ICSE Board
Class X Mathematics
Sample Paper 6 – Solution

SECTION – A (40 Marks)

Q. 1

a)

$A^2 = 9A + mI$

$\Rightarrow A^2 - 9A = mI \quad ...(1)$

Now, $A^2 = AA$

$$= \begin{bmatrix} 2 & 0 \\ -1 & 7 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ -1 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 \\ -9 & 49 \end{bmatrix}$$

Substituting A^2 in (1), we have

$A^2 - 9A = mI$

$$\Rightarrow \begin{bmatrix} 4 & 0 \\ -9 & 49 \end{bmatrix} - 9\begin{bmatrix} 2 & 0 \\ -1 & 7 \end{bmatrix} = m\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & 0 \\ -9 & 49 \end{bmatrix} - \begin{bmatrix} 18 & 0 \\ -9 & 63 \end{bmatrix} = \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -14 & 0 \\ 0 & -14 \end{bmatrix} = \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}$$

$\Rightarrow m = -14$

b)

Let the account be held for n months.
Since
P = Rs. 600 and Rate = 10%

$$\therefore \text{Interest} = P \times \frac{n(n+1)}{2\times 12} \times \frac{r}{100}$$

$$= 600 \times \frac{n(n+1)}{2\times 12} \times \frac{10}{100}$$

$$= \text{Rs.} \frac{5n(n+1)}{2}$$

Since money deposited + Interest = Maturity value

$$600 \times n + \frac{5n(n+1)}{2} = 24{,}930$$

$$5n^2 + 5n + 1200n = 49{,}860$$

$$n^2 + 241n = 9972$$

$$n(n+277) - 36(n+277) = 0$$

$$(n+277)(n-36) = 0$$

$$n = -277 \text{...months can't be negative}$$

$$n = 36$$

The time for which the account was held = 36 months = 3 years

c)

Given that the die has 6 faces marked by the given numbers as below:

1	2	3	–1	–2	–3

When a die is rolled, total number of possible outcomes $= 6$

(i) For getting a positive integer, the favourable outcomes are: 1, 2, 3

$\Rightarrow$ Number of favourable outcomes $= 3$

$\Rightarrow$ Required probability $= \frac{3}{6} = \frac{1}{2}$

(ii) For getting an integer greater than -3, the favourable outcomes are: $-2, -1, 1, 2, 3$

$\Rightarrow$ Number of favourable outcomes $= 5$

$\Rightarrow$ Required probability $= \frac{5}{6}$

(iii) For getting a smallest integer, the favourable outcomes are: -3

$\Rightarrow$ Number of favourable outcomes $= 1$

$\Rightarrow$ Required probability $= \frac{1}{6}$

Q. 2

a)

Let the radius of a solid right circular cylinder be $r = 100$ cm

And, let the height of a solid right circular cylinder be $h = 100$ cm

$\therefore$ Volume (original) of a solid right circular cylinder $= \pi r^2 h$

$$= \pi \times (100)^2 \times 100$$

$$= 1000000\pi \text{ cm}^3$$

New radius $= r' = 120$ cm

New height $= h' = 80$ cm

$\therefore$ Volume (New) of a solid right circular cylinder $= \pi r'^2 h'$

$$= \pi \times (120)^2 \times 80$$

$$= 1152000\pi \text{ cm}^3$$

$\therefore$ Increase in Volume = New Volume − Original Volume

$$= 1152000\,\pi \text{ cm}^3 - 1000000\pi \text{ cm}^3$$

$$= 152000\,\pi \text{ cm}^3$$

Thus, Percentage change in volume $= \dfrac{\text{Increase in Volume}}{\text{Original Volume}} \times 100\%$

$$= \frac{152000\,\pi \text{ cm}^3}{1000000\,\pi \text{ cm}^3} \times 100\%$$

$$= 15.2\%$$

b)

Total amount of prize $= S_n =$ Rs. 700

Let the value of the first prize be Rs. a.

Number of prizes $= n = 7$

Let the value of first prize be Rs. a.

