ICSE Board
Class X Mathematics
Sample Paper – 1 Solution

SECTION – A (40 Marks)

Q. 1.

(a) Put $x - 2 = 0 \Rightarrow x = 2$ in $kx^2 - 3x + 6$

By Factor Theorem, the remainder obtained on dividing $kx^2 - 3x + 6$ by $x - 2$

$= k(2)^2 - 3(2) + 6$

Put $x + 3 = 0 \Rightarrow x = -3$ in $3x^2 + 5x - k$

Similarly, the remainder obtained by dividing $3x^2 + 5x - k$ by $x + 3$

$= 3(-3)^2 + 5(-3) - k$

Now, $k(2)^2 - 3(2) + 6 = 2[3(-3)^2 + 5(-3) - k]$

$4k - 6 + 6 = 2[27 - 15 - k]$

$\Rightarrow 4k = 2(12 - k) \Rightarrow 4k = 24 - 2k$

$\Rightarrow 4k + 2k = 24 \Rightarrow 6k = 24$

So, $k = 4$.

(b)

$$AB = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0+0 & 1+0 \\ 0-1 & 0+0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$BA = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0+0 & 0-1 \\ 1+0 & 0-0 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

Thus, $AB \neq BA$

(c) Let the sum of money lent out = Rs. x

Amount received after one year $= P\left(1+\frac{r}{200}\right)^{2n}$

$= x\left(1+\frac{8}{200}\right)^{2\times1}$

$= x\left(1+\frac{1}{25}\right)^{2}$

$= x\left(\frac{26}{25}\right)^{2}$

$= \frac{676}{625}x$

$\therefore$ Compound interest (C.I.) $= \frac{676}{625}x - x = \frac{676x - 625x}{625} = \text{Rs.}\frac{51x}{625}$

And, Simple interest (S.I.) $= \text{Rs.}\frac{x \times 8 \times 1}{100} = \text{Rs.}\frac{2x}{25}$

Given, difference between C.I. and S.I. = Rs. 16

$\Rightarrow \frac{51}{625}x - \frac{2}{25}x = 16 \Rightarrow \frac{51x - 50x}{625} = 16 \Rightarrow x = 16 \times 625 = 10{,}000$

$\therefore$ The sum lent out = Rs. 10,000

Q. 2.

(a) No. of cards removed = 3(face cards of clubs)

Remaining no. of cards = 52 – 3 = 49

i. Probability of getting a club

Remaining no. of cards = 13 – 3 = 10

$\therefore$ P(a club) $= \frac{10}{49}$

ii. a red face card

No. of red face cards = 6 (i.e. 3 of diamonds & 3 of hearts)

P(a red face card) $= \frac{6}{49}$

(b) $2x - \frac{3}{x} = 5 \Rightarrow 2x^2 - 5x - 3 = 0$

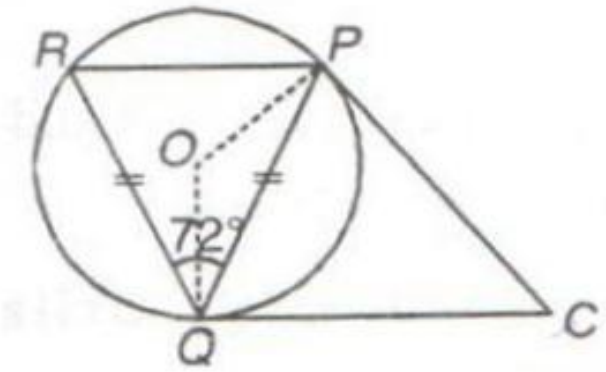
$\Rightarrow 2x + 1 = 0$ or $x - 3 = 0$

$\Rightarrow 2x^2 - 6x + x - 3 = 0$

$\Rightarrow 2x(x - 6) + 1(x - 3) = 0 \Rightarrow x = \frac{-1}{2}$ or $x = 3$

(c)

i. Given: QR = QP



$\Rightarrow \angle QPR = \angle QRP$

$m\angle QPR + m\angle RPQ + m\angle PQR = 180°$

$2m\angle QPR + 72° = 180°$

$2m\angle QPR = 108°$

$m\angle QPR = 54°$

$\therefore m\angle QRP = m\angle RPQ = 54°$

Now, $m\angle POQ = 2m\angle QRP = 108°$

For angles in alternate segment

$\therefore m\angle CQP = m\angle QRP = 54°$

And $m\angle CPQ = m\angle QPC = 54°$

ii. Now, $m\angle CQP + m\angle CPQ + m\angle PCQ = 180°$

$54° + 54° + m\angle PCQ = 180°$

$108° + m\angle PCQ = 180°$

$\Rightarrow m\angle PCQ = 180° - 108° = 72°$

Q. 3.

(a) Total amount deposited by Manu in 5 years = Rs. 240 × 60 = Rs. 14,400

Equivalent principal for 1 month = $\text{Rs.}240 \times \frac{60(60+1)}{2} = \text{Rs.}4,39,200$

