

**ICSE Mathematics**  
**Class X**  
**Sample Paper 1**

Time: 2½ hour

Total Marks: 80

**Solutions**

**SECTION – A (40 Marks)**

**Q. 1**

**Solution**

(a)

Put  $x - 2 = 0 \Rightarrow x = 2$  in  $kx^2 - 3x + 6$  [1]

So by Factor Theorem, the remainder obtained by dividing  $kx^2 - 3x + 6$  by  $x - 2 = k(2)^2 - 3(2) + 6$

Put  $x + 3 = 0 \Rightarrow x = -3$  in  $3x^2 + 5x - k$  [1]

And similarly, the remainder obtained by dividing  $3x^2 + 5x - k$  by  $x + 3 = 3(-3)^2 + 5(-3) - k$

Now,  $k(2)^2 - 3(2) + 6 = 2[3(-3)^2 + 5(-3) - k]$

$4k - 6 + 6 = 2[27 - 15 - k]$

$\Rightarrow 4k = 2(12 - k) \Rightarrow 4k = 24 - 2k$

$\Rightarrow 4k + 2k = 24 \Rightarrow 6k = 24$

So,  $k = 4$ . [1]

(b)

$$AB = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0+0 & 1+0 \\ 0-1 & 0+0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \quad [1]$$

$$BA = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0+0 & 0-1 \\ 1+0 & 0-0 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \quad [1]$$

Thus,  $AB \neq BA$ , proved. [1]

(c)

Let the sum of money lent out = Rs.  $x$

$$\text{Amount received after one year} = \left(1 + \frac{r}{200}\right)^{2n} \quad [1]$$

$$= x \left(1 + \frac{8}{200}\right)^{2 \times 1}$$

$$= x \left(1 + \frac{1}{25}\right)^2$$

$$= x \left(\frac{26}{25}\right)^2$$

$$= \frac{676}{625} x \quad [1]$$

$$\therefore \text{Compound interest (C.I.)} = \frac{676}{625} x - x = \frac{676x - 625x}{625} = \text{Rs. } \frac{51x}{625}$$

$$\text{And, Simple interest (S.I.)} = \text{Rs. } \frac{x \times 8 \times 1}{100} = \text{Rs. } \frac{2x}{25}$$

Given, difference between C.I. and S.I. = Rs. 16 [1]

$$\Rightarrow \frac{51}{625} x - \frac{2}{25} x = 16 \Rightarrow \frac{51x - 50x}{625} = 16 \Rightarrow x = 16 \times 625 = 10,000 \quad [1]$$

$\therefore$  The sum lent out = Rs. 10,000

## Q. 2

### Solution

(a)

No. of cards removed = 3 (face cards of clubs)

Remaining no. of cards =  $52 - 3 = 49$  [1]

(i) a club

Remaining no. of cards =  $13 - 3 = 10$  [1]

$$\therefore P(\text{a club}) = \frac{10}{49}$$

(ii) a red face card

$$P(\text{a red face card}) = \frac{6}{49} \quad [1]$$

(b)

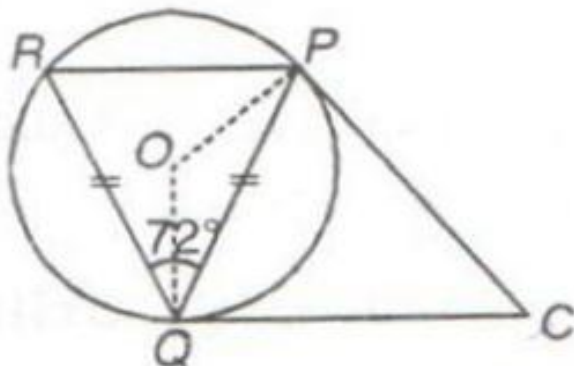
$$2x - \frac{3}{x} = 5 \Rightarrow 2x^2 - 5x - 3 = 0 \quad [1]$$

$$\Rightarrow 2x + 1 = 0 \quad \text{or} \quad x - 3 = 0$$

$$\Rightarrow 2x^2 - 6x + x - 3 = 0$$

$$\Rightarrow 2x(x - 3) + 1(x - 3) = 0 \Rightarrow x = \frac{-1}{2} \quad \text{or} \quad x = 3 \quad [1]$$

(c)



(i) Given:  $QR = QP$

$$\Rightarrow \angle QPR = \angle QRP$$

$\left[ \frac{1}{2} \right]$

$$\angle QPR + \angle RPQ + \angle PQR = 180^\circ$$

$$2\angle QPR + 72^\circ = 180^\circ$$

$$2\angle QPR = 108^\circ$$

$$\angle QPR = 54^\circ$$

$\left[ \frac{1}{2} \right]$

$$\therefore \angle QRP = \angle RPQ = 54^\circ$$

$\left[ \frac{1}{2} \right]$

$$\angle POQ = 2\angle QRP = 108^\circ$$

Take angles in alternate segment

$\left[ \frac{1}{2} \right]$

$$\angle CQP = \angle QRP = 54^\circ$$

$$\text{And } \angle CPQ = \angle QPC = 54^\circ$$

$\left[ \frac{1}{2} \right]$

(ii) Now,  $\angle CQP + \angle CPQ + \angle PCQ = 180^\circ$

$\left[ \frac{1}{2} \right]$

$$54^\circ + 54^\circ + \angle PCQ = 180^\circ$$

$[1]$

$$108^\circ + \angle PCQ = 180^\circ$$

$$\Rightarrow \angle PCQ = 180^\circ - 108^\circ = 72^\circ$$

$\left[ \frac{1}{2} \right]$

**Q. 3**

**Solution**

(a)