Depreciation in next prize $= -$Rs. 20

We have,

$$S_n = \frac{7}{2}\left[2a + (n-1)d\right]$$

$$\Rightarrow 700 = \frac{7}{2}\left[2a + 6(-20)\right]$$

$$\Rightarrow 700 = \frac{7}{2}\left[2a - 120\right]$$

$$\Rightarrow 1400 = 14a - 840$$

$$\Rightarrow 14a = 2240$$

$$\Rightarrow a = 160$$

$\Rightarrow$ Value of 1^{st} prize $=$ Rs. 160

Value of 2^{nd} prize $=$ Rs. $(160 - 20) =$ Rs. 140

Value of 3^{rd} prize $=$ Rs. $(140 - 20) =$ Rs. 120

Value of 4^{th} prize $=$ Rs. $(120 - 20) =$ Rs. 100

Value of 5^{th} prize $=$ Rs. $(100 - 20) =$ Rs. 80

Value of 6^{th} prize $=$ Rs. $(80 - 20) =$ Rs. 60

Value of 7^{th} prize $=$ Rs. $(60 - 20) =$ Rs. 40

c)

(i) In ΔABD,

$\angle DAB + \angle ABD + \angle ADB = 180^\circ$

$\Rightarrow 65^\circ + 70^\circ + \angle ADB = 180^\circ$

$\Rightarrow 135^\circ + \angle ADB = 180^\circ$

$\Rightarrow \angle ADB = 180^\circ - 135^\circ = 45^\circ$

Now, $\angle ADC = \angle ADB + \angle BDC = 45^\circ + 45^\circ = 90^\circ$

Since $\angle ADC$ is the angle of semicircle, so AC is a diameter of the circle.

(ii) $\angle ACB = \angle ADB$ ....(angles in the same segment of a circle)

$\Rightarrow \angle ACB = 45^\circ$

(iii) $\angle ABC = \angle ABD + \angle DBC$ ....(angles in the same segment of a circle)

$\Rightarrow \angle ABC = 90^\circ$

$\Rightarrow \angle ABD + \angle DBC = 90^\circ$

$\Rightarrow 70^\circ + \angle DBC = 90^\circ$

$\Rightarrow \angle DBC = 20^\circ$

Q. 3

a)

Since $(x-2)$ is a factor of polynomial $2x^3+ax^2+bx-14$, we have

$2(2)^3+a(2)^2+b(2)-14=0$

$\Rightarrow 16+4a+2b-14=0$

$\Rightarrow 4a+2b+2=0$

$\Rightarrow 2a+b+1=0$

$\Rightarrow 2a+b=-1 \quad ....(i)$

On dividing by $(x-3)$, the polynomial $2x^3+ax^2+bx-14$ leaves remainder 52,

$\Rightarrow 2(3)^3+a(3)^2+b(3)-14=52$

$\Rightarrow 54+9a+3b-14=52$

$\Rightarrow 9a+3b+40=52$

$\Rightarrow 9a+3b=12$

$\Rightarrow 3a+b=4 \quad ....(ii)$

Subtracting (i) from (ii), we get

$a=5$

Substituting $a=5$ in (i), we get

$2\times 5+b=-1$

$\Rightarrow 10+b=-1$

$\Rightarrow b=-11$

Hence, $a=5$ and $b=-11$.

b)

$$\text{L.H.S.}=(\sin\theta+\cos\theta)(\tan\theta+\cot\theta)$$

$$=(\sin\theta+\cos\theta)\left(\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}\right)$$

$$=(\sin\theta+\cos\theta)\left(\frac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}\right)$$

$$=\frac{\sin\theta+\cos\theta}{\cos\theta\sin\theta}$$

$$=\frac{\sin\theta}{\cos\theta\sin\theta}+\frac{\cos\theta}{\cos\theta\sin\theta}$$

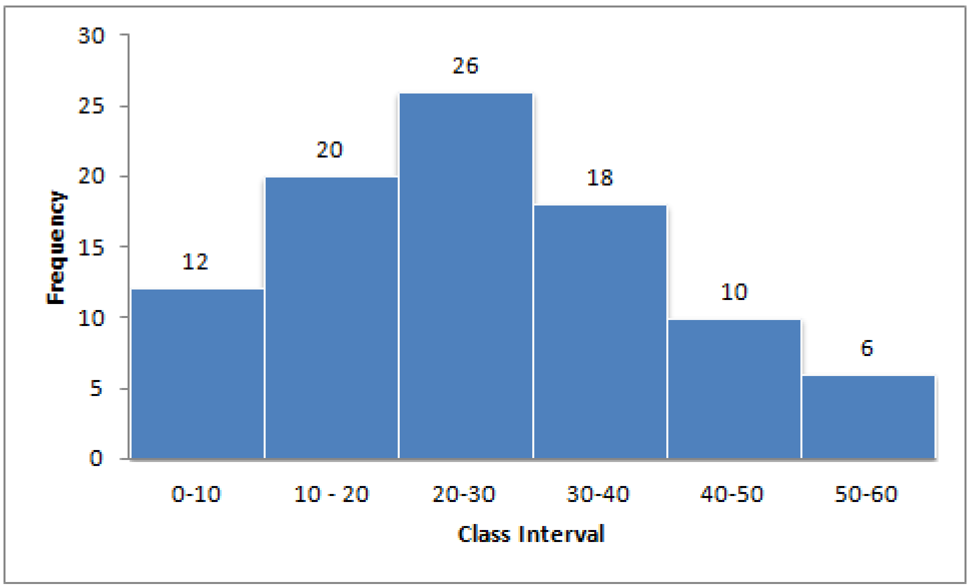
$$=\frac{1}{\cos\theta}+\frac{1}{\sin\theta}$$

$$=\sec\theta+\operatorname{cosec}\theta$$

$$=\text{R.H.S.}$$

c)

