CBSE Board
Class IX Mathematics
Term I
Sample Paper - 1

Time: 3 hour Total Marks: 90

Solution
Section A

1. Correct answer: C

$\pi - 10$

2. Correct answer: C

$p(x) = x^3 + 10x^2 + px$

$(x - \mathbf{1})$ is the factor of $p(x)$

So $p(1) = 0$

$1 + 10 + p = 0$

$p = -11$

3. Correct answer: D

$p(x) = x^3 + 1$

$p(-1) = 0$

Thus, required remainder is 0.

4. Correct answer: B

$x^2 - 16x + 64$

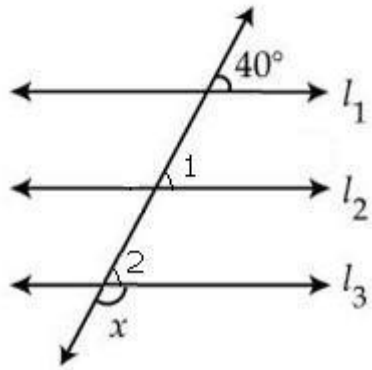
$= x^2 - 8x - 8x + 64$

$= x(x - 8) - 8(x - 8)$

$= (x - 8)(x - 8)$

Thus, the factors of the given polynomial are (x-8) and (x-8).

5. Correct answer: B



We know corresponding angles are equal.
So, $\angle 1 = \angle 2 = 40^o$
Therefore, $x = 180^o - 40^o = 140^o$

6. Correct answer: B
Two triangles will be congruent by SAS axiom if 2 sides and the included angle of one triangle are equal to the two sides and the included angle of the other triangle.Thus, the triangles will be congruent when AC=DE.

7. Correct answer: A

Each side of the equilateral triangle $= \frac{18x}{3} = 6x$.

Area of equilateral triangle $= (\text{side})^2 \frac{\sqrt{3}}{4} = (6x)^2 \frac{\sqrt{3}}{4} = 9x^2\sqrt{3}$.

8. Correct answer: A

$$s = \frac{a+b+c}{2} = \frac{15+25+14}{2} = 27$$

Using Heron's formula,

$$\text{Area of a triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{27(27-15)(27-25)(27-14)}$$
$$= \sqrt{27(12)(2)(13)} = 18\sqrt{26}\ \text{cm}^2$$

Section B

9. $\left(\frac{81}{16}\right)^{-3/4} \times \left(\frac{25}{9}\right)^{-3/2}$

$= \left[\left(\frac{3}{2}\right)^4\right]^{-\frac{3}{4}} \times \left[\left(\frac{5}{3}\right)^2\right]^{-\frac{3}{2}}$

$= \left(\frac{3}{2}\right)^{-3} \times \left(\frac{5}{3}\right)^{-3}$

$= \left(\frac{2}{3}\right)^{3} \times \left(\frac{3}{5}\right)^{3}$

$= \frac{2^3}{3^3} \times \frac{3^3}{5^3} = \frac{2^3}{5^3} = \frac{8}{125}$

10.

$x^2 + \frac{1}{x^2} + 2 - 2x - \frac{2}{x}$

$= \left(x^2 + \frac{1}{x^2} + 2\right) - 2\left(x + \frac{1}{x}\right)$

$= \left(x + \frac{1}{x}\right)^2 - 2\left(x + \frac{1}{x}\right)$

$= \left(x + \frac{1}{x}\right)\left(x + \frac{1}{x} - 2\right)$

11. Let $x = 75$, $y = -25$, $z = -50$

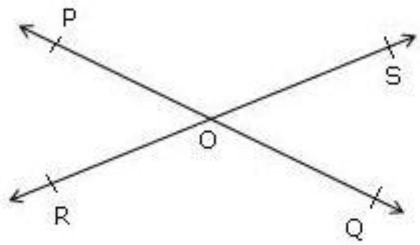
$x + y + z = 75 - 25 - 50 = 0$

We know, if $x + y + z = 0$ then $x^3 + y^3 + z^3 = 3xyz$

$75^3 - 25^3 - 50^3 = 3(75)(-25)(-50)$

$= 281250$

12.