Total amount deposited by Manu in 5 years = Rs. 240 × 60 = Rs. 14,400

$$\text{Equivalent principal for 1 month} = \text{Rs.} 240 \times \frac{60(60 + 1)}{2} = \text{Rs.} 4,39,200 \quad [1]$$

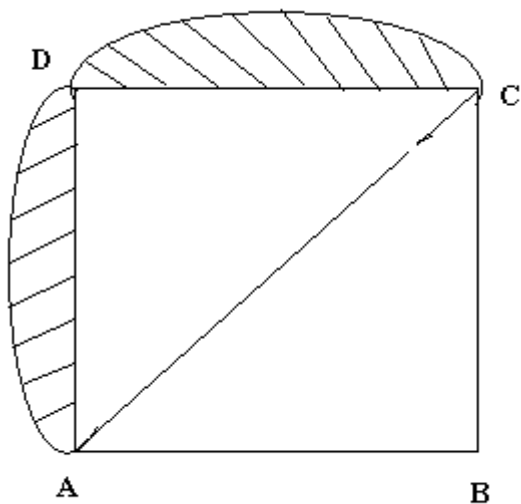
Let the rate of interest be r%

$$\text{Interest on Rs. 4,39,200 for 1 month} = \text{Rs.} 439200 \times \frac{1}{12} \times \frac{r}{100} = \text{Rs.} 366r \quad [1]$$

Maturity amount = 17694

$$\text{Or } 14400 + 366r = 17694 \Rightarrow r = 9\% \quad [1]$$

(b)



$$AC = \sqrt{2}a = 58\sqrt{2}m$$

$$r = \frac{58\sqrt{2}}{2} = 29\sqrt{2}m \quad \left[ \frac{1}{2} \right]$$

Area of whole lawn = area of  $\Delta ABC$  + Area of semi- circle on AC as diameter. [1]

$$= \frac{1}{2} \times 58 \times 58 + \frac{\pi(29\sqrt{2})^2}{2} \quad \left[ \frac{1}{2} \right]$$

$$= 29 \times 58 + \frac{22 \times 841 \times 2}{7 \times 2} \quad \left[ \frac{1}{2} \right]$$

$$= 4325.14m^2 \quad \left[ \frac{1}{2} \right]$$

(c)

(i) Coordinates of centroid,  $x = \frac{x_1 + x_2 + x_3}{3} = \frac{1+3+7}{3} = \frac{11}{3}$  [1]

And,  $y = \frac{y_1 + y_2 + y_3}{3} = \frac{4+2+5}{3} = \frac{11}{3}$  [1]

So,  $G = \left(\frac{11}{3}, \frac{11}{3}\right)$

(ii) the equation of a line, through G and parallel to AB.

Sol: Slope of line AB =  $\frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 4}{3 - 1} = -\frac{2}{2} = -1$  [1]

Now, equation of line through G and parallel to AB.

$\Rightarrow y - y_1 = m(x - x_1)$  [1]

$\Rightarrow y - \frac{11}{3} = -1\left(x - \frac{11}{3}\right)$

$\Rightarrow 3y - 11 = -3x + 11$

$\Rightarrow 3x + 3y - 22 = 0$

**Q. 4**

**Solution**

(a)

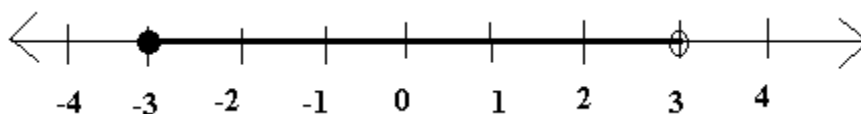
$-2\frac{2}{3} \leq x + \frac{1}{3} < 3\frac{1}{3}; x \in \mathbb{R}.$

$\Rightarrow \frac{-8}{3} \leq x + \frac{1}{3} < \frac{10}{3}$  [by subtracting  $\frac{1}{3}$ , we get] [1]

$\frac{-8}{3} - \frac{1}{3} \leq x + \frac{1}{3} - \frac{1}{3} < \frac{10}{3} - \frac{1}{3}$  [1]

$\frac{-9}{3} \leq x < \frac{9}{3} \Rightarrow -3 \leq x < 3$

Solution is  $\{x : x \in \mathbb{R}, -3 \leq x < 3\}$  [1]



(b)

$\frac{\sin(90^\circ - 70^\circ)}{\sin 20^\circ} + \frac{\sin(90^\circ - 59^\circ)}{\sin 31^\circ} - 8\left(\frac{1}{2}\right)^2$  [1]

Since,  $\cos \theta = \sin(90^\circ - \theta)$  [1]

