Goa Board
Class IX Mathematics
Term 1
Sample Paper – 5 Solution

Time: 3 hours **Total Marks: 90**

Section A

1. Correct answer: A

   Since AOB is a straight line,

   $\angle$ AOB = 180°

   $\Rightarrow$ x + 10° + x + x + 20° = 180°

   $\Rightarrow$ 3x = 150°

   $\Rightarrow$ x = 50°

2. Correct answer: B

3. Correct answer: B

   $$\frac{56}{1000} = 0.056$$

4. Correct answer: C

   The abscissa or x-coordinate of any point on Y-axis is zero.

Section B

5. a = 2+ $\sqrt{3}$

   $$\Rightarrow \frac{1}{a} = \frac{1}{2+\sqrt{3}}$$

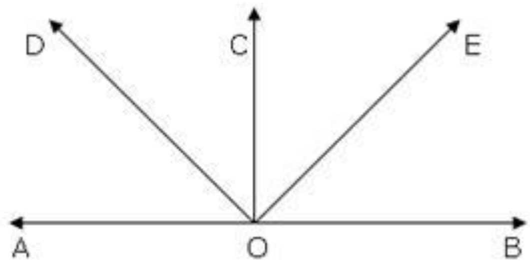
   $$= \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$$

   $$= \frac{2-\sqrt{3}}{2^2 - \left(\sqrt{3}\right)^2}$$

   $= 2 - \sqrt{3}$

   So, $a + \frac{1}{a} = (2+\sqrt{3}) + (2-\sqrt{3}) = 4$

6.



OD and OE are the bisectors of ∠AOC and ∠BOC, respectively.
Therefore, ∠AOD=∠COD and ∠BOE = ∠COE
Also, ∠DOE = 90° (By the given condition)
Now, ∠AOC+ ∠BOC = ∠AOD+∠COD+∠BOE+∠COE
= ∠COD+∠COD+∠COE+∠COE
= 2(∠COD+∠COE)
=2(∠DOE) =2 × 90^o = 180^o
Hence, points A, B and C are collinear.

7. Let $f(z) = 3z^3 + 8z^2 - 1$
The possible integral zeros of f(z) are -1 and 1.
$f(z) = 3z^3 + 8z^2 - 1$
$f(-1) = 3(-1)^3 + 8(-1)^2 - 1 \neq 0$
$\Rightarrow$ -1 is not a zero of f(z)
$f(1) = 3(1)^3 + 8(1)^2 - 1 \neq 0$
$\Rightarrow$ 1 is not a zero of f(z)
Therefore, f(z) has no integral zero.

8. (A) A line segment is a part of line between two points.
(B) Radius of a circle is defined as the distance between centre and a point on its circumference.

9. $(-2x + 5y - 3z)^2$
$= (-2x)^2 + (5y)^2 + (-3z)^2 + 2(-2x)(5y) + 2(5y)(-3z) + 2(-2x)(-3z)$
$= 4x^2 + 25y^2 + 9z^2 - 20xy - 30yz + 12zx$

10. Perimeter = 36 cm = 10 cm + 2(Length of an equal side)

$\Rightarrow$ Length of an equal side = 13 cm

Here, $s = \frac{36}{2} = 18$, and the sides are 10, 13 and 13.

By Heron's formula,

Area = $\sqrt{18 \times 8 \times 5 \times 5} = 60$ sq cm

Section C

11. Given: x = 2y + 6 or x - 2y - 6 = 0

We know that if a + b + c = 0, then $a^3 + b^3 + c^3 = 3xyz$

Therefore, we have:

$(x)^3 + (-2y)^3 + (-6)^3 = 3x(-2y)(-6)$

Or, $x^3 - 8y^3 - 36xy - 216 = 0$

12. In $\triangle$ POR and $\triangle$ QOS

$\angle$ QPR = $\angle$ PQS (given)

OP = OQ (O is the mid-point of PQ)