$\angle POR + \angle ROQ = 180^0$ (Linear Pair)

But $\angle POR : \angle ROQ = 5 : 7$

$\therefore \angle POR = \frac{5}{12} \times 180^0 = 75^0$

$\angle ROQ = \frac{7}{12} \times 180^0 = 105^0$

$\angle POR = \angle SOQ = 75^0$ (Vertically opposite angles)

$\angle POS = \angle ROQ = 105^0$ (Vertically opposite angles)

OR

In ΔABC

$\angle ABC + 100^0 = 180^0$ (linear pair)

$\angle ABC = 80^0$

$x + 80^0 = 115^0$ (Exterior angle property)

$x = 35^0$

13. AD is the bisector of $\angle A$

$\therefore \angle BAD = \angle CAD$

Exterior $\angle BDA > \angle CAD$

$\Rightarrow \angle BDA > \angle BAD$

$\Rightarrow AB > BD$ (side opposite the bigger angle is longer)

14.

(A) Point of the form (a, 0) lie on the x axis.

The point (-4, 0) will lie on the negative side of the x axis.

(B) (-,+) are the sign of the coordinates of points in the II quadrant.

∴ The point (-10, 2) lies in the II quadrant.

(C) Point of the form (0, a) lie on the y axis.

So, the point (0, 8) will lie on the positive side of the y axis.

(D) (+,+) are the sign of the coordinates of points in the I quadrant.

∴ The point (10, 4) lies in the I quadrant.

Section C

15.

$$\frac{(25)^{\frac{3}{2}} \times (343)^{\frac{3}{5}}}{16^{\frac{5}{4}} \times 8^{\frac{4}{3}} \times 7^{\frac{3}{5}}}$$

$$= \frac{\left(5^2\right)^{\frac{3}{2}} \times \left(7^3\right)^{\frac{3}{5}}}{\left(2^4\right)^{\frac{5}{4}} \times \left(2^3\right)^{\frac{4}{3}} \times 7^{\frac{3}{5}}}$$

$$= \frac{5^3 \times 7^{\frac{9}{5}}}{2^5 \times 2^4 \times 7^{\frac{3}{5}}}$$

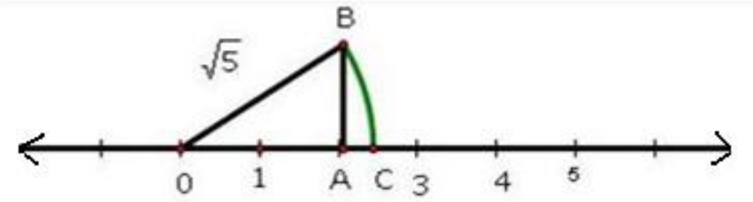
$$= \frac{5^3 \times 7^{\frac{9}{5}}}{2^9 \times 7^{\frac{3}{5}}}$$

$$= \frac{5^3 \times 7^{\frac{6}{5}}}{2^9}$$

16. Using Pythagoras theorem: $5 = 2^2 + 1^2$

Taking positive square root, we get, $\sqrt{5} = \sqrt{(2)^2 + (1)^2}$

(i) We mark a point 'A' representing 2 units on number line.
(ii) Construct AB of unit length perpendicular to OA. Then, join OB.
(iii) Taking O as centre and OB as radius, draw an arc intersecting number line at point C.
(iv) Point C represents $\sqrt{5}$ on number line.



17.

When $p(x) = ax^3 + 3x^2 - 3$ is divided by $(x - 4)$, the remainder is given by

$R_1 = a(4)^3 + 3(4)^2 - 3 = 64a + 45$

When $q(x) = 2x^3 - 5x + a$ is divided by $(x - 4)$, the remainder is given by

$R_2 = 2(4)^3 - 5(4) + a = 108 + a$

Given: $R_1 + R_2 = 0$

$\Rightarrow 65a + 153 = 0$

$\Rightarrow a = \frac{-153}{65}$

OR

By hit and trial we find $x = 3$ is a factor of given polynomial, as
$2(3)^3 - 9 - 39 - 6 = 54 - 54 = 0$
By dividing $2x^3 - x^2 - 13x - 6$ by $x - 3$ we get
$2x^2 + 5x + 2$ as quotient.
Factorising this further

$2x^2 + 5x + 2 = 2x^2 + 4x + x + 2$ $[1 \times 4 = 5]$ $[1 + 4 = 5]$
$= 2x(x + 2) + 1(x + 2)$
$= (2x + 1)(x + 2)$

So, $2x^3 - x^2 - 13x - 6 = (2x + 1)(x + 2)(x - 3)$

18.