Let the rate of interest be r%

Interest on Rs. 4,39,200 for 1 month $= \text{Rs.}439200 \times \frac{1}{12} \times \frac{r}{100} = \text{Rs.}366r$

Maturity amount = 17694

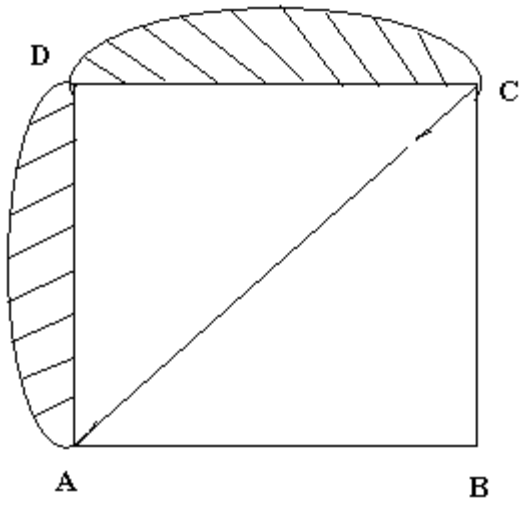
Or 14400 + 366r = 17694

$\Rightarrow$ r = 9%

(b)

Length of diagonal = $AC = \sqrt{2}a = 58\sqrt{2}$ m

$r = \frac{58\sqrt{2}}{2} = 29\sqrt{2}$ m



Area of whole lawn = area of ΔABC + Area of semi-circle on AC as diameter.

$= \frac{1}{2} \times 58 \times 58 + \frac{\pi(29\sqrt{2})^2}{2}$

$= 29 \times 58 + \frac{22 \times 841 \times 2}{7 \times 2}$

$= 4325.14 \text{ m}^2$

(c)

i. Co-ordinates of centroid, $x = \frac{x_1 + x_2 + x_3}{3} = \frac{1+3+7}{3} = \frac{11}{3}$

And, $y = \frac{y_1 + y_2 + y_3}{3} = \frac{4+2+5}{3} = \frac{11}{3}$

So, $G = \left(\frac{11}{3}, \frac{11}{3}\right)$

ii. The equation of a line, through G and parallel to AB.

Slope of line $AB = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-4}{3-1} = -\frac{2}{2} = -1$

Now, equation of line through G and parallel to AB,

$\Rightarrow y - y_1 = m(x - x_1)$

$\Rightarrow y - \frac{11}{3} = -1\left(x - \frac{11}{3}\right)$

$\Rightarrow 3y - 11 = -3x + 11$

$\Rightarrow 3x + 3y - 22 = 0$

Q. 4.

(a) $-2\frac{2}{3} \le x + \frac{1}{3} < 3\frac{1}{3}; x \in R.$

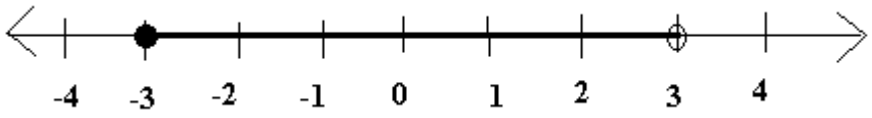
$\Rightarrow \frac{-8}{3} \le x + \frac{1}{3} < \frac{10}{3}$ [by subtracting $\frac{1}{3}$, we get]

$\Rightarrow \frac{-8}{3} - \frac{1}{3} \le x + \frac{1}{3} - \frac{1}{3} < \frac{10}{3} - \frac{1}{3}$

$\Rightarrow \frac{-9}{3} \le x < \frac{9}{3}$

$\Rightarrow -3 \le x < 3$

Solution is $\{x : x \in R, -3 \le x < 3\}$



(b) $\frac{\sin(90^\circ-70^\circ)}{\sin 20^\circ}+\frac{\sin(90^\circ-59^\circ)}{\sin 31^\circ}-8\left(\frac{1}{2}\right)^2$

