ... (3)

⇒ PQ = SR

iii. From equations (3), we have
PQ || SR and PQ = SR
Clearly one pair of opposite sides of quadrilateral PQRS is parallel and equal
Hence, PQRS is a parallelogram.

24. Total number of families = 475 + 814 + 211 = 1500

i. Number of families having 2 girls = 475

$$\text{Required probability} = \frac{\text{Number of families having 2 girls}}{\text{Total number of families}} = \frac{475}{1500} = \frac{19}{60}$$

ii. Number of families having 1 girl = 814

$$\text{Required probability} = \frac{\text{Number of families having 1 girls}}{\text{Total number of families}} = \frac{814}{1500} = \frac{407}{750}$$

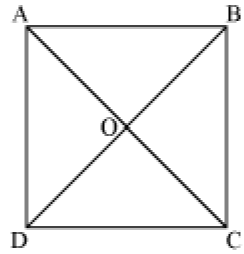
iii. Number of families having no girl = 211

$$\text{Required probability} = \frac{\text{Number of families having no girl}}{\text{Total number of families}} = \frac{211}{1500}$$

(SECTION – D)

25. Let ABCD be a square. Let the diagonals AC and BD intersect each other at a point O.

To show diagonals of a square are equal and bisect each other at right angles, we need to prove AC = BD, OA = OC, OB = OD and m∠AOB = 90°.



Now, in ΔABC and ΔDCB

AB = DC (sides of square are equal to each other)
∠ABC = ∠DCB (all interior angles are of 90°)
BC = CB (common side)
∴ ΔABC ≅ ΔDCB (by SAS congruency)
∴ AC = DB (by CPCT)

Hence, the diagonals of a square are equal in length.

Now in ΔAOB and ΔCOD

∠AOB = ∠COD (vertically opposite angles)
∠ABO = ∠CDO (alternate interior angles)
AB = CD (sides of square are always equal)
∴ ΔAOB ≅ ΔCOD (by AAS congruence rule)
∴ AO = CO and OB = OD (by CPCT)

Hence, the diagonals of a square bisect each other

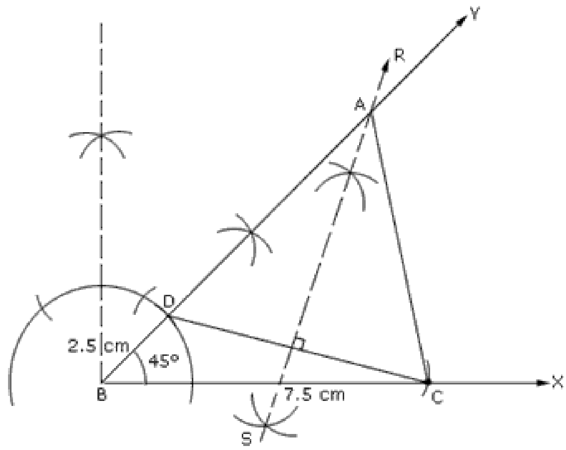
Now in ΔAOB and ΔCOB

Now as we had proved that diagonals bisect each other

So, AO = CO
AB = CB (sides of square are equal)
BO = BO (common)

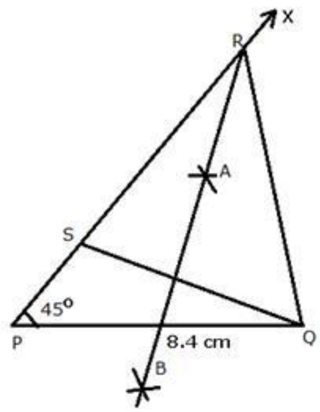
∴ ΔAOB ≅ ΔCOB (by SSS congruence)
∴ ∠AOB = ∠COB (by CPCT)
But, m∠AOB + m∠COB = 180° (linear pair)
2m∠AOB = 180°
m∠AOB = 90°
Hence, the diagonals of a square bisect each other at right angle.

26. Let AB > AC
AB – AC = 2.5 cm
Steps of construction:
i. Draw a ray BX and cut off a line segment BC = 7.5 cm from it.
ii. Construct ∠YBC = 45°.
iii. Cut off a line segment BD = 2. 5 cm from BY.
iv. Join CD.
v. Draw a perpendicular bisectors RS of CD intersecting BY at a point A.
vi. Join AC. Then ABC is the required triangle.



OR

Steps of Construction:
1. Draw the base PQ = 8.4 cm and at point P make an angle of 45°.
2. Cut the line segment PS equal to the length of PR – QR, that is 2.8 cm
3. Join SQ and draw the perpendicular bisector, say AB of SQ.
4. Let it intersect BX at a point R. Join RQ. Then, PQR is the required triangle.