Class Interval	Frequency
0–10	12
10–20	20
20–30	26
30–40	18
40–50	10
50–60	06



Steps of construction:

(a) Taking suitable scales, mark class intervals on the x-axis and frequency on the y-axis.

(b)Construct rectangles with class intervals as bases and corresponding frequencies as heights.

Q. 4

a)

$$-3(x-7) \geq 15-7x > \frac{x+1}{3}, x \in R$$

$$\Rightarrow -3(x-7) \geq 15-7x \text{ and } 15-7x > \frac{x+1}{3}$$

$$\Rightarrow -3x + 21 \geq 15-7x \text{ and } 45-21x > x+1$$

$$\Rightarrow -3x + 7x \geq 15-21 \text{ and } 45-1 > x+21x$$

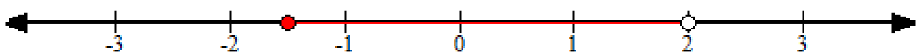
$$\Rightarrow 4x \geq -6 \text{ and } 44 > 22x$$

$$\Rightarrow x \geq \frac{-3}{2} \text{ and } 2 > x$$

$$\Rightarrow x \geq -1.5 \text{ and } 2 > x$$

$\therefore$ The solution set is $\{x : x \in R, -1.5 \leq x < 2\}$.

The solution set is represented on the number line as follows:



b)

(i) The equation of the line passing through the points $P(-1,4)$ and $Q(5,-2)$ is

$$y-4=\frac{-2-4}{5-(-1)}[x-(-1)]$$

i.e. $y-4=\frac{-6}{6}(x+1)$

i.e. $y-4=-1(x+1)$

i.e. $y-4=-x-1$

i.e. $x+y=3$

(ii) The line $x+y=3$ cuts x-axis at point A. Hence, its y-coordinate is 0.

And, x-coordinate is given by

$x+0=3 \Rightarrow x=3$

So, the coordinates of A are (3,0).

The line $x+y=3$ cuts y-axis at point B. Hence, its x-coordinate is 0.

And, y-coordinate is given by

$0+y=3 \Rightarrow y=3$

So, the coordinates of B are (0,3).

(iii) Since M is the mid-point of line segment AB,

So, coordinates of $M=\left(\frac{3+0}{2},\frac{0+3}{2}\right)=\left(\frac{3}{2},\frac{3}{2}\right)$

c)

$x^2 - 3(x + 3) = 0$

$\Rightarrow x^2 - 3x - 9 = 0$

Comparing with $ax^2 + bx + c$, we get

$a = 1, b = -3, c = -9$

$$\text{Now, } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-9)}}{2(1)}$$

$$\Rightarrow x = \frac{3 \pm \sqrt{9 + 36}}{2}$$

$$\Rightarrow x = \frac{3 \pm \sqrt{45}}{2}$$

$$\Rightarrow x = \frac{3 \pm \sqrt{9 \times 5}}{2}$$

$$\Rightarrow x = \frac{3 \pm 3\sqrt{5}}{2}$$

$$\Rightarrow x = \frac{3 + 3\sqrt{5}}{2} \text{ or } x = \frac{3 - 3\sqrt{5}}{2}$$

$$\Rightarrow x = \frac{3 + 3 \times 2.236}{2} \text{ or } x = \frac{3 - 3 \times 2.236}{2}$$

$$\Rightarrow x = \frac{3 + 6.708}{2} \text{ or } x = \frac{3 - 6.708}{2}$$

$$\Rightarrow x = \frac{9.708}{2} \text{ or } x = \frac{-3.708}{2}$$

$\Rightarrow x = 4.854$ or $x = -1.854$

$\Rightarrow x = 4.9$ or $x = -1.9$

SECTION – B (40 Marks)

Q. 5

a)