Since, $\cos\theta = \sin(90^\circ - \theta)$

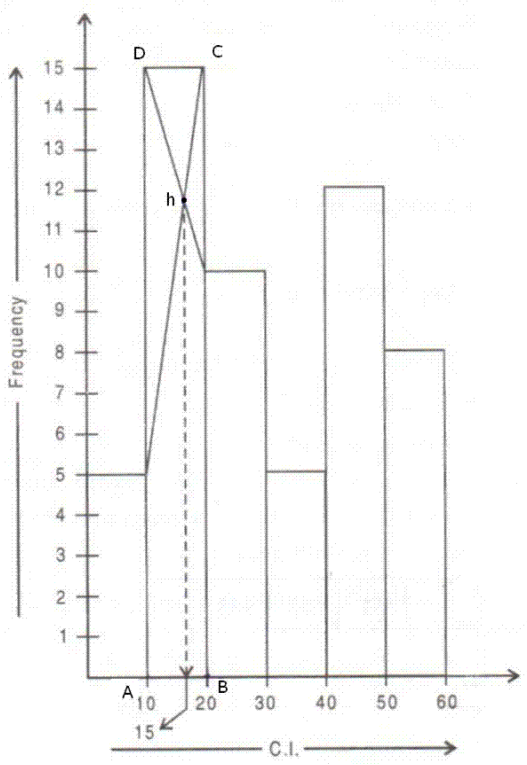
$=\frac{\sin 20^\circ}{\sin 20^\circ}+\frac{\sin 31^\circ}{\sin 31^\circ}-8\times\frac{1}{4}=1+1-2=0$

(c) To locate the mode from the histogram, we proceed as follows:

i Find the modal class. Rectangle ABCD is the largest rectangle. It represents the modal class, that is, the mode lies in this rectangle. The modal class is 10–20.

ii Draw two lines diagonally from the vertices C and D to the upper corners of the two adjacent rectangles. Let these rectangles intersect at point H.

iii The x-value of the point H is the mode. Thus, mode of the given data is approximately 15.



So, mode = 15

SECTION – B (40 Marks)

Q. 5.

(a) M.P. = Rs. x, Discount = 20% of x = $\text{Rs.}\frac{x}{5}$

New M.P. after discount = $\text{Rs.}\, x - \frac{x}{5} = \text{Rs.}\frac{4x}{5}$

Again discount = 5% of $\frac{4x}{5} = \frac{1}{20}\left(\frac{4x}{5}\right) = \text{Rs.}\frac{x}{25}$

Changed M.P. after 2nd discount = $\frac{4x}{5} - \frac{x}{25} = \text{Rs.}\frac{19x}{25}$

Now, $2298.24 = \frac{19x}{25} + 8\%\text{ of }\frac{19x}{25}$

$2298.24 = \frac{19x}{25} + \frac{8}{100} \times \frac{19x}{25}$

$\Rightarrow 2298.24 = \frac{19x}{25} + \frac{38x}{625}$

$\Rightarrow 2298.24 = \frac{475x + 38x}{625}$

$\Rightarrow 2298.24 = \frac{513x}{625}$

$\Rightarrow \frac{2298.24 \times 625}{513} = x$

$\Rightarrow x = \text{Rs.}\,2800$

$\therefore$ M.P. = Rs. 2800

(b)

Length of the wire used in one round = 3 mm = 0.3 cm

Number of rounds required to cover 12 cm of length = $\frac{12}{0.3} = 40$

Diameter of the cylinder = 10 cm $\Rightarrow$ Radius of the cylinder = 5 cm

Length of wire required for one round = $2\pi r = 2\pi \times 5$ cm $= 10\pi$ cm

$\therefore$ Length of wire required for 40 rounds = $10\pi \times 40$

$= 400\pi$ cm $= 400 \times 3.14 = 1256$ cm

Now, radius of the wire = $\frac{3}{2}$ mm $= \frac{3}{20}$ cm

Volume of the wire = $\pi r^2 h = \pi\left(\frac{3}{20}\right)^2 \times 1256$ cm^3