$\angle$ POS = $\angle$ QOR (given)

$\angle$ POS + x° = $\angle$ QOR + x°

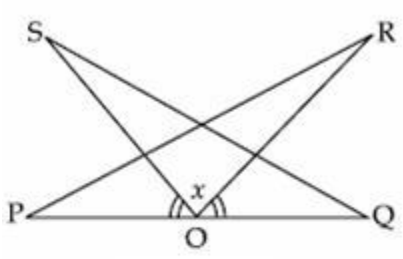
$\angle$ POR = $\angle$ QOS

By ASA congruence rule,

$\triangle$ PQR $\cong$ $\triangle$ QOS

$\Rightarrow$ PR = QS (By CPCT)

Hence, proved.



13. Let $x = 0.\overline{001}$

Then, x= 0.001001001……….. (i)

Therefore, 1000x = 1.001001001…………… (ii)

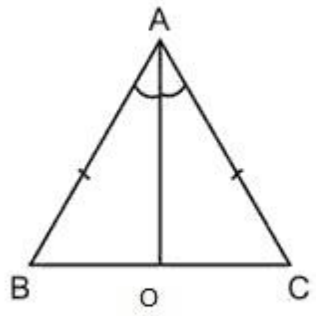
Subtracting (i) from (ii), we get,

$999x = 1 \Rightarrow x = \frac{1}{999}$

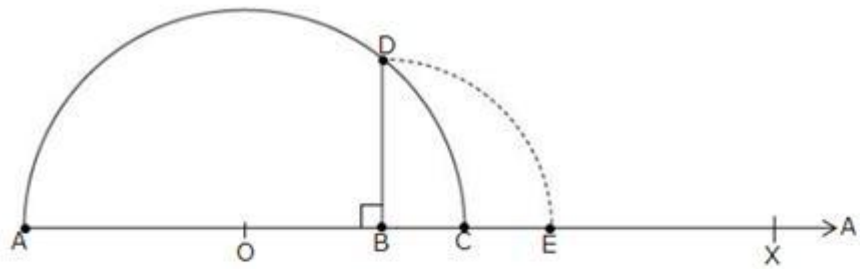
Hence, $0.\overline{001} = \frac{1}{999}$

14. Let ABC be an isosceles triangle with AB = AC.

Construction: Draw the bisector AO of ∠A.
In ΔABO and ΔACO, we have:
AB=AC (Given)
AO=OA (Common)
∠BAO=∠CAO (By Construction)
ΔABO ≅ ΔACO (By SAS congruence criteria)
∴ ∠B=∠C (By C.P.C.T)

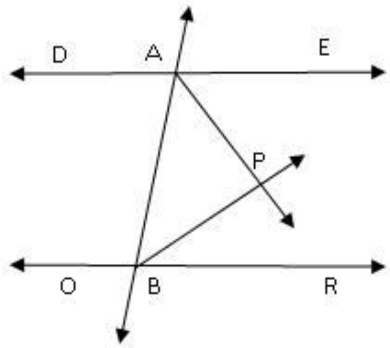


15. Steps of construction:
Draw a line segment AB = 3.2 units and extend it to C such that BC = 1 units.
Find the midpoint O of AC.
With O as centre and OA as radius, draw a semicircle.
Now, draw BD ⊥ AC, intersecting the semicircle at D.
Then, BD = $\sqrt{3.2}$ units.
With B as centre and BD as radius, draw an arc meeting AC produced at E.
Then, BE = BD = $\sqrt{3.2}$ units.



16. Let p(z) = $az^3 + 4z^2 + 3z - 4$ and q(z) = $z^3 - 4z + a$
When p(z) is divided by z-3, the remainder is given by:
p(3) = $a \times 3^3 + 4 \times 3^2 + 3 \times 3 - 4$
= 27a + 36 + 9 - 4
= 27a + 41 ……………….(i)
When q(z) is divided by z-3 the remainder is given by:
q(3)= $3^3 - 4 \times 3 + a$
= 27 – 12 + a
= 15 + a ……………..(ii)
Given that p(3) = q(3). So, from (i) and (ii), we have:
27a + 41 = 15 + a
27a - a = -41 + 15
26a = -26
a = -1

17.