$= \frac{\sin 20^\circ}{\sin 20^\circ} + \frac{\sin 31^\circ}{\sin 31^\circ} - 8 \times \frac{1}{4} = 1 + 1 - 2 = 0$  [1]

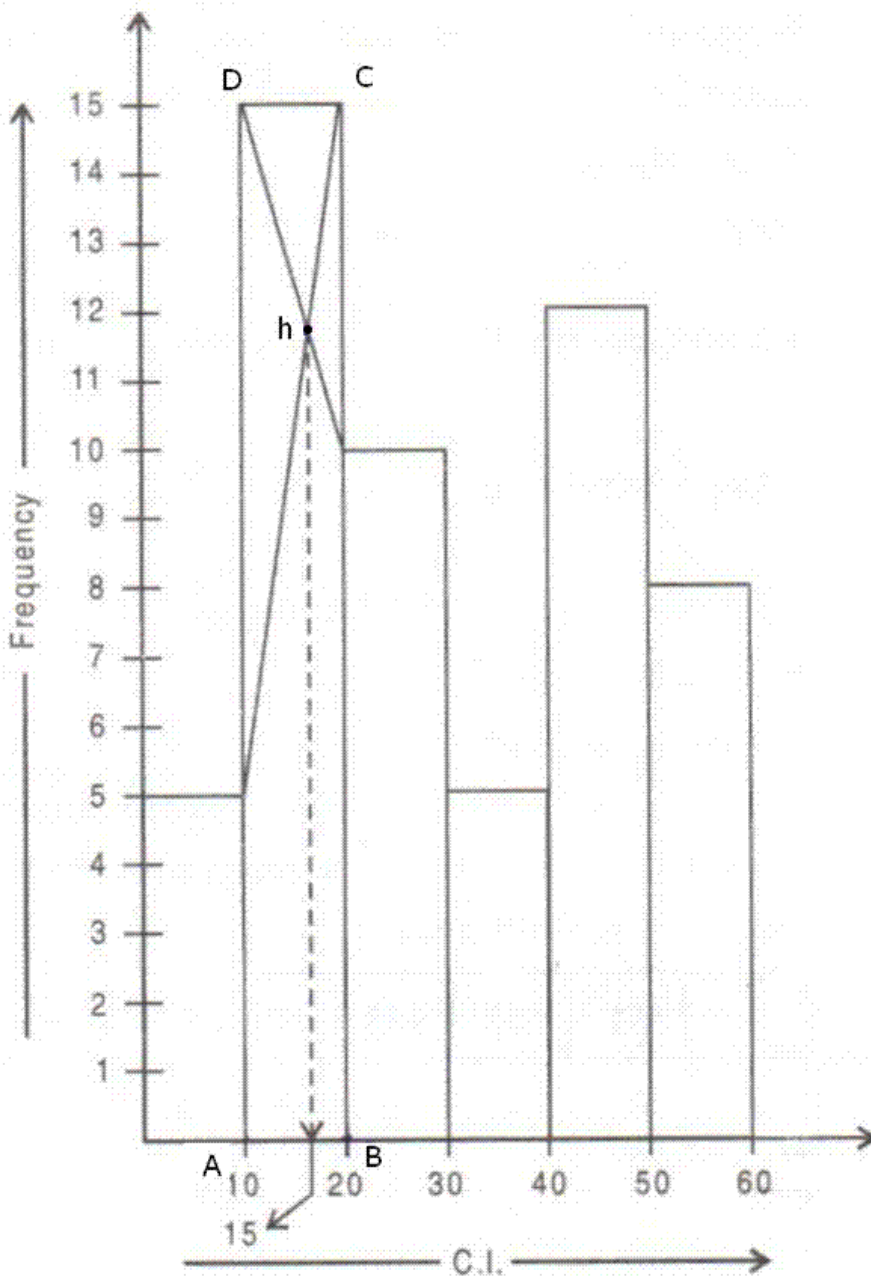
(c)

To locate the mode from the histogram, we proceed as follows:

- i Find the modal class. Rectangle ABCD is the largest rectangle. It represents the modal class, that is, the mode lies in this rectangle. The modal class is 10 – 20. [1]
- ii Draw two lines diagonally from the vertices C and D to the upper corners of the two adjacent rectangles. Let these rectangles intersect at point H. [1]
- iii The x-value of the point H is the mode. Thus, mode of the given data is approximately 15. [1]

So, the histogram is

[1]



So, mode = 15.