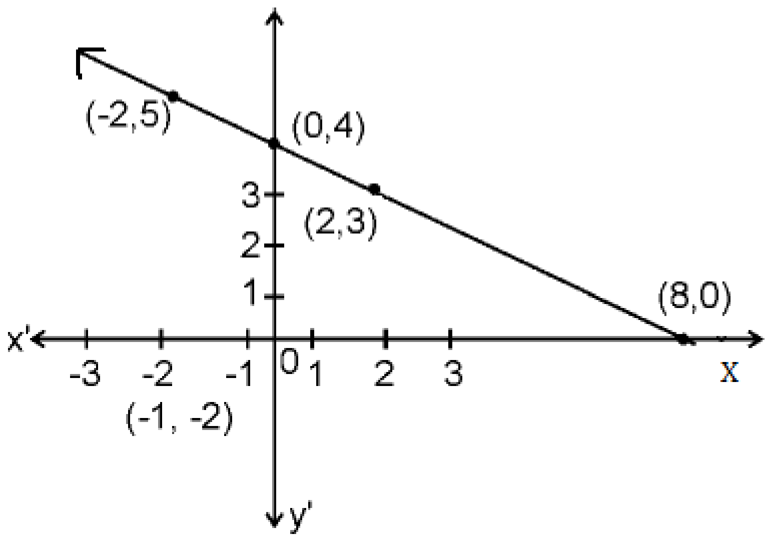


Hence, the diagonals of a square bisect each other at right angle.

27. $x + 2y = 8$

$\Rightarrow y = \frac{1}{2}(8 - x)$

x	−2	0	2
y	5	4	3



From the graph it is clear that (−1, −2) does not lie on the line.
Therefore, (−1, −2) is not a solution of line $x + 2y = 8$.

28. i. Radius of 1 solid iron sphere = r

Volume of 1 solid iron sphere $= \frac{4}{3}\pi r^3$

Volume of 27 solid iron spheres $= 27 \times \frac{4}{3}\pi r^3$

It is given that 27 iron spheres are melted to from 1 iron sphere. So, volume of this iron sphere will be equal to volume of 27 iron spheres.
Radius of the new sphere = r'.

Volume of the new sphere $= \frac{4}{3}\pi r'^3$

$\frac{4}{3}\pi r'^3 = 27 \times \frac{4}{3}\pi r^3$

$\Rightarrow r'^3 = 27r^3$

$\Rightarrow r' = 3r$

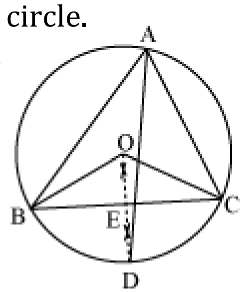
ii. Surface area of 1 solid iron sphere of radius r = $4\pi r^2$
Surface area of iron sphere of radius r' = $4\pi (r')^2$ $= 4\pi (3r)^2 = 36\pi r^2$

$\therefore \frac{S}{S'} = \frac{4\pi r^2}{36\pi r^2} = \frac{1}{9} = 1:9$

29. Let the perpendicular bisector of side BC and angle bisector of ∠A meet at point D.

Let the perpendicular bisector of side BC intersect it at E.

Perpendicular bisector of side BC will pass through circumcentre O of the circle.



Now, ∠BOC and ∠BAC are the angles subtended by arc BC at the centre and a point A on the remaining part of the circle respectively.

We also know that the angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

∠BOC = 2 ∠BAC = 2 ∠A ... (1)

In ΔBOE and ΔCOE

OE = OE (common)

OB = OC (radii of same circle)

∠OEB = ∠OEC (Each 90° as OD ⊥ BC)

∴ ΔBOE ≅ ΔCOE (R.H.S. congruence rule)

∠BOE = ∠COE (by CPCT) ... (2)

But ∠BOE + ∠COE = ∠BOC

⇒ ∠BOE +∠BOE = 2 ∠A [Using equations (1) and (2)]

⇒ 2 ∠BOE = 2 ∠A

⇒ ∠BOE = ∠A

∴ ∠BOE = ∠COE = ∠A

The perpendicular bisector of side BC and angle bisector of ∠A meet at point D.