Weight of the wire = Volume of the wire × 8.88 gm

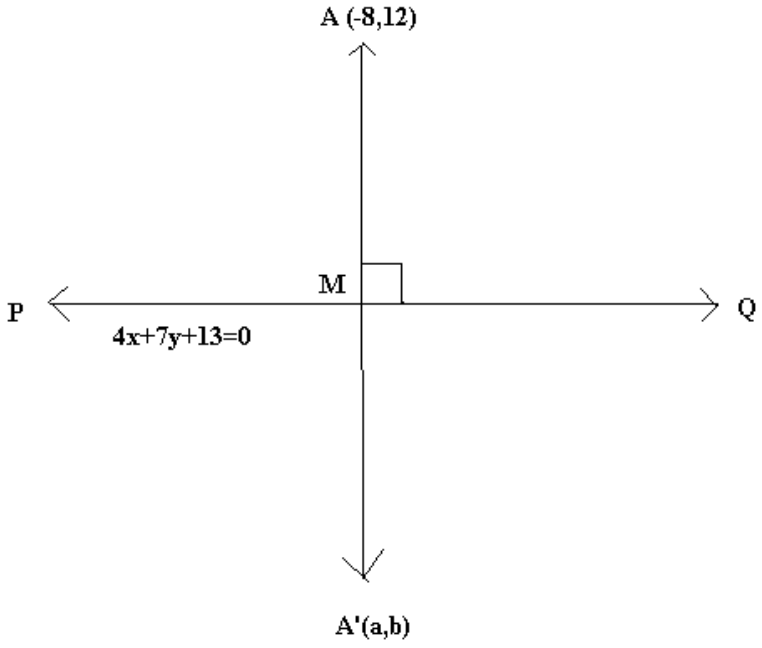
$= 3.14 \times \left(\frac{3}{20}\right)^2 \times 1256 \times 8.88 = 787.98$ gm

(c) $M = \left(\frac{a-8}{2}, \frac{b+12}{2}\right)$

It lies on $4x + 7y + 13 = 0$

$\therefore 4\left(\frac{a-8}{2}\right) + 7\left(\frac{b+12}{2}\right) + 13 = 0$

$\Rightarrow 4a + 7b + 78 = 0$



Since AA' $\perp$ PQ, so

Slope (PQ) × Slope (AA') = −1

$\frac{-4}{7} \times \frac{b-12}{a+8} = -1$

$\Rightarrow 7a - 4b + 104 = 0$

$4a + 7b = -78$ ....(1)

$7a - 4b = -104$ ....(2)

Multiplying equation (1) by 4 and equation (2) by 7 we get

$16a + 28b = -312$

$49a - 28b = -728$

$65a = -1040 \Rightarrow a = \frac{-1040}{65} = -16,$

Put a = −16 in equation, we get

$4a + 7b + 78 = 0$

$\Rightarrow 4 \times -16 + 7b + 78 = 0$

$\Rightarrow -64 + 7b + 78 = 0$

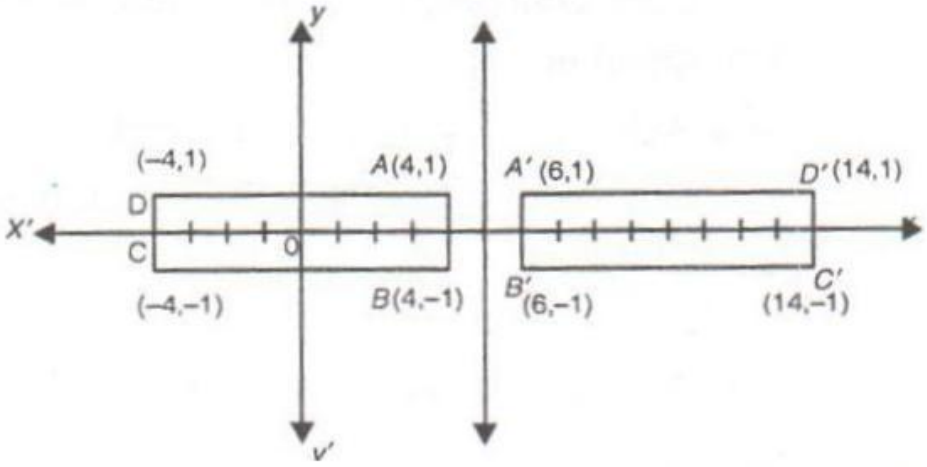
$\Rightarrow 7b + 14 = 0 \Rightarrow 7b = -14 \Rightarrow b = -2$

Image = (−16, −2)

Q. 6.

(a) First the coordinates A(4, 1), B(4, −1), D(−4, 1) and C(−4, −1) are plotted to form a rectangle.