**SECTION – B (40 Marks)**

**Q. 5**

**Solution**

(a)

$$\text{M.P.} = \text{Rs. } x, \text{ Discount} = 20\% \text{ of } x = \text{Rs. } \frac{x}{5}$$

$$\text{New M.P. after discount} = \text{Rs. } x - \frac{x}{5} = \text{Rs. } \frac{4x}{5}$$

$$\text{Again discount} = 5\% \text{ of } \frac{4x}{5} = \frac{1}{20} \left( \frac{4x}{5} \right) = \text{Rs. } \frac{x}{25} \quad [1]$$

$$\text{Changed M.P. after 2}^{\text{nd}} \text{ discount} = \frac{4x}{5} - \frac{x}{25} = \text{Rs. } \frac{19x}{25}$$

$$\text{Now, } 2298.24 = \frac{19x}{25} + 8\% \text{ of } \frac{19x}{25} \quad [1]$$

$$2298.24 = \frac{19x}{25} + \frac{8}{100} \times \frac{19x}{25}$$

$$\Rightarrow 2298.24 = \frac{19x}{25} + \frac{38x}{625} \Rightarrow 2298.24 = \frac{475x + 38x}{625}$$

$$\Rightarrow 2298.24 = \frac{513x}{625} \quad [1]$$

$$\Rightarrow \frac{2298.24 \times 625}{513} = x \Rightarrow x = \text{Rs. } 2800$$

$$\therefore \text{M.P.} = \text{Rs. } 2800$$

(b)

$$\text{Length of wire for one round} = 3\text{mm} = 0.3\text{cm}$$

$$\text{Number of rounds required to cover the } 12 \text{ cm length} = \frac{12}{0.3} = 40$$

$$\text{Diameter of the cylinder} = 10\text{cm} \Rightarrow \text{Radius of the cylinder} = 5 \text{ cm}$$

$$\text{Length of wire required for one round} = 2\pi r = 2\pi \times 5\text{cm} = 10\pi\text{cm}$$

$$\therefore \text{Length of wire required for } 40 \text{ rounds} = 10\pi \times 40 \quad [1]$$

$$= 400\pi\text{cm} = 400 \times 3.14 = 1256\text{cm}$$

$$\text{Now, radius of the wire} = \frac{3}{2} \text{ mm} = \frac{3}{20} \text{ cm} \quad [1]$$

$$\text{Volume of the wire} = \pi r^2 h = \pi \left( \frac{3}{20} \right)^2 \times 1256\text{cm}^3$$

$$\text{Weight of the wire} = \text{Volume of the wire} \times 8.88 \text{ gm}$$

$$= 3.14 \times \left( \frac{3}{20} \right)^2 \times 1256 \times 8.88 = 787.98\text{gm} \quad [1]$$

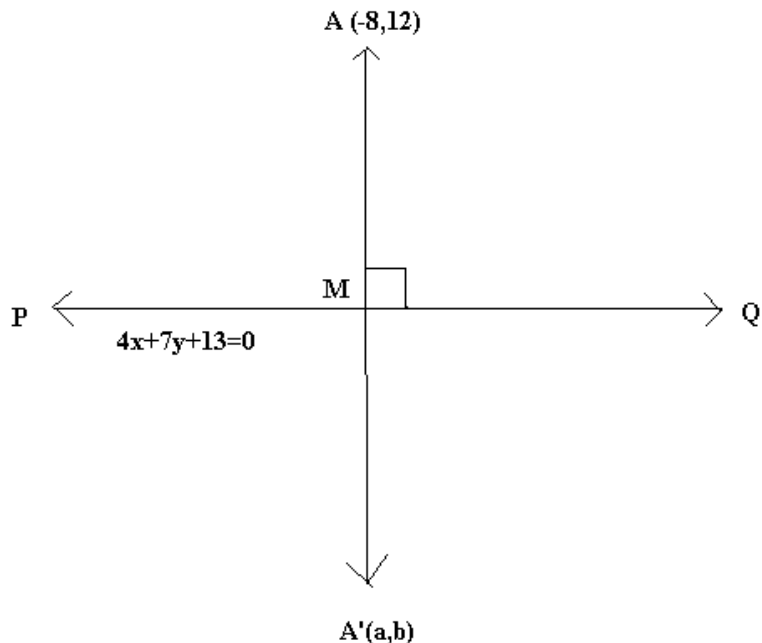
(c)

$$M = \left( \frac{a-8}{2}, \frac{b+12}{2} \right)$$

It lies on  $4x + 7y + 13 = 0$

$$\therefore 4 \left( \frac{a-8}{2} \right) + 7 \left( \frac{b+12}{2} \right) + 13 = 0 \quad [1]$$