Given,

First term, $a = 27$

8^{th} term $= ar^7 = \frac{1}{81}$

$n = 10$

Now,

$$\frac{ar^7}{a} = \frac{1/81}{27}$$

$$\Rightarrow r^7 = \frac{1}{2187}$$

$$\Rightarrow r^7 = \left(\frac{1}{3}\right)^7$$

$$\Rightarrow r = \frac{1}{3}\ (r < 1)$$

$$\therefore S_n = \frac{a(1-r^n)}{1-r}$$

$$\Rightarrow S_{10} = \frac{27\left(1-\left(\frac{1}{3}\right)^{10}\right)}{1-\frac{1}{3}}$$

$$= \frac{27\left(1-\frac{1}{3^{10}}\right)}{\frac{2}{3}}$$

$$= \frac{81}{2}\left(1-\frac{1}{3^{10}}\right)$$

$$= \frac{81}{2}(1-3^{-10})$$

b)

$\text{Investment} = \text{Rs. } 131040$

$\text{N.V. of 1 share} = \text{Rs. } 100$

$\text{Discount} = 9\% \text{ of Rs. } 100 = \text{Rs. } 9$

$\therefore \text{M.V. of 1 share} = \text{Rs. } 100 - \text{Rs. } 9 = \text{Rs. } 91$

$\therefore \text{Number of shares purchased} = \dfrac{\text{Investment}}{\text{M.V. of 1 share}} = \dfrac{131040}{91} = 1440$

$\text{Number of shares worth Rs. } 72000 = \dfrac{72000}{100} = 720$

$\therefore$ Mrs. Kulkarni sells 720 shares at a premium of 10%

$\text{M.V. of 1 share} = \text{Rs.}100 + \text{Rs. } 10 = \text{Rs. } 110$

$\therefore \text{Selling price of 720 shares} = 720 \times \text{Rs. } 110 = \text{Rs. } 79200$

$\text{Number of remaining shares} = 1440 - 720 = 720$

She sells 720 shares at a discount of 5%

$\text{M.V. of 1 share} = \text{Rs. } 100 - \text{Rs.}5 = \text{Rs. } 95$

$\therefore \text{Selling price of 720 shares} = 720 \times \text{Rs. } 95 = \text{Rs. } 68400$

$\therefore \text{Total selling price} = \text{Rs. } (79200 + 68400) = \text{Rs. } 147600$

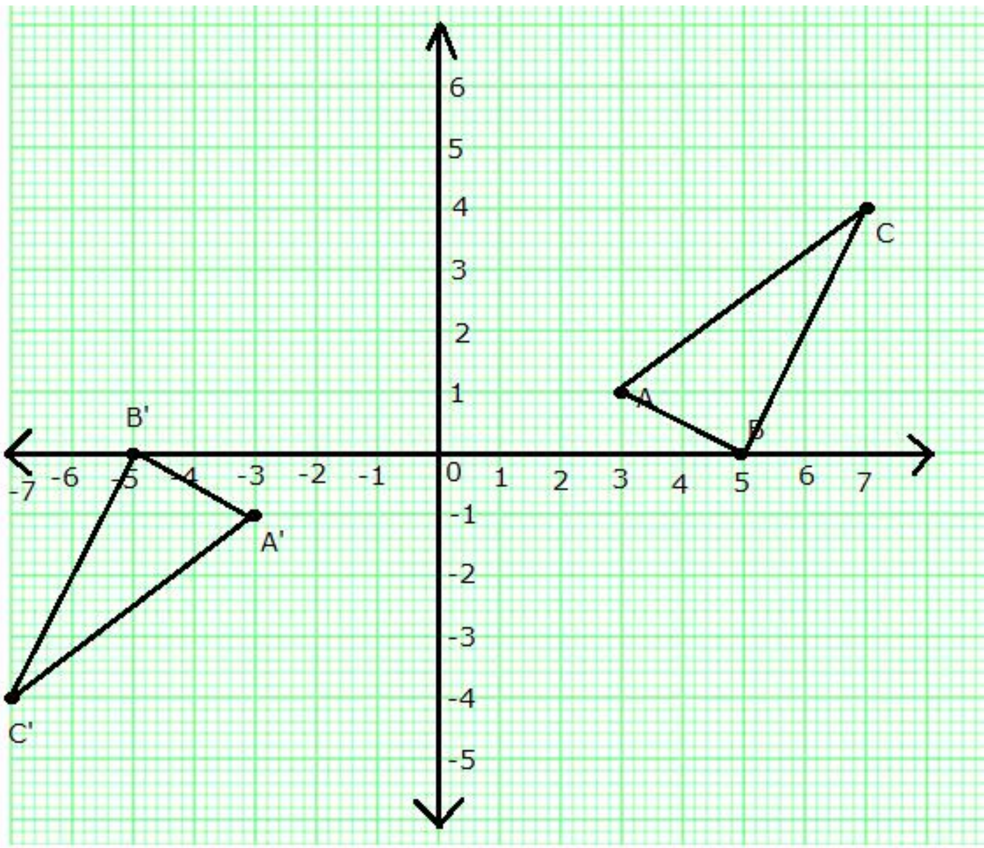
$\therefore \text{Total gain} = \text{Total selling price} - \text{Total investment}$

$= \text{Rs. } (147600 - 131040)$

$= \text{Rs. } 16560$

c)

The graph shows triangle ABC and triangle A'B'C' which is obtained when ABC is reflected in the origin.