$x^3 + 13x^2 + 32x + 20$ is a cubic polynomial, so it has three factors of which one is $(x + 2)$

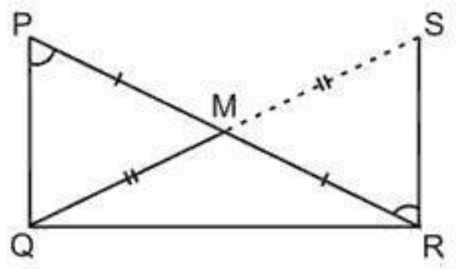
$$
\begin{array}{r|l}
 & x^2 + 11x + 10 \\
\hline
x + 2 & x^3 + 13x^2 + 32x + 20 \\
 & x^3 + 2x^2 \\
 & \underline{(-)\ (-)} \\
 & \quad 11x^2 + 32x \\
 & \quad 11x^2 + 22x \\
 & \quad \underline{(-)\quad (-)} \\
 & \qquad 10x + 20 \\
 & \qquad \underline{10x + 20} \\
 & \qquad \underline{\quad 0 \quad}
\end{array}
$$

We can factorise, $x^2 + 11x + 10 = x^2 + 10x + x + 10$

$= (x + 1)(x + 10)$

$\Rightarrow x^3 + 13x^2 + 32x + 20 = (x + 1)(x + 2)(x + 10)$

19.



Produce QM to S such that QM = MS. Join SR

In ΔPMQ and ΔRMS

PM = MR (M is the mid point)

QM = MS (By construction)

$\angle PMQ = \angle RMS$...vertically opposite angles

$\therefore \Delta PMQ \cong \Delta RMS$ (SAS congruence criterion)

$\therefore PQ = SR$ and $\angle QPM = \angle SRM$ (c.p.c.t)

$\angle QPM = \angle SRM$ (alternate angles) $\therefore RS \parallel PQ$

$\angle PQR + \angle QRS = 180°$ (Co–interior angles)

$\Rightarrow 90° + \angle QRS = 180°$

$\Rightarrow \angle QRS = 90°$

In ΔPQR and ΔQRS

QR = RQ (common)

PQ = SR

$\angle PQR = \angle QRS$ (90° each)

$\therefore \Delta PQR \cong \Delta SRQ$ (SAS congruence criterion)

$\therefore PR = QS \Rightarrow \frac{1}{2}SQ = \frac{1}{2}PR$

$\Rightarrow \frac{1}{2}SQ = QM = \frac{1}{2}PR$

Hence, proved

20.

LM = MN...given

$\Rightarrow \angle MLN = \angle MNL$ (angles opposite equal sides are equal)

$\Rightarrow \angle MLQ = \angle MNP$

LP = QN (given)

$\Rightarrow LP + PQ = PQ + QN$ (adding PQ on both sides)

$\Rightarrow LQ = PN$

In ΔLMQ and ΔNMP

LM = MN

$\angle MLQ = \angle MNP$

LQ = PN

$\Delta LMQ \cong \Delta NMP$ (SAS congruence rule)

OR

In ΔDCB, $\angle DBC = \angle DCB$ (given)

DC = DB [Side opp. to equal $\angle$'s areequal]___(i)

In ΔABD and ΔACD

AB = AC (given)

BD = CD [From(i)]

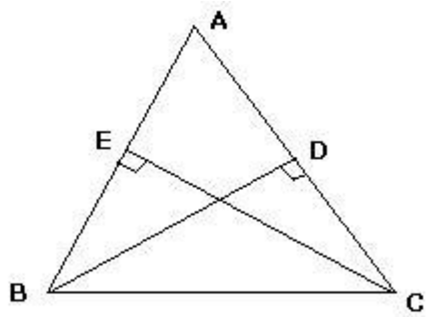
AD = AD common

$\Delta ABD \cong \Delta ACD$ [SSS Rule]

$\angle BAD = \angle CAD$ (CPCT)

Hence, AD is bisector of $\angle BAC$

21.