Since interior angles on the same side of transversal are supplementary.

Therefore, $\angle EAB + \angle RBA = 180^\circ$

$\frac{1}{2}\angle EAB + \frac{1}{2}\angle RBA = \frac{1}{2} x 180^\circ$ ... (i)

As AP and BP are bisectors of $\angle EAB$ and $\angle RBA$ respectively

$\angle PAB = \frac{1}{2}\angle EAB$ and $\angle PBA = \frac{1}{2}\angle RBA$ ... (ii)

From (i) and (ii), we get,

$\angle PAB + \angle PBA = 90^\circ$

In ΔAPB we have,

$\angle PAB + \angle PBA + \angle APB = 180^\circ$

$90^\circ + \angle APB = 180^\circ$

$\angle APB = 180^\circ - 90^\circ = 90^\circ$

18. Let the length of the smallest side = x

Then, the other two sides are x+4 and 2x-6.

Perimeter of the triangle = x+x+4+2x-6=50

$4x-2 = 50 \Rightarrow 4x=52 \Rightarrow x=13$

Thus, the sides of the triangle are 13, 13 + 4 and $2 \times 13 - 6$, i.e. 13 cm, 17 cm and 20 cm, respectively.

Let a = 13, b = 17 and c = 20

$s = \frac{a+b+c}{2} = \frac{13+17+20}{2} = 25$

Area of the triangle

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{25(25-13)(25-17)(25-20)}$

$= \sqrt{25 \times 12 \times 8 \times 5}$

$= 20\sqrt{3}$

$= 20 \times 5.48 = 109.6 \text{ cm}^2$

19. Given, $x=1+\sqrt{2}$

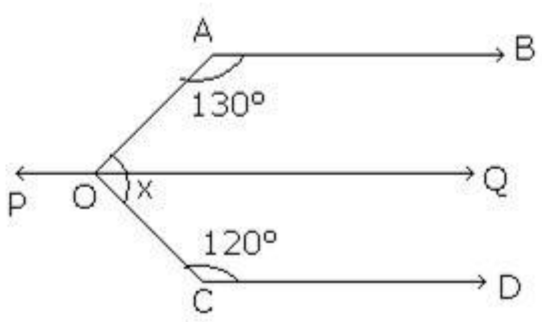
And, $\frac{1}{x}=\frac{1}{1+\sqrt{2}}$

$$\frac{1}{x} = \frac{1}{1+\sqrt{2}} \times \frac{1-\sqrt{2}}{1-\sqrt{2}} = \frac{1-\sqrt{2}}{1^2-\left(\sqrt{2}\right)^2} = -1+\sqrt{2}$$

Now, $\left(x-\frac{1}{x}\right) = \sqrt{2}+1-(\sqrt{2}-1) = 2$

$$\left(x-\frac{1}{x}\right)^3 = 2^3 = 8$$

20.



Through O, draw a line POQ parallel to AB.
Now PQ ||AB and CD||AB
So, CD||PQ
Now, AB||PQ and AO is a transversal, so, we have:
$\angle AOQ + \angle OAB = 180°$
(Co-interior angles are supplementary)
$\angle AOQ + 130° = 180°$
$\therefore \angle AOQ = 50°$
Similarly, PQ||CD and OC is a transversal.
$\therefore \angle QOC + \angle DCO = 180°$
$\angle QOC + 120° = 180°$
$\therefore \angle QOC = 60°$
Therefore, $x = \angle AOC = \angle AOQ + \angle QOC = 50° + 60° = 110°$