Q. 6

a)

Let $\frac{x}{a}=\frac{y}{b}=\frac{z}{c}=k$

$\Rightarrow x=ak, y=bk, z=ck$

$$\text{L.H.S.}=\frac{x^3}{a^3}+\frac{y^3}{b^3}+\frac{z^3}{c^3}$$

$$=\frac{(ak)^3}{a^3}+\frac{(bk)^3}{b^3}+\frac{(ck)^3}{c^3}$$

$$=\frac{a^3k^3}{a^3}+\frac{b^3k^3}{b^3}+\frac{c^3k^3}{c^3}$$

$$=k^3+k^3+k^3$$

$$=3k^3$$

$$\text{R.H.S.}=\frac{3xyz}{abc}$$

$$=\frac{3(ak)(bk)(ck)}{abc}$$

$$=3k^3$$

$\Rightarrow$ L.H.S. = R.H.S.

i.e. $\frac{x^3}{a^3}+\frac{y^3}{b^3}+\frac{z^3}{c^3}=\frac{3xyz}{abc}$

b)

$$\begin{bmatrix} a+3b & 3c+d \\ 2a-b & c-2d \end{bmatrix} = \begin{bmatrix} 5 & 8 \\ 3 & 5 \end{bmatrix}$$

$a+3b=5 \quad -(1)$

$2a-b=3 \quad - \ 2$

$\Rightarrow b=2a-3$

putting this value of b in 1

$a+6a-9=5$

$\Rightarrow 7a=14$

$\Rightarrow a=2$

from(1)

$2+3b=5$

$\Rightarrow 3b=3$

$\Rightarrow b=1$

$3c+d=8 \quad - \ 3$

$c-2d=5 \quad - \ 4$

$c=5+2d$

putting the value of c in 3

$15+6d+d=8$

$\Rightarrow 7d=-7$

$\Rightarrow d=-1$

from 4

$c+2=5$

$\Rightarrow c=3$

c)

$$\text{L.H.S.} = \frac{\tan A}{(1-\cot A)} + \frac{\cot A}{(1-\tan A)}$$

$$= \frac{\tan A}{1-\frac{1}{\tan A}} + \frac{\frac{1}{\tan A}}{1-\tan A}$$

$$= \frac{\tan A}{\frac{\tan A-1}{\tan A}} + \frac{1}{\tan A(1-\tan A)}$$

$$= \frac{\tan^2 A}{\tan A-1} - \frac{1}{\tan A(\tan A-1)}$$

$$= \frac{\tan^3 A-1}{\tan A(\tan A-1)}$$

$$= \frac{(\tan A-1)(\tan^2 A+\tan A+1)}{\tan A(\tan A-1)} \quad ....[a^3-b^3=(a-b)(a^2+ab+b^2)]$$

$$= \frac{\tan^2 A+\tan A+1}{\tan A}$$

$$= \frac{\tan^2 A}{\tan A} + \frac{\tan A}{\tan A} + \frac{1}{\tan A}$$

$$= \tan A + 1 + \cot A$$

$$= 1 + \tan A + \cot A$$

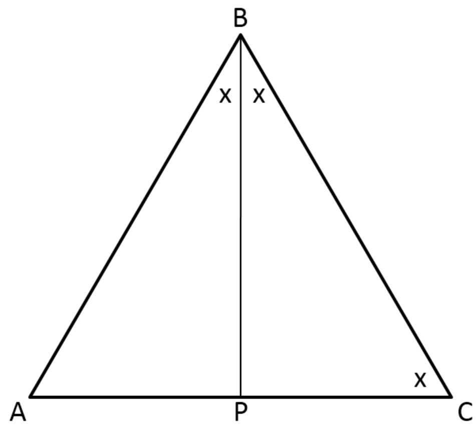
$$= \text{R.H.S.}$$

Q.7

a)