In △ABD and △ACE

∠ADB= ∠AEC= 90^0

∠BAD= ∠CAE (common angle)

BD = CE (given)

By AAS Criteria

△ABD ≅ △ACE

AB = AC (CPCT)

Hence, △ABC is isosceles.

22. In △AOD and △AOB

AD = AB (given)

AO =AO (common side)

OD = OB (given)

∴ △AOD ≅ △AOB (SSS congruence rule)

∴ ∠AOD = ∠AOB (c.p.c.t)

Similarly, △DOC ≅ △BOC (SSS congruence rule)

∴ ∠DOC = ∠BOC (c.p.c.t)

∠AOD + ∠AOB + ∠DOC + ∠BOC = 360^o (angles at a point)

2∠AOD + 2∠DOC = 360^o

∠AOD + ∠DOC = 180^o

Hence, AO and OC are in one and the same straight line.

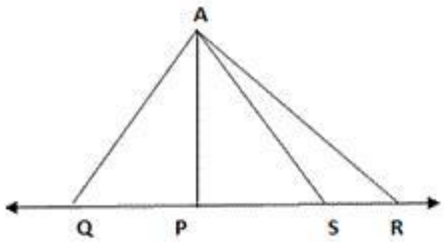
23.

Given: $AP \perp l$ & $PR > PQ$

To show: $AR > AQ$

Const: Take a point S on PR such that PS = PQ

Join AS



In ΔAPQ & ΔAPS

$\angle APQ = \angle APS$ (each 90°)

$AP = AP$ (common)

$PQ = PS$ (by const)

$\Delta APQ \cong \Delta APS$ (by SAS cong them)

$\angle AQP = \angle ASP$ (by cpct)

$\angle ASP > \angle ARS$ (exterior angle of ΔASR)

So, $\angle AQP > \angle ARS$ (as $\angle AQP = \angle ASP$)

$AR > AQ$ (side opposite to greater angle in ΔAQR is greater)

OR

In ΔPQR, $PQ > PR$.

$\Rightarrow \angle PRQ > \angle PQR$ (Angle opposite to longer side is greater)

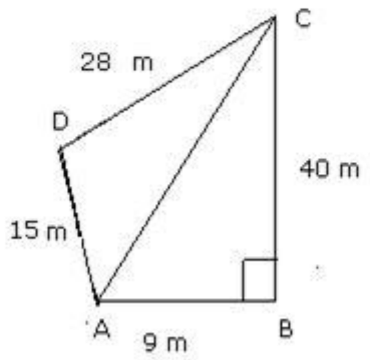
$\Rightarrow \frac{1}{2}\angle PRQ > \frac{1}{2}\angle PQR$

$\Rightarrow \angle SRQ > \angle SQR$ (RS and QS are the bisectors of $\angle PRQ$ and $\angle PQR$)

In ΔSQR, $\angle SRQ > \angle SQR$ (Proved above)

$\Rightarrow SQ > SR$ (Side opposite to greater angle is longer)

24.



Let ABCD be the garden.

In $\triangle ABC$,

$AC^2 = 9^2 + 40^2 = 1681$

$AC = 41$

Area of the garden = Area of $\triangle ABC$ + area of $\triangle ACD$

Area of $\triangle ABC = \frac{1}{2} \times b \times h = \frac{1}{2} \times 9 \times 40 = 180\,m^2$

Area of $\triangle ACD = \sqrt{s(s-a)(s-b)(s-c)}$

$\left[\because s = \frac{15 + 28 + 41}{2} = 42\text{ m}\right]$

$\therefore$ Area of $\triangle ACD = \sqrt{42 \times 27 \times 14 \times 1} = 7 \times 3 \times 3 \times 2 = 126\,m^2$

$\therefore$ Area of the garden = 180 + 126 = 306 m^2

Section D

25.