The rectangle so formed and the reflected rectangle are shown in the figure:



It is reflected through x = 5, then the coordinates of the new rectangle become A'(6, −1), B'(6, 1), C'(14, −1) and D'(14, 1)

Perimeter of A'B'C'D' = A'B' + B'C' + C'D' + D'A'

= 2 + 8 + 2 + 8 = 20 units

Area = L × B = B'C' × C'D' = 8 × 2 = 16 sq.units

(b) $\frac{x^4+1}{2x^2}=\frac{17}{8}$

Using componendo and dividend,

$\Rightarrow \frac{x^4+2x^2+1}{x^4-2x^2+1}=\frac{17+8}{17-8}$

$\Rightarrow \frac{(x^2+1)^2}{(x^2-1)^2}=\frac{25}{9}$

$\Rightarrow \frac{(x^2+1)}{(x^2-1)}=\frac{5}{3}$

Using componendo and dividendo again,

$\Rightarrow \frac{x^2+1+x^2-1}{x^2+1-x^2+1}=\frac{5+3}{5-3}$

$\Rightarrow \frac{2x^2}{2}=\frac{8}{2}$

$\Rightarrow x^2=4$

$\Rightarrow x=\pm 2$

(c)

Min. balance for the month of June = 8680.00
Min. balance for the month of July = 6180.00
Min. balance for the month of Aug = 6180.00
Min. balance for the month of Sept = 6180.00
Min. balance for the month of Oct = 6180.00
Min. balance for the month of Nov = 6560.00
Min. balance for the month of Dec = 00.00

Total = 39, 960.00
P = Rs. 39,960
I = Rs. (5940.80 – 5741) = Rs. 199.80
R = ?, $T=\frac{1}{12}$ years

Use, $I=\frac{P\times R\times T}{100}$

$\Rightarrow 199.80=\frac{39{,}960\times R\times 1}{12\times 100}$

$\Rightarrow R=\frac{199.80\times 12\times 100}{39960}=6\%$

Q.7.

(a) Given, $\frac{\sqrt{a+x}+\sqrt{a-x}}{\sqrt{a+x}-\sqrt{a-x}}=\frac{b}{1}$

By using the componendo and dividendo, we get

$$\frac{2\sqrt{a+x}}{2\sqrt{a-x}}=\frac{b+1}{b-1}$$

Now squaring both sides, we get

$$\frac{a+x}{a-x}=\frac{b^2+1+2b}{b^2+1-2b},$$

Again use componendo and dividend, to get the value of x.

$$\frac{a+x+a-x}{a+x-a+x}=\frac{b^2+1+2b+b^2+1-2b}{b^2+1+2b-b^2-1+2b}$$

$$\Rightarrow \frac{2a}{2x}=\frac{2b^2+2}{4b}$$

$$\Rightarrow \frac{a}{x}=\frac{b^2+1}{2b}$$

$$\Rightarrow x=\frac{2ab}{b^2+1}$$

(b)

$$A^2=A.A=\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}=\begin{bmatrix}7 & 12\\ 4 & 7\end{bmatrix}$$

$$A^3=A^2.A=\begin{bmatrix}7 & 12\\ 4 & 7\end{bmatrix}\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}=\begin{bmatrix}26 & 45\\ 15 & 26\end{bmatrix}$$

Now, L.H.S. = $A^3 - 4A^2 + A$

$$\Rightarrow \begin{bmatrix}26 & 45\\ 15 & 26\end{bmatrix}-4\begin{bmatrix}7 & 12\\ 4 & 7\end{bmatrix}+\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}26 & 45\\ 15 & 26\end{bmatrix}-\begin{bmatrix}28 & 48\\ 16 & 28\end{bmatrix}+\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}=0=\text{R.H.S.}$$

(c) Here we need to find the mean by the mean deviation method.

x_i	f_i	$u_i = \frac{x_i - A}{h}, h = 5$	$f_i u_i$
15	5	−4	−20
20	8	−3	−24
25	11	−2	−22
30	20	−1	−20
A = 35	23	0	0
40	18	1	18
45	13	2	26
50	3	3	9
55	1	4	4
	$\sum f_i = 102$		$\sum f_i u_i = -29$