$$\Rightarrow 4a + 7b + 78 = 0$$



Since  $AA' \perp PQ$ , so

$$\text{Slope (PQ)} \times \text{Slope (AA')} = -1 \quad [1]$$

$$\frac{-4}{7} \times \frac{b-12}{a+8} = -1$$

$$\Rightarrow 7a - 4b + 104 = 0$$

$$4a + 7b = -78 \dots (1)$$

$$7a - 4b = -104 \dots (2)$$

Multiplying equation (1) by 4 and equation (2) by 7 we get

$$16a + 28b = -312$$

$$49a - 28b = -728$$

$$65a = -1040 \Rightarrow a = \frac{-1040}{65} = -16, \quad [1]$$

Put  $a = -16$  in equation, we get

$$4a + 7b + 78 = 0$$

$$\Rightarrow 4 \times -16 + 7b + 78 = 0$$

$$\Rightarrow -64 + 7b + 78 = 0$$

$$\Rightarrow 7b + 14 = 0 \Rightarrow 7b = -14 \Rightarrow b = -2$$

$$\text{Image} = (-16, -2) \quad [1]$$

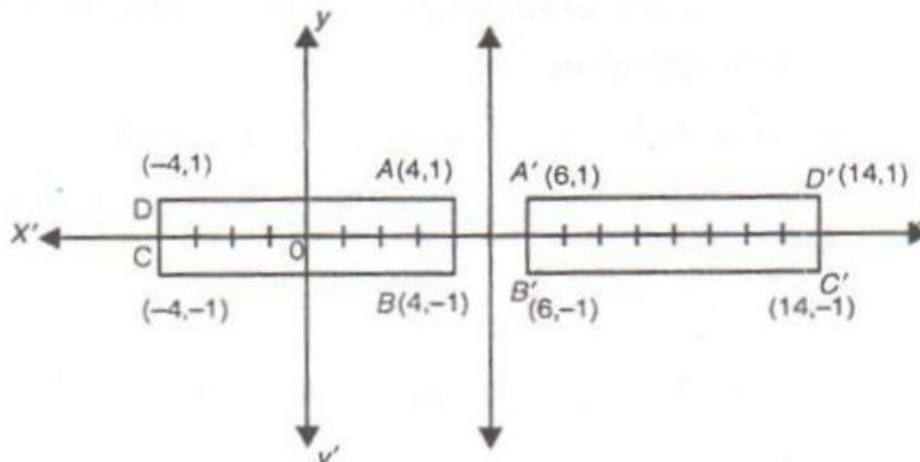


**Q. 6**

**Solution**

(a)

First the coordinates A(4,1), B(4,-1), D(-4,1) and C(-4,-1) are plotted to form a rectangle. The rectangle so formed and the reflected rectangle is shown in fig:



[2]

It is reflected through  $x = 5$ , then the coordinates of the new rectangle become  $A'(6,1)$ ,  $B'(6,-1)$ ,  $C'(14,-1)$  and  $D'(14,1)$

[1]

$$\begin{aligned} \text{Perimeter of } A'B'C'D' &= A'B' + B'C' + C'D' + D'A' \\ &= 2 + 8 + 2 + 8 = 20 \text{ units} \end{aligned}$$

[1]

$$\text{Area} = L \times B = B'C' \times C'D' = 8 \times 2 = 16 \text{ sq.units}$$

[1]

(b)

$$\left. \begin{aligned} \text{Min. balance for the month of June} &= 8680.00 \\ \text{Min. balance for the month of July} &= 6180.00 \\ \text{Min. balance for the month of Aug} &= 6180.00 \\ \text{Min. balance for the month of Sept} &= 6180.00 \\ \text{Min. balance for the month of Oct} &= 6180.00 \\ \text{Min. balance for the month of Nov} &= 6560.00 \\ \text{Min. balance for the month of Dec} &= 00.00 \end{aligned} \right\}$$

$$\text{Total} = 39,960.00$$

[1]

[1]

$$P = \text{Rs. } 39,960$$

$$I = \text{Rs. } (5940.80 - 5741) = \text{Rs. } 199.80$$

$$R = ?, \quad T = \frac{1}{12} \text{ years}$$

[1]

$$\text{Use, } I = \frac{P \times R \times T}{100}$$

[1]

$$\Rightarrow 199.80 = \frac{39,960 \times R \times 1}{12 \times 100}$$

[1]

$$\Rightarrow R = \frac{199.80 \times 12 \times 100}{39960} = 6\%$$

**Q.7**

**Solution**

(a)

$$\text{Given, } \frac{\sqrt{a+x} + \sqrt{a-x}}{\sqrt{a+x} - \sqrt{a-x}} = \frac{b}{1}$$

By using the componendo and dividendo, we get [1]

$$\frac{2\sqrt{a+x}}{2\sqrt{a-x}} = \frac{b+1}{b-1}$$

Now squaring both sides, we get [1]

$$\frac{a+x}{a-x} = \frac{b^2+1+2b}{b^2+1-2b}$$

Again use componendo and dividendo, to get the value of x. [1]

$$\frac{a+x+a-x}{a+x-a+x} = \frac{b^2+1+2b+b^2+1-2b}{b^2+1+2b-b^2-1+2b}$$

$$\Rightarrow \frac{2a}{2x} = \frac{2b^2+2}{4b}$$

$$\Rightarrow \frac{a}{x} = \frac{b^2+1}{2b}$$

$$\Rightarrow x = \frac{2ab}{b^2+1}$$

(b)

$$A^2 = A.A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} \quad [1]$$

$$A^3 = A^2.A = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix} \quad [1]$$

Now, L.H.S. =  $A^3 - 4A^2 + A$

$$\Rightarrow \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix} - 4 \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

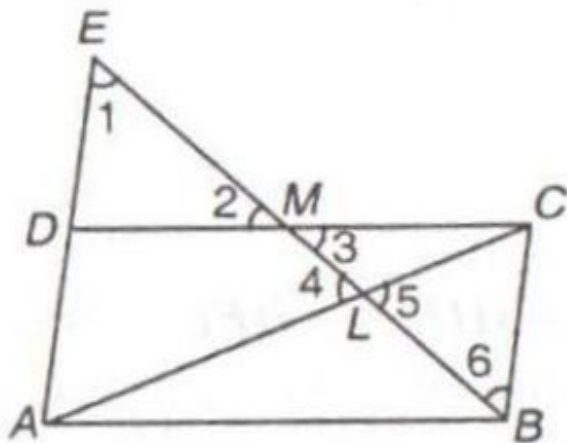
$$\Rightarrow \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix} - \begin{bmatrix} 28 & 48 \\ 16 & 28 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0 = \text{R.H.S.} \quad \dots \text{proved} \quad [1]$$