$$\frac{1}{3-\sqrt{8}} = \frac{1}{3-\sqrt{8}} \times \frac{3+\sqrt{8}}{3+\sqrt{8}} = \frac{3+\sqrt{8}}{9-8} = 3+\sqrt{8}$$

$$\frac{1}{\sqrt{8}-\sqrt{7}} = \frac{1}{\sqrt{8}-\sqrt{7}} \times \frac{\sqrt{8}+\sqrt{7}}{\sqrt{8}+\sqrt{7}} = \frac{\sqrt{8}+\sqrt{7}}{8-7} = \sqrt{8}+\sqrt{7}$$

$$\frac{1}{\sqrt{7}-\sqrt{6}} = \frac{1}{\sqrt{7}-\sqrt{6}} \times \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}} = \frac{\sqrt{7}+\sqrt{6}}{7-6} = \sqrt{7}+\sqrt{6}$$

$$\frac{1}{\sqrt{6}-\sqrt{5}} = \frac{1}{\sqrt{6}-\sqrt{5}} \times \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}} = \frac{\sqrt{6}+\sqrt{5}}{6-5} = \sqrt{6}+\sqrt{5}$$

$$\frac{1}{\sqrt{5}-2} = \frac{1}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2} = \frac{\sqrt{5}+2}{5-4} = \sqrt{5}+2$$

$$\frac{1}{3-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-2}$$

$$= 3+\sqrt{8}-\left(\sqrt{8}+\sqrt{7}\right)+\left(\sqrt{7}+\sqrt{6}\right)-\left(\sqrt{6}+\sqrt{5}\right)+\left(\sqrt{5}+2\right)$$

$$= 5$$

OR

$$\frac{1}{2+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{6}} + \frac{1}{\sqrt{6}+\sqrt{7}} + \frac{1}{\sqrt{7}+\sqrt{8}}$$

$$= \frac{2-\sqrt{5}}{(2)^2-\left(\sqrt{5}\right)^2} + \frac{\sqrt{5}-\sqrt{6}}{\left(\sqrt{5}\right)^2-\left(\sqrt{6}\right)^2} + \frac{\sqrt{6}-\sqrt{7}}{\left(\sqrt{6}\right)^2-\left(\sqrt{7}\right)^2} + \frac{\sqrt{7}-\sqrt{8}}{\left(\sqrt{7}\right)^2-\left(\sqrt{8}\right)^2}$$

$$= \frac{2-\sqrt{5}}{-1} + \frac{\sqrt{5}-\sqrt{6}}{-1} + \frac{\sqrt{6}-\sqrt{7}}{-1} + \frac{\sqrt{7}-\sqrt{8}}{-1}$$

$$= \left(\sqrt{5}-2\right)+\left(\sqrt{6}-\sqrt{5}\right)+\left(\sqrt{7}-\sqrt{6}\right)+\left(\sqrt{8}-\sqrt{7}\right)$$

$$= -2+\sqrt{8}$$

$$= -2+2\sqrt{2}$$

$$= 2\left(\sqrt{2}-1\right)$$

26.

On rationalising, $\frac{3-\sqrt{5}}{3+2\sqrt{5}}$

$= \frac{3-\sqrt{5}}{3+2\sqrt{5}} \times \frac{3-2\sqrt{5}}{3-2\sqrt{5}}$

$= \frac{19-9\sqrt{5}}{9-20}$

$= \frac{19-9\sqrt{5}}{-11}$

$= \frac{-19}{11} + \frac{9\sqrt{5}}{11}$

And $, a\sqrt{5} - b = \frac{9}{11}\sqrt{5} - \frac{19}{11}$

$\Rightarrow a = \frac{9}{11}$ and $b = \frac{19}{11}$

27. Let $f(x) = x^3 + 2x^2 - 5ax - 8$ and
$g(x) = x^3 + ax^2 - 12x - 6$
When divided by (x-2) and (x-3), f(x) and g(x) leave remainder p and q respectively

$f(x) = x^3 + 2x^2 - 5ax - 8$

$\therefore f(2) = 2^3 + 2 \times 2^2 - 5a \times 2 - 8$

$= 8 + 8 - 10a - 8$

$p = 8 - 10a \quad -----(1)$

$g(x) = x^3 + ax^2 - 12x - 6$

$g(3) = 3^3 + a \times 3^2 - 12 \times 3 - 6$

$= 27 + 9a - 36 - 6$

$\therefore q = -15 + 9a \quad -----(2)$

If $q - p = 10$

$\Rightarrow -15 + 9a - 8 + 10a = 10$

$\Rightarrow 19a - 23 = 10$

$\Rightarrow 19a = 33$

$\therefore \quad a = \frac{33}{19}$

28.