From the table, $A = 35, \sum f_i = 102, h = 5, \sum f_i u_i = -29$

So, Mean $\left(\bar{x}\right) = A + h\frac{\sum f_i u_i}{\sum f_i}$

$= 35 + 5 \times \frac{-29}{102}$

$= 35 - \frac{145}{102}$

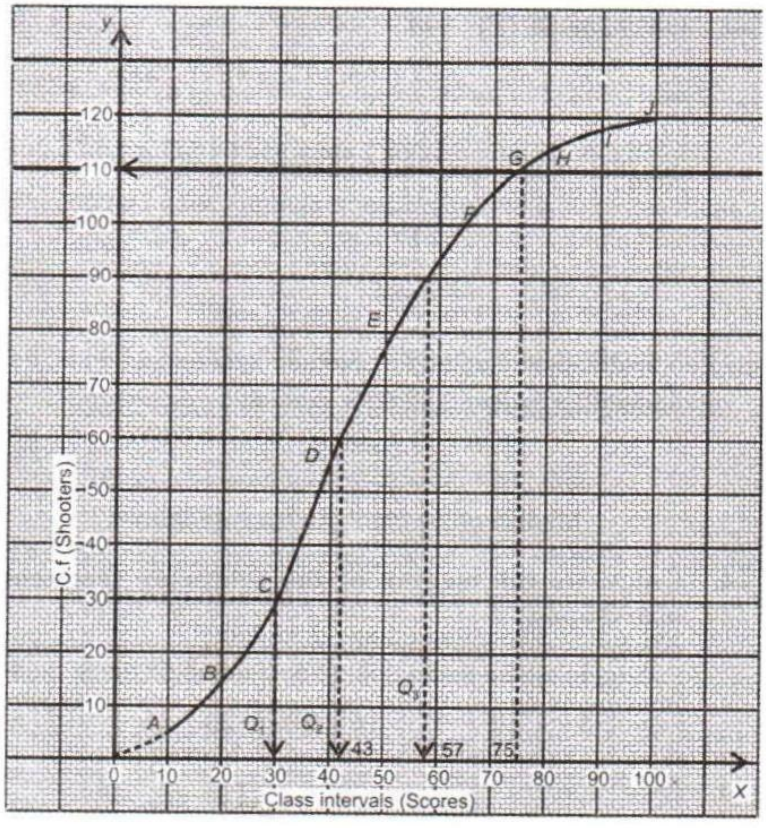
$= 35 - 1.42$

$= 33.58$

Q.8.

(a)

Scores Obtained	Number of Shooters	C.f.
0-10	5	5
10-20	9	14
20-30	16	30
30-40	22	52
40-50	26	78
50-60	18	96
60-70	11	107
70-80	6	113
80-90	4	117
90-100	3	120
	n=120	



i. n = 120 (even)

The position of the median is given by $\frac{n}{2} = \frac{120}{2} = 60$

So, from the graph, median = 43 scores

ii. The positions of Q_1 is given by $\frac{n}{4} = \frac{120}{4} = 30$

So, from the graph, Q_1 = 30 scores

The position of Q_3 is given by $\frac{3n}{4} = \frac{3 \times 120}{4} = 90$

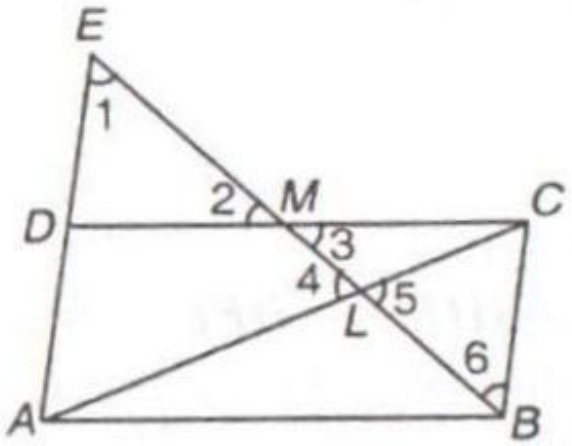
So, from the graph, Q_3 = 57 scores

Now, Inter-quartile range = $Q_3 - Q_1 = 57 - 30$

= 27 scores

iii. number of shooters who obtained more than 75% scores = 120 – 110 = 10

(b) Construction: Draw the fig.



$\angle 1 = \angle 6$ [Alternate interior $\angle$'s]

$\angle 2 = \angle 3$ [Vertically opposite $\angle$'s]

DM = MC

$\therefore$ $\Delta DEM \cong \Delta BMC$ [AAS congruency]

$\Rightarrow$ DE = BC [by c.p.c.t]

Also AD = BC [opposite sides of a || gm]

$\Rightarrow$ AE = AD + DE = 2BC

Now, $\angle 1 = \angle 6$ and $\angle 4 = \angle 5$ $\therefore$ $\Delta ELA \sim \Delta BLC$

$\Rightarrow \frac{EL}{BL} = \frac{AE}{BC} \Rightarrow \frac{EL}{BL} = \frac{2BC}{BC} \Rightarrow EL = 2BL$

Q. 9.

(a) Market value of 1 share = Rs. $\left(\frac{26 \times 110}{100}\right)$ = Rs.28.80

