Mathematics
Class X
Sample Paper - 1

Time: **2½ hrs** Total Marks: 100

Instructions: The answers of the questions of Part I are to be given at the beginning of the answer script mentioning the question numbers in the serial order. Necessary calculations and drawing, if any, must be given in the right hand side by drawing margin at the first few pages in the answer script. Tables or calculators cannot be used. If necessary, take $\pi = \frac{22}{7}$. Graph paper will be supplied as required.

PART I

The answers of all questions of this paper are to be written at the beginning of the answer script.

1. Answer all questions:

(i) Find the ratio of 100 cm to 10 m. [1]

(ii) The LCM of a^6 and a^{2+k} is
(a) a^2 (b) a^{2+k} (c) a^{6+k} (d) a^{8+k} [1]

(iii)If the digits in the unit's place and ten's place of an integer be y and x respectively, then the integer is:
(a) xy (b) x + y (c) 10x + y (d) 10y + x [1]

(iv) What is the distance between the points (-6, 7) and (-1,-5)?
(a) 15 units (b) 12 units (c) 13 units (d) 10 units [1]

(v)AD is a median of a triangle ABC and G is its centroid. Find the ratio AD: AG. [1]

(vi)Express 1 radian in terms of right angle. [1]

2. Answer all questions:

(i)If $a \propto b$, $b \propto \frac{1}{c}$ and $c \propto d$ then find the variational relationship between a and d. [2]

(ii) If a: b = 5: 3, find $\frac{5a-3b}{5a+3b}$. [2]

(iii) If $3x - 5 \leq x - 2$, then find the greatest value of 10x. [2]

(iv) Internal bisectors of angles $\angle B$ and $\angle C$ of ΔABC meet at O. If $\angle A = 80^o$, find the value of $\angle BOC$. [2]

(v) The mid points of AB and AC of a triangle ABC are respectively X and Y. If BC + XY = 12 units, then BC - XY = ? [2]

(vi) Find the volume of a cube whose total surface area is 1014 m^2. [2]

(vii) If $\sin 4\theta = \cos 5\theta$ $(0^o < \theta < 90^o)$, find the value of θ. [2]

PART – II

3. Answer any TWO questions (Algebraic methods may be applied): **$2 \times 5 = 10$**

(a) Two vessels contain syrup and water in ratios 4: 5 and 5: 1 respectively. In what ratio should we take the mixtures of syrup and water from the two vessels, so that the ratio of syrup and water in the new mixture may become 5: 4?

(b) A shopkeeper sold a stove for Rs. 1692 and incurred a loss of 20% For how much should he has sold the stove to gain 20%.

(c) The simple interest and compound interest of a certain sum of money for two years are Rs. 400 and Rs. 410 respectively. Find the sum of money and the rate of interest.

(d) The rate of increase of population of a town at the end of a year is 2% of the population at the beginning of the year. If the present population of the town be 2000000, what will be the population after 3 years?

4. Resolve into factors (any ONE). [4]

(a) $6(x - y)^2 - x + y - 15$

(b) $a^4 + a^2 + 1$

OR

Find the L.C.M.:

$3x^2 - 24y^2$, $4(x^2 + 2xy - 3y^2)$.

5. Solve (any ONE): [4]

(a) $\frac{x}{3} - \frac{y}{4} = 0$, $5y - 7x + 2 = 0$

(b) $\frac{x}{x+1} + \frac{x+1}{x} = 2\frac{1}{12}$

6. **Answer any One question:** [4]

(a) The area of rectangle gets reduced by 9 m^2, if length is reduced by 5 m and breadth is increased by 3 m. If we increase the length by 3 m and breadth by 2 m, the area is increased by 67 m^2. Find the length and breadth of the rectangle.

(b) The difference of squares of two natural numbers is 45. The square of the smaller number is four times the larger number. Find the numbers.

7. **Draw the graphs of the inequations and indicate the solution region (any One):** [4]

(a) $x + y \leq 15; x \geq 2 ; y \geq -3$

(b) Show the solutions of the following system of inequations through graphs:
$x \geq 0; y \geq 0; x + y \leq 1.$

8. **Answer any One question:** [3]

(a) If (a - b): (a + b) = 1: 11, find the ratio (5a + 4b + 15): (5a - 4b + 3).