(c)

Construction: Draw the fig

[1]



$\angle 1 = \angle 6$  [Alternate interior  $\angle$ 's]

$\angle 2 = \angle 3$  [Vertically opposite  $\angle$ 's]

and  $DM = MC$

$\therefore \triangle DEM \cong \triangle BMC$  [AAS congruency]

[1]

$\Rightarrow DE = BC$  [by c.p.c.t]

Also  $AD = BC$  [opposite sides of a || gm]

$\Rightarrow AE = AD + DE = 2 BC$

[1]

Now,  $\angle 1 = \angle 6$  and  $\angle 4 = \angle 5 \therefore \triangle ELA \sim \triangle BLC$

$\Rightarrow \frac{EL}{BL} = \frac{AE}{BC} \Rightarrow \frac{EL}{BL} = \frac{2BC}{BC} \Rightarrow EL = 2BL$  Proved.

[1]

**Q.8**

**Solution**

(a)

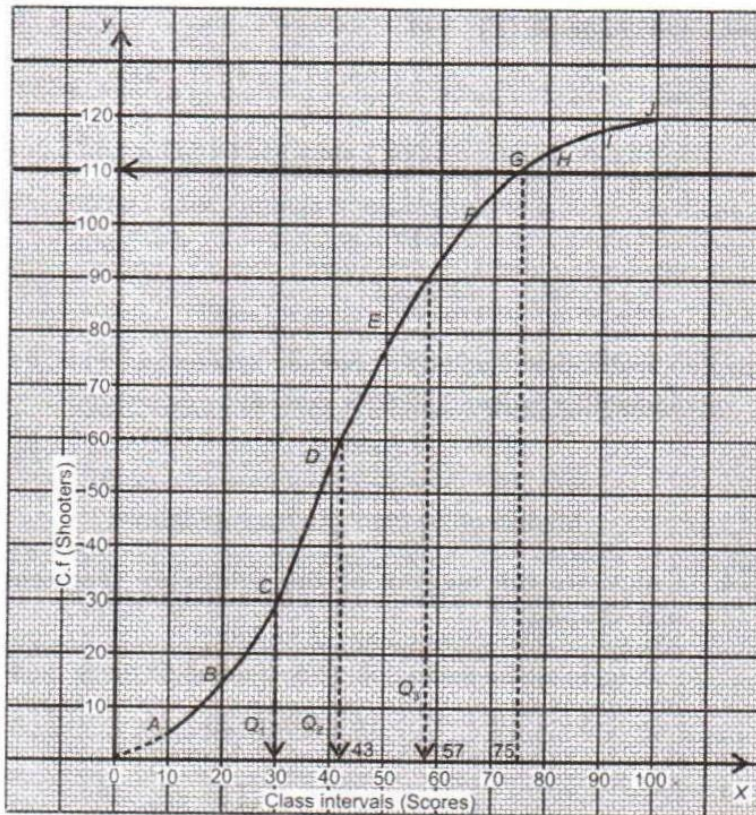
Drawing table

[1]

Scores Obtained	Number of Shooters	C.f.
0-10	5	5
10-20	9	14
20-30	16	30
30-40	22	52
40-50	26	78
50-60	18	96
60-70	11	107
70-80	6	113
80-90	4	117
90-100	3	120
	n=120	

Plotting Ogive

[1]



(i)  $n = 120$  (even)

The position of median is given by  $\frac{n}{2} = \frac{120}{2} = 60$ .

So, from the graph, median = 43 scores

[1]

(ii) The positions of  $Q_1$  is given by  $\frac{n}{4} = \frac{120}{4} = 30$

So, from the graph,  $Q_1 = 30$  scores

[1]

The position of  $Q_3$  is given by  $\frac{3n}{4} = \frac{3 \times 120}{4} = 90$

So, from the graph,  $Q_3 = 57$  scores

Now, Inter-quartile range =  $Q_3 - Q_1 = 57 - 30$

$\left[ \frac{1}{2} \right]$

= 27 scores

(iii) number of shooters who obtained more than 75% scores =  $120 - 110 = 10$

$\left[ \frac{1}{2} \right]$

(b)

Drawing Table and calculating values

[2]

$x_i$	$f_i$	$u_i = \frac{x_i - A}{h}, h = 5$	$f_i u_i$
15	5	-4	-20
20	8	-3	-24
25	11	-2	-22
30	20	-1	-20
A=35	23	0	0
40	18	1	18
45	13	2	26
50	3	3	9
55	1	4	4
	$\sum f_i = 102$		$\sum f_i u_i = -29$