Consider $\dfrac{(a^2-b^2)^3+(b^2-c^2)^3+(c^2-a^2)^3}{(a-b)^3+(b-c)^3+(c-a)^3}$

We know that,

If $x+y+z=0$ then $x^3+y^3+z^3=3xyz$

Now, $a^2-b^2+b^2-c^2+c^2-a^2=0$

And, $a-b+b-c+c-a=0$

$\therefore \dfrac{(a^2-b^2)^3+(b^2-c^2)^3+(c^2-a^2)^3}{(a-b)^3+(b-c)^3+(c-a)^3}$

$= \dfrac{3(a^2-b^2)(b^2-c^2)(c^2-a^2)}{3(a-b)(b-c)(c-a)}$

$= \dfrac{3(a-b)(a+b)(b-c)(b+c)(c-a)(c+a)}{3(a-b)(b-c)(c-a)}$

$= (a+b)(b+c)(c+a)$

29. Let $p(x) = x^3 + 13x^2 + 32x + 20$

$p(-1) = -1 + 13 - 32 + 20 = -33 + 33 = 0$

Therefore $(x + 1)$ is a factor of $p(x)$.

On dividing $p(x)$ by $(x + 1)$ we get

$p(x) \div (x + 1) = x^2 + 12x + 20$

Thus,

$x^3 + 13x^2 + 32x + 20 = (x + 1)(x^2 + 12x + 20)$

$= (x + 1)(x^2 + 10x + 2x + 20)$

$= (x + 1)[x(x + 10) + 2(x + 10)]$

$= (x + 1)(x + 2)(x + 10)$

Hence, $x^3 + 13x^2 + 32x + 20 = (x + 1)(x + 2)(x + 10)$.

OR

Given: $8x^3 + 27y^3 = 730$

$2x^2y + 3xy^2 = 15$

Now, $(2x + 3y)^3 = (2x)^3 + (3y)^3 + 3(2x)(3y)(2x + 3y)$

$= 8x^3 + 27y^3 + 18xy(2x + 3y)$

So, $(2x + 3y)^3 = 8x^3 + 27y^3 + 18(2x^2y + 3xy^2)$

$(2x + 3y)^3 = 730 + 18(15)$ [Using the given conditions]

$= 730 + 270$

$(2x + 3y)^3 = 1000$

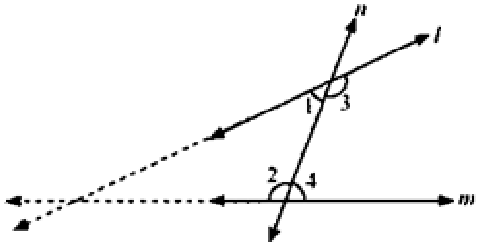
Therefore, $2x + 3y = 10$

30. Euclid's 5th postulate states that:

If a straight line falling on two straight lines makes the interior angles on the same side of it taken together less than two right angles, then the two straight lines, if produced indefinitely, meet on that side on which the sum of angles is less than two right angles.

This implies that if n intersects lines l and m and if

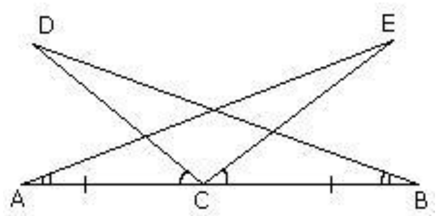
$\angle 1 + \angle 2 < 180^\circ$, then $\angle 3 + \angle 4 > 180^\circ$. In that case, producing line l and mfurther will meet in the side of $\angle 1$ and $\angle 2$ which is less than 180^o



If $\angle 1 + \angle 2 = 180^\circ$, then $\angle 3 + \angle 4 = 180^\circ$

In that case, the lines l and m neither meet at the side of $\angle 1$ and $\angle 2$ nor at the side of $\angle 3$ and $\angle 4$ implying that the lines l and m will never intersect each other.Therefore, the lines are parallel.

31.