(b) If a: b = b: c, prove that a: c = $(a^2 + b^2) : (b^2 + c^2)$

9. **Answer any One question:** [3]

(a) If 5 farmers can harvest just cultivated in 10 bighas of land in 12 days, then how many farmers can harvest jute cultivated in 18 bighas in 9 days? Apply the theory of variation .

(b) The volume of a sphere varies directly as cube of its radius. Three solid spheres of radii 3 cm, 4 cm and 5 cm are melted and a new solid sphere is made. If no volume is lost for melting, find the diameter of the new sphere.

10. **Answer any One question:** [3]

(a) If $x = \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$ and $y = \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$ find the value of $x^2 + xy + y^2$.

(b) Show that $\frac{1}{(3-\sqrt{8})} - \frac{1}{(\sqrt{8}-\sqrt{7})} + \frac{1}{(\sqrt{7}-\sqrt{6})} - \frac{1}{(\sqrt{6}-\sqrt{5})} + \frac{1}{(\sqrt{5}-2)} = 5.$

11. **Answer any Two questions:** $(2 \times 5 = 10)$

(a) Prove that the angle subtended by an arc at the center is double the angle subtended by it at any point on the remaining part of the circle.

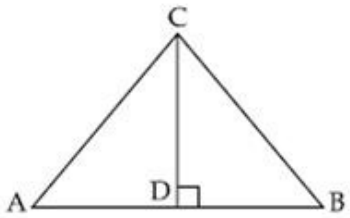
(b) Prove that tangent at any point of a circle is perpendicular to the radius through the point of contact.

(c) If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio. Prove it.

12. Answer any One question: [3]

(a) ABCD is a cyclic quadrilateral. The bisectors of $\angle DAB$ and $\angle BCD$ intersect the circle at the points X and Y respectively. Prove that XY is a diameter of that circle.

(b) In figure, $\angle ACB = 90^o$ and $CD \perp AB$. Prove that $\frac{BC^2}{AC^2} = \frac{BD}{AD}$.



13. Answer any One question: [5]

(a) Construct circumcircle of the ΔABC, given AB = 5 cm, BC = 7 cm and $\angle ABC = 50^o$.

(b) Draw a square equal in area to a rectangle of length 9 cm and breadth 4 cm.

14. Answer any One question: [3]

(a) The base of a pyramid is a rectangle of length 12 metre and breadth 9 metre. Each of the slant edge of the pyramid is of length 8.5 metre. Find the volume of the pyramid.

(b) The radius of a spherical balloon increases from 7 cm to 14 cm as air is being pumped into it. Find the ratio of surface areas of the balloon in the two cases.

15. Answer any One question: [4]

(a) Outer and inner diameter of a hollow cylinder of height 25 cm, made of iron are 14 cm and 10 cm respectively. The cylinder is melted and a solid right circular cone of height half of that of the cylinder, is made. What is the diameter of the cone?

(b) The diameter of a sphere is decreased by 25%. By what per cent does its curved surface area decrease?

16. Answer any Two questions: $(2 \times 3 = 6)$

(a) Two angles of triangle are 75^o and 45^o. Find the value of the third angle in circular measure.

(b) If $\cot\theta = \frac{x}{y}$, prove that

$$\frac{x\cos\theta - y\sin\theta}{x\cos\theta + y\sin\theta} = \frac{x^2 - y^2}{x^2 + y^2}$$

(c) Find the value of $\theta\left(0^o \le \theta \le 90^o\right)$, when $\sin^2\theta - 3\sin\theta + 2 = 0$

(d) If $\cos 43^o = \frac{x}{\sqrt{x^2 + y^2}}$ what is the value of $\tan 47^o$?

17. Answer any One question: [5]

(a) An aeroplane at an altitude of 200 m observes the angles of depression of two opposite points on two banks of the river to be 45^o and 60^o. Find in metres, the width of the river. (use $\sqrt{3}$ = 1.732)

(b) From a window, 60 m high above the ground, of a house in a street, the angles of elevation and depression of the top and foot of another house on the opposite side of the street are 60^o and 45^o respectively. Show that the height of the opposite house is 60 $(1 + \sqrt{3})$ metres.