Section D

21. Let $p(x) = 2x^4 + x^3 - 14x^2 - 19x - 6$ and $q(x) = x^2 + 3x + 2$

$q(x) = x^2 + 3x + 2 = (x + 1)(x + 2)$

Now, $p(-1) = 2(-1)^4 + (-1)^3 - 14(-1)^2 - 19(-1) - 6$

$= 2 - 1 - 14 + 19 - 6 = 21 - 21 = 0$

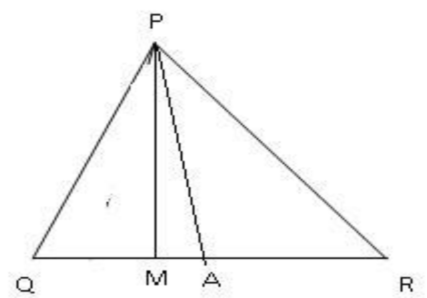
And, $p(-2) = 2(-2)^4 + (-2)^3 - 14(-2)^2 - 19(-2) - 6$

$= 32 - 8 - 56 + 38 - 6 = 70 - 70 = 0$

Therefore, (x+1) and (x+2) are the factors of p(x), so p(x) is divisible by (x+1) and (x+2).

Hence, p(x) is divisible by $x^2 + 3x + 2$.

22.



Since, PA is the bisector of $\angle QPR$,

$\angle QPA = \angle RPA$ ………… (i)

In ΔPQM, we have:

$\angle PQM + \angle PMQ + \angle QPM = 180°$

$\angle PQM + 90° + \angle QPM = 180°$

$\angle PQM = 90° - \angle QPM$ ………. (ii)

In ΔPMR, we have:

$\angle PMR + \angle PRM + \angle RPM = 180°$

$90° + \angle PRM + \angle RPM = 180°$

$\angle PRM = 90° - \angle RPM$ …………. (iii)

Subtracting (iii) from (ii), we get

$\angle Q - \angle R = (90° - \angle QPM) - (90° - \angle RPM)$

$\angle Q - \angle R = \angle RPM - \angle QPM$

$\angle Q - \angle R = (\angle RPA + \angle APM) - (\angle QPA - \angle APM)$

$\angle Q - \angle R = \angle RPA + \angle APM - \angle QPA + \angle APM$

$\angle Q - \angle R = 2\angle APM$ ($\because$ using (i))

Hence, $\angle APM = \frac{1}{2}(\angle Q - \angle R)$

23.

$$\frac{1}{3-\sqrt{8}}=\frac{1}{3-\sqrt{8}}\times\frac{3+\sqrt{8}}{3+\sqrt{8}}=\frac{3+\sqrt{8}}{9-8}=3+\sqrt{8}$$

$$\frac{1}{\sqrt{8}-\sqrt{7}}=\frac{1}{\sqrt{8}-\sqrt{7}}\times\frac{\sqrt{8}+\sqrt{7}}{\sqrt{8}+\sqrt{7}}=\frac{\sqrt{8}+\sqrt{7}}{8-7}=\sqrt{8}+\sqrt{7}$$

$$\frac{1}{\sqrt{7}-\sqrt{6}}=\frac{1}{\sqrt{7}-\sqrt{6}}\times\frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}}=\frac{\sqrt{7}+\sqrt{6}}{7-6}=\sqrt{7}+\sqrt{6}$$

$$\frac{1}{\sqrt{6}-\sqrt{5}}=\frac{1}{\sqrt{6}-\sqrt{5}}\times\frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}}=\frac{\sqrt{6}+\sqrt{5}}{6-5}=\sqrt{6}+\sqrt{5}$$