From the table,  $A = 35, \sum f_i = 102, h = 5, \sum f_i u_i = -29$

[1]

$$\text{So, Mean } (\bar{x}) = A + h \frac{\sum f_i u_i}{\sum f_i}$$

[1]

$$= 35 + 5 \times \frac{-29}{102}$$

$$= 35 - \frac{145}{102}$$

[1]

$$= 35 - 1.42$$

$$= 33.58$$

**Q. 9**

**Solution**

(a)

$$\text{Market value of 1 share} = \text{Rs.} \left( \frac{26 \times 110}{100} \right) = \text{Rs.} 28.80$$

$$(i) \text{ Number of shares bought} = \frac{20,020}{28.60} = \frac{20,02,000}{2860} = 700$$

[1]

$$(ii) \text{ Dividend on one share} = \text{Rs.} \frac{26 \times 15}{100} = \text{Rs.} \frac{39}{10}$$

$$\therefore \text{ dividend on 700 shares} = 700 \times \frac{39}{10} = \text{Rs.} 2730$$

[1]

$$(iii) \text{ Rate of interest} = \frac{2730}{20020} \times 100 = 13.63\%$$

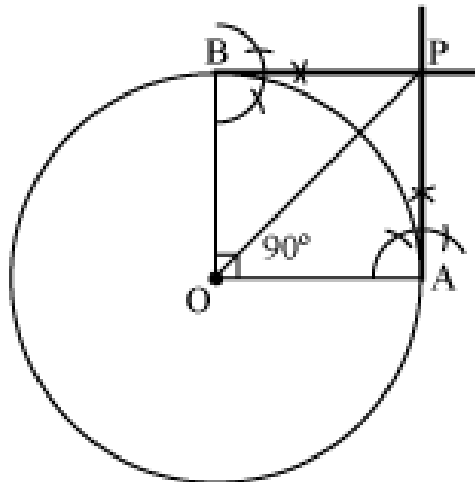
[1]

(b)

The steps of construction are as follows:

- (i) Draw a circle of any convenient radius with O as centre.
- (ii) Take a point A on the circumference of the circle and join OA. Draw a perpendicular to OA at point A.
- (iii) Draw a radius OB, making an angle of  $90^\circ$  with OA.
- (iv) Draw a perpendicular to OB at point B. Let both the perpendiculars intersect at point P.
- (v) Join OP.

PA and PB are the required tangents, which make an angle of  $45^\circ$  with OP. [1]



[2]

(c)

L.H.S.

$$= \frac{\tan A}{1 - \frac{1}{\tan A}} + \frac{1}{\tan A(1 - \tan A)} \quad [1]$$

$$= \frac{\tan^2 A}{(\tan A - 1)} - \frac{1}{\tan A(\tan A - 1)} \quad [1]$$

$$= \frac{\tan^3 A - 1}{\tan A(\tan A - 1)} \quad [1]$$

Use  $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$$= \frac{(\tan A - 1)(\tan^2 A + \tan A + 1)}{\tan A(\tan A - 1)} \quad [1]$$

$$= \tan A + 1 + \cot A$$

$$= 1 + \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}$$

$$= 1 + \frac{\sin^2 A + \cos^2 A}{\sin A \cos A} = 1 + \frac{1}{\sin A \cos A} \quad [1]$$

$$= 1 + \operatorname{cosec} A \sec A$$

= R.H.S

Hence Proved

**Q.10**

**Solution**

(a)

Let the numbers be x and y;

$$\text{So } xy = 196 \Rightarrow x = \frac{196}{y} \text{ and } 112x = y^2 \quad [1]$$

Put  $x = \frac{196}{y}$  in  $112x = y^2$ , we get

$$\Rightarrow \frac{112 \times 196}{y} = y^2 \quad [1]$$

$$\Rightarrow y^3 = 112 \times 196$$

$$\Rightarrow y = \sqrt[3]{112 \times 196} \Rightarrow y = 28 \quad [1]$$

$$\text{And } x = \frac{196}{28} = 7$$

So, numbers are = 7, 14, 28.

(b)

Let principal = Rs. P then amount = Rs.  $\frac{125}{64}P$

$$\text{Now, } A = P \left(1 + \frac{R}{100}\right)^n \quad [2]$$

$$\text{Or } \frac{125}{64}P = P \left(1 + \frac{R}{100}\right)^3 \quad [1]$$

$$\text{Or } \frac{125}{64} = \left(1 + \frac{R}{100}\right)^3$$

$$\Rightarrow \left(\frac{125}{64}\right) = \left(1 + \frac{R}{100}\right)^3$$

$$\Rightarrow \left(\frac{5}{4}\right)^3 = \left(1 + \frac{R}{100}\right)^3 \quad [1]$$