$$\frac{x}{3}+\frac{3}{6-x}=\frac{2(6+x)}{15}$$
$$\Rightarrow \frac{x(6-x)+3\times 3}{3(6-x)}=\frac{12+2x}{15}$$
$$\Rightarrow \frac{x(6-x)+3\times 3}{6-x}=\frac{12+2x}{5}$$
$$\Rightarrow \frac{6x-x^2+9}{6-x}=\frac{12+2x}{5}$$
$$\Rightarrow 30x-5x^2+45=72+12x-12x-2x^2$$
$$\Rightarrow 30x-5x^2+45=72-2x^2$$
$$\Rightarrow 3x^2-30x+27=0$$
$$\Rightarrow x^2-10x+9=0$$
$$\Rightarrow x^2-9x-x+9=0$$
$$\Rightarrow x(x-9)-1(x-9)=0$$
$$\Rightarrow (x-9)(x-1)=0$$
$$\Rightarrow x-9=0 \text{ or } x-1=0$$
$$\Rightarrow x=9 \text{ or } x=1$$

b)



In ΔABC,

$\angle ABC = 2\angle ACB = 2x$

Let $\angle ACB = x$

$\Rightarrow \angle ABC = 2\angle ACB = 2x$

Given BP is bisector of $\angle ABC$.

Hence $\angle ABP = \angle PBC = x$.

Using the angle bisector theorem,

that is, the bisector of an angle divides the side opposite to it

in the ratio of other two sides.

Hence, $CB : BA = CP : PA$.

Consider ΔABC and ΔAPB,

$\angle APB = 2x$ ....[Exterior angle property]

$\angle ABC = \angle APB$

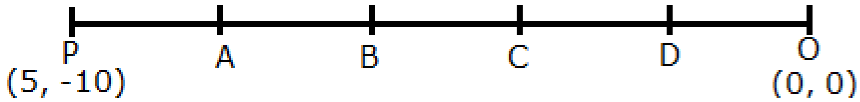
$\angle BCP = \angle ABP$ ....[Given]

$\therefore \Delta ABC \sim \Delta APB$ [AA criterion for Similarity]

$\frac{CA}{AB} = \frac{BC}{BP}$ ....(Corresponding sides of similar triangles are proportional.)

$\Rightarrow AB \times BC = BP \times CA$

c)



Point A divides PO in the ratio 1:4.
Co-ordinates of point A are

$$\left(\frac{1\times0+4\times5}{1+4},\frac{1\times0+4\times(-10)}{1+4}\right)=\left(\frac{20}{5},\frac{-40}{5}\right)=(4,-8)$$

Point B divides PO in the ratio 2:3.
Co-ordinates of point B are

$$\left(\frac{2\times0+3\times5}{2+3},\frac{2\times0+3\times(-10)}{2+3}\right)=\left(\frac{15}{5},\frac{-30}{5}\right)=(3,-6)$$

Point C divides PO in the ratio 3:2.
Co-ordinates of point C are

$$\left(\frac{3\times0+2\times5}{3+2},\frac{3\times0+2\times(-10)}{3+2}\right)=\left(\frac{10}{5},\frac{-20}{5}\right)=(2,-4)$$

Point D divides PO in the ratio 4:1.
Co-ordinates of point D are

$$\left(\frac{4\times0+1\times5}{4+1},\frac{4\times0+1\times(-10)}{4+1}\right)=\left(\frac{5}{5},\frac{-10}{5}\right)=(1,-2)$$

Q.8

a)

Work done by A in one day = $\frac{1}{x}$

Work done by B in one day = $\frac{1}{x+16}$

Together A and B can do the work in 15 days. Therefore, we have

$$\frac{1}{x}+\frac{1}{x+16}=\frac{1}{15}$$
$$\frac{x+16+x}{x(x+16)}=\frac{1}{15}$$
$$\frac{2x+16}{x^2+16x}=\frac{1}{15}$$
$$30x+240=x^2+16x$$
$$x^2-14x-240=0$$
$$(x-24)(x+10)=0$$
$$x=24,-10$$

Since x cannot be negative.

Thus, x = 24.

b)

We have,

x	f	fx
0	46	0
50	f_1	$50f_1$
100	f_2	$100f_2$
150	25	3750
200	10	2000
250	5	1250
	$\sum f = 86 + f_1 + f_2$	$\sum fx = 7000 + 50f_1 + 100f_2$

Given, $\sum f = 200$

$\Rightarrow 86 + f_1 + f_2 = 200$

$\Rightarrow f_1 + f_2 = 114$ ....(i)

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\Rightarrow 73 = \frac{7000 + 50f_1 + 100f_2}{200}$$

$\Rightarrow 7000 + 50f_1 + 100f_2 = 14600$

$\Rightarrow 50f_1 + 100f_2 = 7600$

$\Rightarrow f_1 + 2f_2 = 152$ ....(ii)

Subtracting (ii) from (i), we get

$f_2 = 38$

$\Rightarrow f_1 = 114 - 38 = 76$

Hence, $f_1 = 76$ and $f_2 = 38$

c)

Steps for construction:

(i) Draw AB = 5 cm using a ruler.