∴ ∠BOD = ∠BOE = ∠A ... (3)

Since AD is the bisector of angle ∠A

∠BAD = $\frac{\angle A}{2}$

⇒ 2 ∠BAD = ∠A ... (4)

From equations (3) and (4), we have

∠BOD = 2 ∠BAD

It is possible only if BD will be a chord of the circle. For this the point D lies on circum circle.

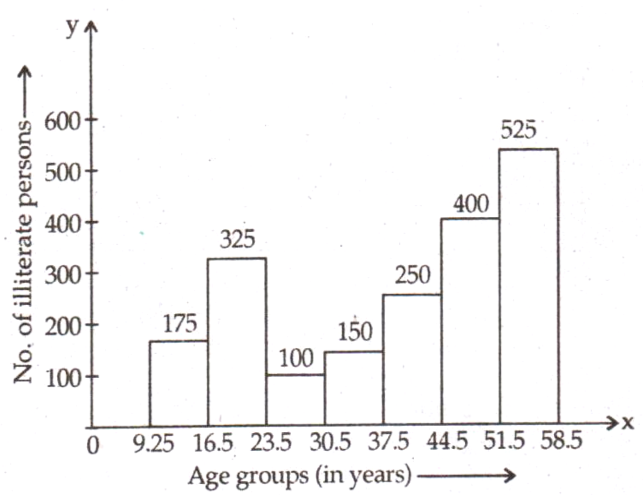
Therefore, the perpendicular bisector of side BC and angle bisector of ∠A meet on the circum circle of triangle ABC.

30. Given frequency distribution is as below:

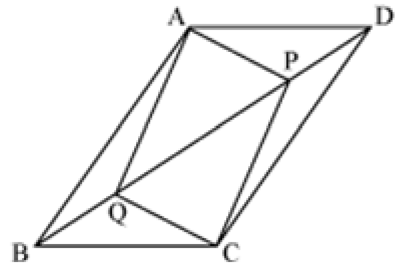
Age group (in years)	10-16	17-23	24-30	31-37	38-44	45-51	52-58
Number of illiterate persons	175	325	100	150	250	400	525

Here, class intervals, i.e. age groups (in years) are in the inclusive form. So, we first have to convert them into exclusive form as given below:

Age group(in years)	9.5-16.5	16.5-23.5	23.5-30.5	30.5-37.5	37.5-44.4	44.5-51.5	51.5-58.5
No of illiterate	175	325	100	150	250	400	525



31. In parallelogram ABCD, two points P and Q are taken on diagonal BD such that DP = BQ (see the given figure). Show that:



i. In ΔAPD and ΔCQB

∠ADP = ∠CBQ	(alternate interior angles for BC || AD)
AD = CB	(opposite sides of parallelogram ABCD)
DP = BQ	(given)
∴ ΔAPD ≅ ΔCQB	(using SAS congruence rule)

ii. As we had observed that ΔAPD ≅ ΔCQB

∴ AP = CQ (CPCT)

iii. In ΔAQB and ΔCPD

∠ABQ = ∠CDP	(alternate interior angles for AB || CD)
AB = CD	(opposite sides of parallelogram ABCD)
BQ = DP	(given)
∴ ΔAQB ≅ ΔCPD	(using SAS congruence rule)

iv. As we had observed that ΔAQB ≅ ΔCPD

∴ AQ = CP (CPCT)

OR

Given that

XY || BC ⇒ EY || BC

BE || AC ⇒ BE || CY

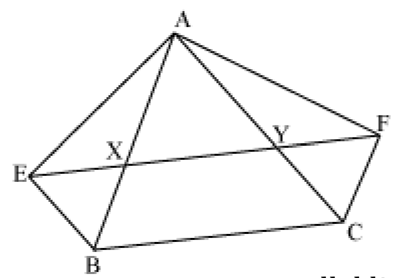
So, EBCY is a parallelogram.

It is given that

XY || BC ⇒ XF || BC

FC || AB ⇒ FC || XB

So, BCFX is a parallelogram.



Now parallelogram EBCY and parallelogram BCFX are on the same base BC and between the same parallel liness BC and EF

∴ area (EBCY) = area (BCFX) ... (1)

Consider parallelogram EBCY and ΔAEB

These are on same base BE and are between same parallel lines BE and AC

∴ area (ΔABE) = $\frac{1}{2}$ area (EBCY) ... (2)

Also parallelogram BCFX and ΔACF are on same base CF and between same parallel lines CF and AB

∴ area (ΔACF) = $\frac{1}{2}$ area (BCFX) ... (3)

From equations (1), (2), and (3), we have

Area (ΔABE) = area (ΔACF)