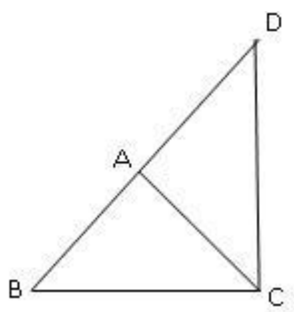
$$\frac{1}{\sqrt{5}-2}=\frac{1}{\sqrt{5}-2}\times\frac{\sqrt{5}\times 2}{\sqrt{5}+2}=\frac{\sqrt{5}+2}{5-4}=\sqrt{5}+2$$

$$\frac{1}{3-\sqrt{8}}-\frac{1}{\sqrt{8}-\sqrt{7}}+\frac{1}{\sqrt{7}-\sqrt{6}}-\frac{1}{\sqrt{6}-\sqrt{5}}+\frac{1}{\sqrt{5}-2}$$

$$=3+\sqrt{8}-\left(\sqrt{8}+\sqrt{7}\right)+\left(\sqrt{7}+\sqrt{6}\right)-\left(\sqrt{6}+\sqrt{5}\right)+\left(\sqrt{5}+2\right)$$

$$=5$$

24.



AB=AC $\Rightarrow$ $\angle$ABC=$\angle$ACB

AC=AD (Since, AB = AC and AB = AD)

$\Rightarrow$ $\angle$ADC = $\angle$ACD

Therefore, $\angle$ABC + $\angle$ADC = $\angle$ACB + $\angle$ACD = $\angle$BCD

$\angle$DBC + $\angle$BDC = $\angle$BCD

$\angle$DBC + $\angle$BDC + $\angle$BCD = 2$\angle$BCD

2$\angle$BCD = 180° (Angle sum property of triangles)

$\angle$BCD =90°

25. Let $p(x) = x^3 - 6x^2 + 11x - 6$

Factor of – 6 are $\pm 1, \pm 2, \pm 3, \pm 6$

Putting x = 1 in p(x), we have:

$P(1) = 1 - 6(1) + 11(1) - 6$

$= 0$

$\therefore (x - 1)$ is factor of p(x)

By long division, we have :

$$\begin{array}{r} x^2 - 5x + 6 \\ x-1 \overline{\smash{)}\, x^3 - 6x^2 + 11x - 6} \\ \underline{x^3 - x^2 \qquad\qquad} \\ -5x^2 + 11x - 6 \\ \underline{-5x^2 + 5x \qquad} \\ 6x - 6 \\ \underline{6x - 6} \\ 0 \end{array}$$

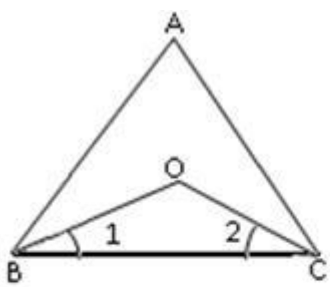
Thus, $f(x) = (x - 1)(x^2 - 5x + 6)$

$\Rightarrow f(x) = (x - 1)(x^2 - 3x - 2x + 6)$

$\Rightarrow f(x) = (x - 1)(x - 2)(x - 3)$

Thus, their capital shares are (x - 1), (x - 2) and (x - 3). They are not the same. The value depicted by this question is that friends in a locality can come together and solve the problem of unemployment by starting business even with small amount as their capitals put together. It also indicates need of sharing responsibilities jointly.

26. In Δ ABC we have,



$\angle A + \angle B + \angle C = 180°$ (sum of the angles of a Δ is 180°)

$\frac{1}{2}\angle A + \frac{1}{2}\angle B + \frac{1}{2}\angle C =$ x 180°

$\frac{1}{2}(\angle A + \angle 1 + \angle 2) = 90°$

$\angle 1 + \angle 2 = 90° - \frac{1}{2}\angle A$ ....(i)