(i) Number of shares bought = $\frac{20,020}{28.60} = \frac{20,02,000}{2860} = 700$

(ii) Dividend on one share = Rs. $\frac{26 \times 15}{100}$ = Rs. $\frac{39}{10}$

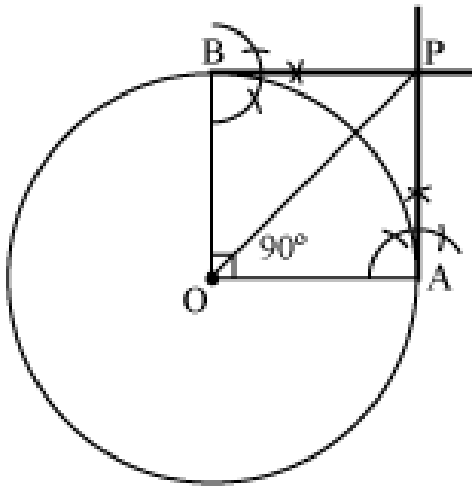
$\therefore$ Dividend on 700 shares = $700 \times \frac{39}{10}$ = Rs.2730

(iii) Rate of interest = $\frac{2730}{20020} \times 100 = 13.63\%$

(b) The steps of construction are as follows:

i. Draw a circle of any convenient radius with centre O.
ii. (Take a point A on the circumference of the circle and join OA. Draw a perpendicular to OA at point A.
iii. Draw radius OB, making an angle of 90° with OA.
iv. Draw a perpendicular to OB at point B. Let both the perpendiculars intersect at point P.
v. Join OP.

PA and PB are the required tangents, which make an angle of 45° with OP.



(c)

L.H.S.

$$= \frac{\tan A}{1 - \frac{1}{\tan A}} + \frac{1}{\tan A(1 - \tan A)}$$

$$= \frac{\tan^2 A}{(\tan A - 1)} - \frac{1}{\tan A(\tan A - 1)}$$

$$= \frac{\tan^3 A - 1}{\tan A(\tan A - 1)}$$

Use $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$$= \frac{(\tan A - 1)(\tan^2 A + \tan A + 1)}{\tan A(\tan A - 1)}$$

$$= \tan A + 1 + \cot A$$

$$= 1 + \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}$$

$$= 1 + \frac{\sin^2 A + \cos^2 A}{\sin A \cos A} = 1 + \frac{1}{\sin A \cos A}$$

$$= 1 + \operatorname{cosec} A \sec A$$

$$= \text{R.H.S}$$

Hence Proved

Q.10.

(a)

Let the numbers be x and y;

So $xy = 196 \Rightarrow x = \frac{196}{y}$ and $112x = y^2$

Put $x = \frac{196}{y}$ in $112x = y^2$, we get

$$\Rightarrow \frac{112 \times 196}{y} = y^2$$

$$\Rightarrow y = \sqrt{112 \times 196} \text{ Þ } y = 28$$

And $x = \frac{196}{28} = 7$

So, numbers are = 71,428.

(b) Let principal = Rs. P then amount = Rs. $\frac{125}{64}$P

Now, $A = P\left(1 + \frac{R}{100}\right)^n$

$\Rightarrow \frac{125}{64}P = P\left(1 + \frac{R}{100}\right)^3$

$\Rightarrow \frac{125}{64} = \left(1 + \frac{R}{100}\right)^3$

$\Rightarrow \left(\frac{125}{64}\right) = \left(1 + \frac{R}{100}\right)^3$

$\Rightarrow \left(\frac{5}{4}\right)^3 = \left(1 + \frac{R}{100}\right)^3$

Taking cube of both the sides

$\frac{5}{4} = 1 + \frac{R}{100}$ or $\frac{5}{4} = \frac{100 + R}{100}$

Or $500 = 400 + 4R$

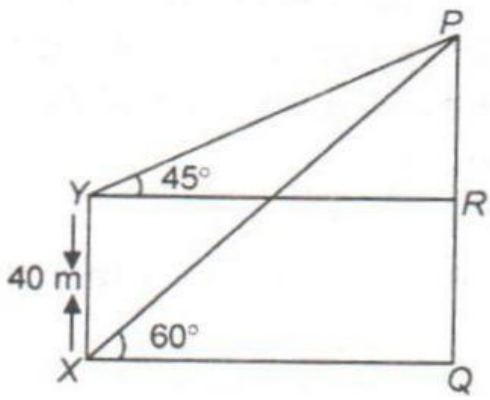
Or $4R = 100 \Rightarrow R = 25$

$\therefore$ Rate = 25%

(c)