32. Diameter= 24 m ⇒ radius =12 m

Radius of the conical part = Radius of the cylindrical part (r) = 12 m

Height of cylindrical part (h) =11 m, height of the cone (h) =5 m

For the conical part of the tent,

$l^2 = r^2 + h^2$

$\Rightarrow l = \sqrt{r^2 + h^2}$

$\Rightarrow l = \sqrt{12^2 + 5^2} = 13m$

Thus, l = 13 m

Surface area of the tent = Curved surface area of the conical part + curved surface area of the cylindrical part

$= \pi rl + 2\pi rh$

$= \pi r(l + 2h)$

$= \frac{22}{7} \times 12\ (13 + 22)$

$= \frac{22}{7} \times 12 \times 35$

$= 1320\ m^2$

Breadth of tarpaulin (B) = 5 m, Let length of tarpaulin = L

Area of tarpaulin required= Surface area of the tent

L × B = 1320

$\Rightarrow L = \frac{1320}{5} = 264\ m$

Thus 264 m long tarpaulin is required.

33. $\pi x + y = 9$

For x = 0

$\pi\ (0) + y = 9$

$\Rightarrow y = 9$

So (0, 9) is a solution of this equation.

For x = 1

$\Rightarrow \pi\ (1) + y = 9$

$\Rightarrow y = 9 - \pi$

So, (1, 9 – π) is a solution of this equation.

For x = 2

$\Rightarrow \pi(2) + y = 9$

$\Rightarrow y = 9 - 2\pi$

So, (2, 9 –2 π) is a solution of this equation.

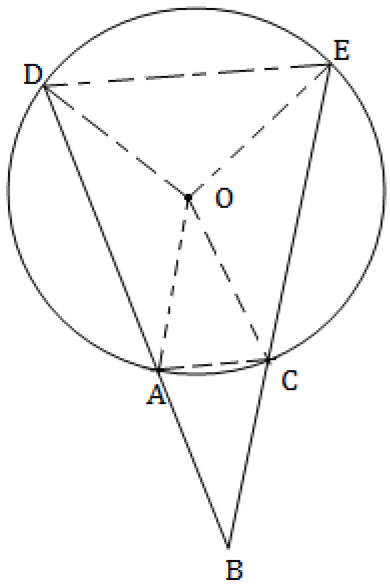
For x = –1

$\pi(-1) + y = 9$

$y = 9 + \pi$

So, $(-1, 9 + \pi)$ is a solution of this equation.

34.



In ΔAOD and ΔCOE

OA = OC	(radii of same circle)	
OD = OE	(radii of same circle)	
AD = CE	(given)	
∴ ΔAOD ≅ ΔCOE	(SSS congruence rule)	
∴ ∠OAD = ∠OCE	(by CPCT)	... (1)
∴ ∠ODA = ∠OEC	(by CPCT)	... (2)

We also have

∠OAD = ∠ODA (As OA = OD) ... (3)

From equations (1), (2) and (3), we have

∠OAD = ∠OCE = ∠ODA = ∠OEC

Let ∠OAD = ∠OCE = ∠ODA = ∠OEC = *x*

In Δ OAC,

OA = OC

∴ ∠OCA = ∠OAC = a (let *a*)

In Δ ODE,

OD = OE

∴ ∠OED = ∠ODE = y (let *y*)

ADEC is a cyclic quadrilateral.

∴ m∠CAD + m∠DEC = 180° (opposite angles are supplementary)

∴ x + a + x + y = 180°

$\therefore 2x + a + y = 180°$

$\therefore y = 180° - 2x - a$ ... (4)

But $m\angle DOE = 180° - 2y$

And $m\angle AOC = 180° - 2a$

Now, $\angle DOE - \angle AOC = 2a - 2y = 2a - 2(180° - 2x - a) = 4a + 4x - 360°$ ... (5)

Now, $m\angle BAC + m\angle CAD = 180°$ (Linear pair)

$\Rightarrow m\angle BAC = 180° - m\angle CAD = 180° - (a + x)$

Similarly, $m\angle ACB = 180° - (a + x)$

Now, in ΔABC

$m\angle ABC + m\angle BAC + m\angle ACB = 180°$ (Angle sum property of a triangle)

$m\angle ABC = 180° - m\angle BAC - m\angle ACB$

$= 180° - (180° - a - x) - (180° - a - x)$

$= 2a + 2x - 180°$

$= \frac{1}{2}[4a + 4x - 360°]$

$m\angle ABC = \frac{1}{2}[m\angle DOE - m\angle AOC]$ [Using equation (5)]