In Δ OBC, we have,

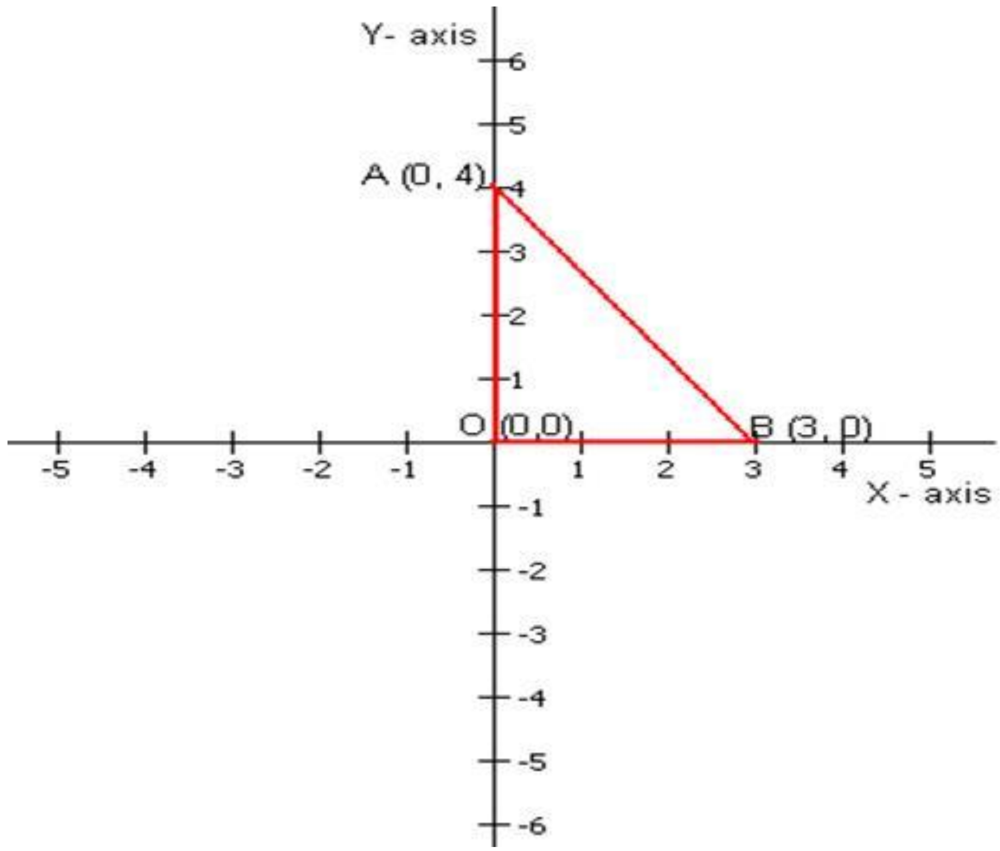
$\angle 1 + \angle 2 + \angle BOC = 180°$ (sum of the angles of a Δ is 180°)

$\angle BOC = 180° - (\angle 1 + \angle 2)$

$\angle BOC = 180° - (90° - \frac{1}{2}\angle A)$ (using (i))

$\angle BOC = 90° + \frac{1}{2}\angle A$

27. The given points A(0,4),O(0,0),B(3,0) can be plotted as follows:



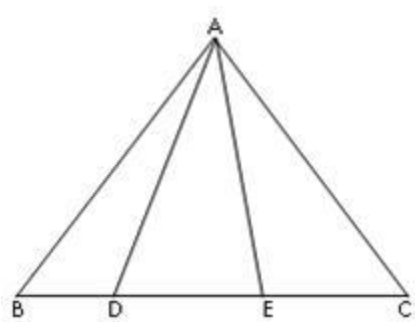
Clearly, AOB is a right-angled triangle.
OA = 4 units, OB = 3 units.

Area of $\Delta AOB = \frac{1}{2} \times \text{Base} \times \text{Height}$

$= \frac{1}{2} \times 3 \times 4 = 6$ square units

28.