Let XQ = x m and PR = h m then YR = XQ = x m

And YX = RQ = 40 m



In ΔPRY, $\frac{h}{x} = \tan 45° \Rightarrow h = x$

In ΔPQX, $\frac{h+40}{x} = \tan 60° \Rightarrow h + 40 = \sqrt{3}x$

$\Rightarrow h + 40 = \sqrt{3}h$ [$\because h = x$]

$40 = \sqrt{3}h - h$

$40 = h(\sqrt{3} - 1)$

Or $h = \frac{40}{\sqrt{3} - 1}$

$= \frac{40}{(\sqrt{3} - 1)} \times \frac{(\sqrt{3} + 1)}{(\sqrt{3} + 1)}$

$= \frac{40(1.73 + 1)}{2}$

$= 20 \times 2.730$

$= 2 \times 27.3$

$= 54.6$

i. Height PQ = h + 40 = 54.6 + 40 = 94.6 m

ii. The distance XQ = x = h = 54.6 m

Q.11.

(a)

i. AM = MP [tangents from an external point]

$\Rightarrow \angle MAP = \angle MPA$

Similarly, MP = MB

$\therefore \angle MBP = \angle MPB$

Now in ΔABP,

$m\angle PAB + m\angle ABP + m\angle APB = 180°$

$m\angle MPA + m\angle MPB + m\angle MPA + m\angle MPB = 180°$

$(m\angle MPA + m\angle MPB) = 180°$

$\Rightarrow m\angle APB = 90°$

ii. AM = MP and MP = MB

$\Rightarrow$ AM = MB

$\therefore$ Tangent at P bisects AB.

(b) Total money for expenses = Rs. 360
Original duration of tour = x days
Final duration of tour = x + 4 days

Originally, 1 day expenses = $Rs.\frac{360}{x}$

Finally, 1 day expenses = $Rs.\frac{360}{x+4}$

According to the question,

$\frac{360}{x} = \frac{360}{x+4} + 3$

$\Rightarrow 360x + 1440 - 360x = 3x^2 + 12x$

$\Rightarrow 3x^2 + 12x - 1440 = 0$

$\Rightarrow x^2 + 4x - 480 = 0$

$\Rightarrow x^2 + 24x - 20x - 480 = 0$

$\Rightarrow x(x + 24) - 20(x + 24) = 0$

$\Rightarrow (x - 20)(x + 24) = 0$

Thus, x = 20, −24

Days cannot be negative, So, x = 20.

(c) Let, $\angle ACX = \theta$ and $\angle ABX = \phi$

i. Given, equation of AB, $x - \sqrt{3}y + 1 = 0$

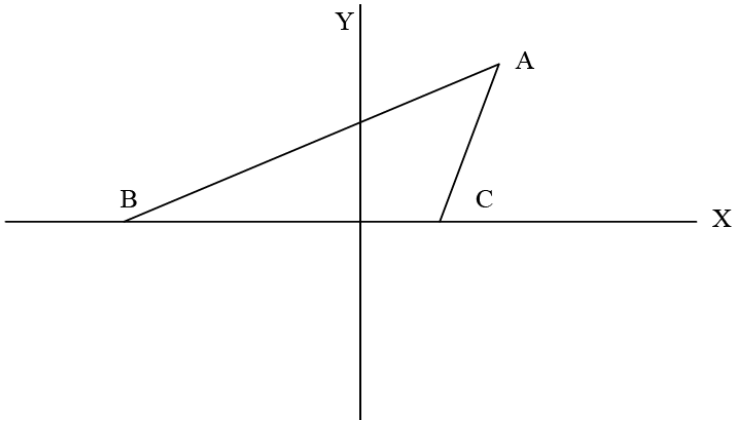
$\therefore$ Slope of AB $= \frac{-\text{coefficient of } x}{\text{coefficient of } y}$

$= \frac{-1}{-\sqrt{3}} = \frac{1}{\sqrt{3}}$

So, $(m_1) = \tan\theta = \frac{1}{\sqrt{3}}$

$\Rightarrow \theta = 30°$

$\therefore$ Angle made by the line AB = 30°



Now, equation of AC, x – y – 2 = 0

$\therefore$ Slope of AC = $= \frac{-\text{coefficient of } x}{\text{coefficient of } y} = \frac{-1}{-1} = 1$

So, $(m_2) = \tan\phi = 1$

$\Rightarrow \phi = 45°$

Angle made by the line AC = 45°

ii. $m\angle BAC = m\angle C - \angle B = 45° - 30° = 15°$