(ii) With A as the centre, cut an arc of 3 cm on AB to obtain C.

(iii) With A as the centre and radius 2.5 cm, draw an arc above AB.

(iv) With the same radius and C as the centre, draw an arc to cut the previous arc and mark the intersection as O.

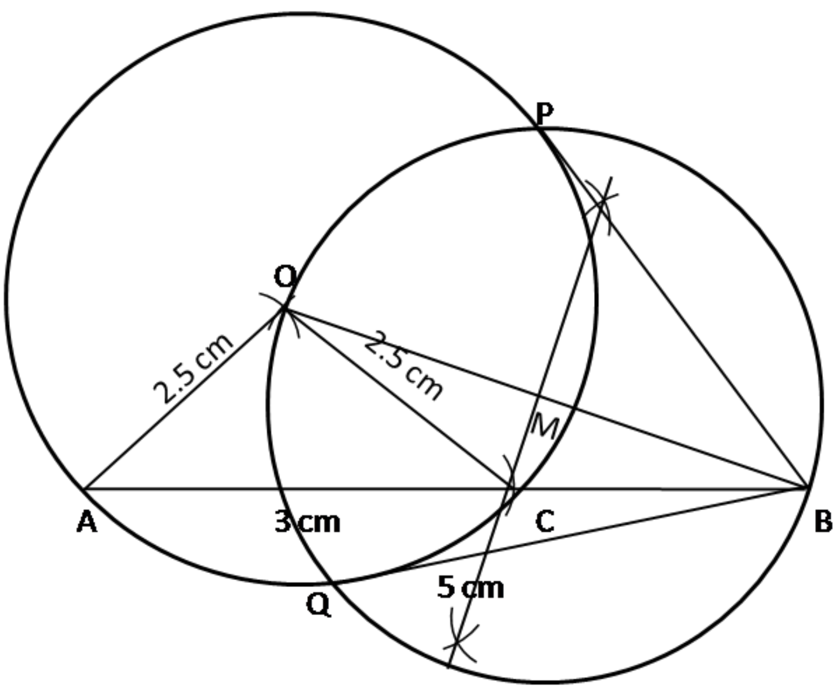
(v) With O as the centre and radius 2.5 cm, draw a circle so that points A and C lie on the circle formed.

(vi) Join OB.

(vii) Draw the perpendicular bisector of OB to obtain the mid-point of OB, M.

(viii) With M as the centre and radius equal to OM, draw a circle to cut the previous circle at points P and Q.

(ix) Join PB and QB. PB and QB are the required tangents to the given circle from the exterior point B.



QB = PB = 3 cm

That is, the length of each tangent is 3 cm.

Q. 9

a)

Given that the recurring deposit per month = Rs 900, period = 3 years = 36 months, R = 8%

Money deposited = Monthly value x No. of months

= 900 x 36 = Rs. 32,400 ... (1)

Total principal for 1 month $= \text{Rs}\dfrac{900\times(36)(36+1)}{2} = \text{Rs}5,99,400$

Interest = $\text{Rs}\dfrac{8\times5,99,400}{12\times100} = \text{Rs}3,996$ ... (2)

Hence, the maturity amount = (1) + (2)

= Rs. (32,400 + 3,996)

Hence, the maturity amount = Rs. 36,396

And interest = Rs. 3,996

b)

Height of the conical vessel, h = 24 cm
Radius of the conical vessel, r = 5 cm

$$=\frac{3}{4}\times\frac{1}{3}\times\pi r^2 h$$
$$=\frac{3}{4}\times\frac{1}{3}\times\pi\times25\times24$$
$$=150\pi$$

Let h be the height of the cylindrical vessel which is filled by water of the conical vessel.
Radius of the cylindrical vessel = 10 cm
Volume of the cylindrical vessel = volume of water
$\pi(10)^2 h = 150\pi$
$h = 150\pi \div 100\pi$
h = 1.5 cm
Thus, the height of the cylindrical vessel is 1.5 cm.

c)

(i) Scale factor $k = \frac{1}{300}$

Length of the model of the ship = k × Length of the ship

$$\Rightarrow 2 = \frac{1}{300}\times \text{Length of the ship}$$
$$\Rightarrow \text{Length of the ship} = 600 \text{ m}$$