Given that, AD=AE
Therefore, $\angle ADE = \angle AED$ (angles opposite to equal sides of a triangle are equal)
So, $\angle ADB = \angle AEC$ (remaining angles of linear pair)
In ΔADB and ΔAEC,
AD=AE (given)
$\angle ADB = \angle AEC$ (proved above)
BD=CE (given)
Thus, ΔADB and Δ AEC are congruent.
(By SAS congruence criterion)

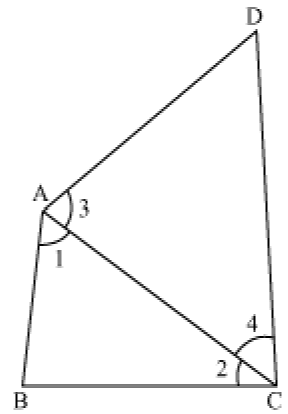


$\therefore \angle ABC = \angle ACB$ (Corresponding parts of congruent triangles)

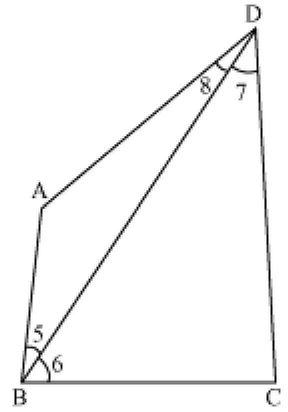
29.

$$x = \frac{1}{3-2\sqrt{2}}$$
$$= \frac{1}{3-2\sqrt{2}} \times \frac{3+2\sqrt{2}}{3+2\sqrt{2}}$$
$$= \frac{3+2\sqrt{2}}{9-8} = 3+2\sqrt{2}$$
$$\text{and} \quad y = \frac{1}{3+2\sqrt{2}}.$$
$$= \frac{1}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}}$$
$$= \frac{3-2\sqrt{2}}{9-8} = 3-2\sqrt{2}$$
$$\Rightarrow x+y = 3+2\sqrt{2}+3-2\sqrt{2} = 6$$
$$xy = \left(3+2\sqrt{2}\right)\left(3-2\sqrt{2}\right) = 9-8 = 1$$
$$x^3+y^3 = (x+y)^3 - 3xy(x+y)$$
$$= 6^3 - 3.1.6 = 216-18 = 198$$

30.



Join AC.
In ΔABC:
AB < BC (AB is smallest side of quadrilateral ABCD)
$\therefore \angle 2 < \angle 1$ (angle opposite to smaller side is smaller) ... (1)
In ΔADC:
AD < CD (CD is the largest side of quadrilateral ABCD)
$\therefore \angle 4 < \angle 3$ (angle opposite to smaller side is smaller) ... (2)
On adding equations (1) and (2), we have:
$\angle 2 + \angle 4 < \angle 1 + \angle 3$
$\Rightarrow \angle C < \angle A$
$\Rightarrow \angle A > \angle C$
Now, join BD.



In ΔABD,
AB < AD (AB is smallest side of quadrilateral ABCD)
∴ ∠8 < ∠5 (angle opposite to smaller side is smaller) ... (3)
In ΔBDC,
BC < CD (CD is the largest side of quadrilateral ABCD)
∴ ∠7 < ∠6 (angle opposite to smaller side is smaller) ... (4)
On adding equations (3) and (4), we have:
∠8 + ∠7 < ∠5 + ∠6
⇒ ∠D < ∠B
⇒ ∠B > ∠D

31.

(a) $x^4 + \frac{1}{x^4} - 2$

$= (x^2)^2 + \frac{1}{(x^2)^2} - 2 \times x^2 \times \frac{1}{x^2}$

$= \left(x^2 - \frac{1}{x^2}\right)^2$

$= \left(x^2 - \frac{1}{x^2}\right)\left(x^2 - \frac{1}{x^2}\right)$

(b) $2x^5 + 432x^2y^3$
$=2x^5 + 432x^2y^3$
$=2x^2(x^3 + 216y^3)$
$= 2x^2[(x)^3 + (6y)^3]$
$= 2x^2(x + 6y)(x^2 + 36y^2 - 6xy)$