Given that $\angle DCA = \angle ECB$

$\Rightarrow \angle DCA + \angle ECD = \angle ECB + \angle ECD$

$\Rightarrow \angle ECA = \angle DCB$ … (i)

Now in Δ DBC and Δ EAC

$\angle DCB = \angle ECA$ [from (i)]

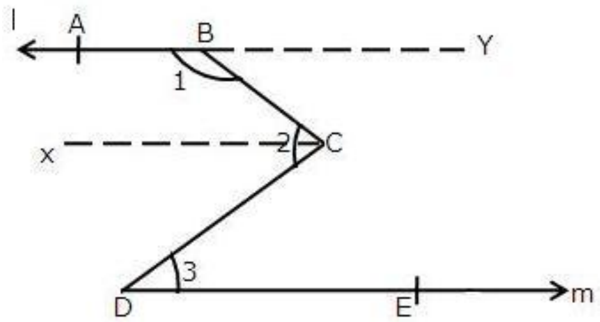
BC = AC (Given)

$\angle DBC = \angle EAC$ (Given)

$\Delta DBC \cong \Delta EAC$ (ASA Congruence)

Therefore, BD =AE (CPCT)

32.



Given: l||m

To prove: $\angle 1 + \angle 2 - \angle 3 = 180^o$

Construction: Draw XC||AB and extend AB to Y.

Proof:

$\angle BCX = 180^o - \angle 1$ (sum of int. $\angle_s$ on same side of transversal is 180^o)

$\angle XCD = \angle 3$ (Alternate int. $\angle_s$)

Now, $\angle 2 = \angle BCX + \angle XCD$

Or, $\angle 2 = (180^o - \angle 1) + \angle 3$

$\therefore \angle 1 + \angle 2 - \angle 3 = 180^o$

33. Given: AC = AE, AB = AD and $\angle BAD = \angle EAC$

To prove: BC = DE

Proof: $\angle BAD = \angle EAC$ (given)

$\Rightarrow \angle BAD + \angle DAC = \angle EAC + \angle DAC$

$\Rightarrow \angle BAC = \angle DAE$

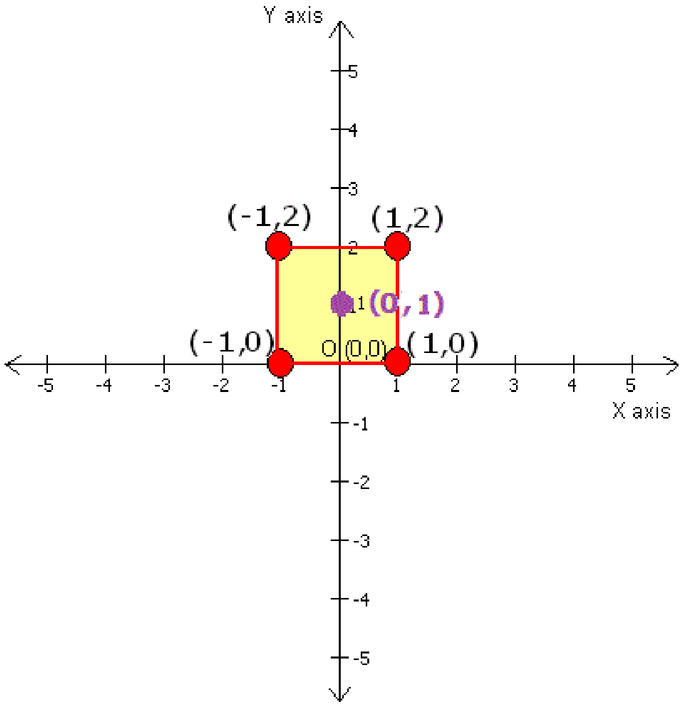
Now ΔABC and ΔADE

$$\left.\begin{matrix} AB = AD \\ \angle BAC = \angle DAE \\ AC = AE \end{matrix}\right|$$

By SAS rule $\Delta ABC \cong \Delta ADE$

$\Rightarrow BC = DE$

34. The points (-1,0) (1,0) (1,2) (-1,2) can be plotted on a graph as follows:



Shape of the lawn is square. Tree is to be planted at the centre of the lawn, so it must be planted at the point of intersection of its diagonals, i.e., at (0,1). Value indicated: Protection of environment