Taking cube of both the sides

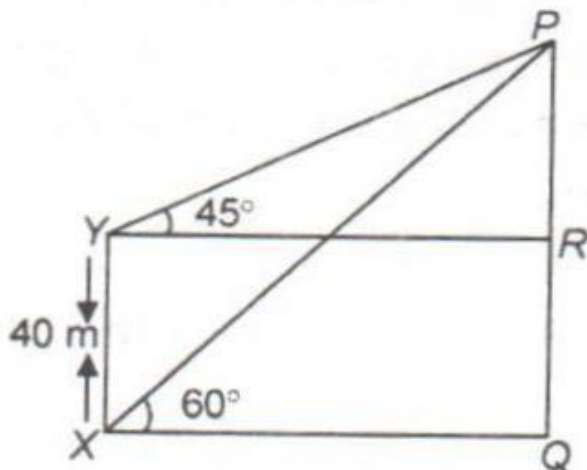
$$\frac{5}{4} = 1 + \frac{R}{100} \text{ or } \frac{5}{4} = \frac{100 + R}{100}$$

$$\text{Or } 500 = 400 + 4R$$

$$\text{Or } 4R = 100 \Rightarrow R = 25$$

∴ Rate = 25%

(c)



Let  $XQ = x$  m and  $PR = h$  m then  $YR = XQ = x$  m  
And  $YX = RQ = 40$  m

$$\text{In } \triangle PRY, \frac{h}{x} = \tan 45^\circ \Rightarrow h = x \quad [1]$$

$$\text{In } \triangle PQX, \frac{h + 40}{x} = \tan 60^\circ \Rightarrow h + 40 = \sqrt{3}x$$

$$\Rightarrow h + 40 = \sqrt{3}h \quad [\because h = x] \quad [1]$$

$$40 = \sqrt{3}h - h$$

$$40 = h(\sqrt{3} - 1)$$

$$\text{Or } h = \frac{40}{\sqrt{3} - 1}$$

$$= \frac{40}{(\sqrt{3} - 1)} \times \frac{(\sqrt{3} + 1)}{(\sqrt{3} + 1)} \quad [1]$$

$$= \frac{40(\sqrt{3} + 1)}{3 - 1}$$

$$= \frac{40(1.73 + 1)}{2}$$

$$= 20 \times 2.730$$

$$= 2 \times 27.3$$

$$= 54.6$$

(i) Height  $PQ = h + 40 = 54.6 + 40 = 94.6$  m [  $\frac{1}{2}$  ]

(ii) The distance  $XQ = x = h = 54.6$  m [  $\frac{1}{2}$  ]



**Q.11**

**Solution**

(a)

(i)  $AM = MP$  [tangents from an external point]

$$\Rightarrow \angle MAP = \angle MPA$$

Similarly,  $MP = MB$

$$\therefore \angle MBP = \angle MPB$$

Now in  $\triangle ABP$ ,

$$\angle PAB + \angle ABP + \angle APB = 180^\circ \quad [1]$$

$$\angle MPA + \angle MPB + \angle MPA + \angle MPB = 180^\circ$$

$$(\angle MPA + \angle MPB) = 180^\circ \quad [1]$$

$$\Rightarrow \angle APB = 90^\circ$$

(ii)  $AM = MP$  and  $MP = MB$  [1]

$$\Rightarrow AM = MB$$

$\Rightarrow$  tangent at P bisects AB.

(b)

Total money for expenses = Rs.360

Original duration of tour = x days

Final duration of tour = x + 4 days

$$\text{Originally, 1 day expenses} = \text{Rs.} \frac{360}{x} \quad \left[ \frac{1}{2} \right]$$

$$\text{Finally, 1 day expenses} = \text{Rs.} \frac{360}{x+4} \quad \left[ \frac{1}{2} \right]$$

According to the question,

$$\frac{360}{x} = \frac{360}{x+4} + 3 \quad \left[ \frac{1}{2} \right]$$

$$\Rightarrow \frac{360}{x} - \frac{360}{x+4} = 3$$

$$\Rightarrow 360x + 1440 - 360x = 3x^2 + 12x$$

$$\Rightarrow 3x^2 + 12x - 1440 = 0$$

$$\Rightarrow x^2 + 4x - 480 = 0$$

$$\Rightarrow x^2 + 24x - 20x - 480 = 0$$

$$\Rightarrow x(x + 24) - 20(x + 24) = 0$$

$$\Rightarrow (x - 20)(x + 24) = 0 \quad \left[ \frac{1}{2} \right]$$

Thus,  $x = 20, -24$

Days cannot be negative, So,  $x = 20$ . [1]

(c)

Let,  $\angle ACX = \theta$  and  $\angle ABX = \phi$

(i) Given, equation of AB,  $x - \sqrt{3}y + 1 = 0$

$$\begin{aligned} \therefore \text{Slope of AB} &= \frac{-\text{coefficient of } x}{\text{coefficient of } y} && [1] \\ &= \frac{-1}{-\sqrt{3}} = \frac{1}{\sqrt{3}} \end{aligned}$$

$$\text{So, } (m_1) = \tan \theta = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \theta = 30^\circ$$

$\therefore$  angle made by the line AB =  $30^\circ$

Now, equation of AC,  $x - y - 2 = 0$

$$\therefore \text{Slope of AC} = = \frac{-\text{coefficient of } x}{\text{coefficient of } y} = \frac{-1}{-1} = 1 \quad [1]$$

$$\text{So, } (m_2) = \tan \phi = 1 \quad [1]$$

$$\Rightarrow \phi = 45^\circ$$

Angle made by the line AC =  $45^\circ$

(ii)  $\angle BAC = \angle C - \angle B \quad [1]$

$$= 45^\circ - 30^\circ = 15^\circ$$