(ii) Area of the deck of the model $= k^2 \times$ Area of the deck of the ship

$$\Rightarrow \text{Area of the deck of the model} = \left(\frac{1}{300}\right)^2\times180{,}000$$
$$=\frac{1}{90000}\times180{,}000$$
$$=2 \text{ m}^2$$

(iii) Volume of the model $= k^3 \times$ Volume of the ship

$$\Rightarrow 6.5 = \left(\frac{1}{300}\right)^3\times \text{Volume of the ship}$$
$$\Rightarrow \text{Volume of the ship} = 6.5\times27000000 = 17{,}55{,}00{,}000 \text{ m}^3$$

Q.10

a)

Since RS is drawn parallel to the tangent PQ,

$\angle SRQ = \angle PQR$

Also, PQ = PR

$\Rightarrow \angle PQR = \angle PRQ$

In ΔPQR,

$\angle PQR + \angle PRQ + \angle QPR = 180°$

$\Rightarrow \angle PQR + \angle PQR + 30° = 180°$

$\Rightarrow 2\angle PQR = 150°$

$\Rightarrow \angle PQR = 75°$

$\Rightarrow \angle SRQ = \angle PQR = 75°$ (alternate angles)

Also, $\angle RSQ = \angle RQP = 75°$ (the angle between a tangent and a chord through the point of contact is equal to an angle in the alternate segment.)

In ΔRSQ,

$\angle RSQ + \angle SRQ + \angle RQS = 180°$

$\Rightarrow 75° + 75° + \angle RQS = 180°$

$\Rightarrow \angle RQS = 30°$

b)

$(2x + 3 = 0) \Rightarrow x = -\frac{3}{2}$ ... (i)

$(x + 2 = 0) \Rightarrow x = -2$ ... (ii)

Putting the value of x from (i) in the polynomial, we get

$$f\left(-\frac{3}{2}\right) = a\times\left(-\frac{3}{2}\right)\times\left(-\frac{3}{2}\right)\times\left(-\frac{3}{2}\right)+3\times\left(-\frac{3}{2}\right)\times\left(-\frac{3}{2}\right)+b\times\left(-\frac{3}{2}\right)-3=0$$

$-27a + 54 - 12b - 24 = 0$

$\Rightarrow 27a = -12b + 30$ ... (iii)

Putting the value of x from (ii) in the polynomial, and the remainder is −3, we get

$f(-2) = ax(-2)x(-2)x(-2) + 3x(-2)x(-2) + bx(-2) - 3 = -3$

$b = 6 - 4a$ ... (iv)

Combining (iii) and (iv), we get

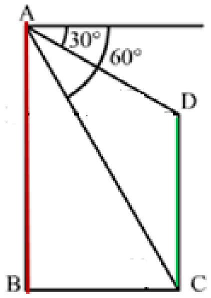
$27a = -12x\ (6 - 4a) + 30$

$\Rightarrow 27a = -72 + 48a + 30$

$\Rightarrow a = 2, b = 6 - 4 \text{ x } 2 = -2$

$a = 2, b = -2$

c)



Given that AB is a building that is 60 m, high.

Let $BC = DE = x$ and $CD = BE = y$

$\Rightarrow AE = AB - BE = 60 - y$

(i) In right ΔAED,

$$\tan 30° = \frac{AE}{DE}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{60-y}{x}$$

$$\Rightarrow x = 60\sqrt{3} - y\sqrt{3} \quad ...(1)$$

In right ΔABC,

$$\tan 60° = \frac{AB}{BC}$$

$$\Rightarrow \sqrt{3} = \frac{60}{x}$$

$$\Rightarrow x = \frac{60}{\sqrt{3}}$$

$$\Rightarrow x = \frac{60}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$\Rightarrow x = \frac{60\sqrt{3}}{3}$$

$$\Rightarrow x = 20\sqrt{3}$$

$$\Rightarrow x = 20 \times 1.732 = 34.64 \text{ m}$$

Thus, the horizontal distance between AB and CD is 34.64 m.

(ii) From (i), we get the height of the lamp post $= CD = y$

$$x = 60\sqrt{3} - y\sqrt{3}$$

$$\Rightarrow 20\sqrt{3} = 60\sqrt{3} - y\sqrt{3}$$

$$\Rightarrow 20 = 60 - y$$

$$\Rightarrow y = 40 \text{ m}$$

Thus, the height of the lamp post is 40 m.

Q.11

a)

i) divisible by 3

3, 6, 9,...

Now,

n = 40, a = 3, d = 3

$$S_n = \frac{n}{2}\left[2\times a + (n-1)d\right]$$

$$S_{40} = \frac{40}{2}\left[2\times 3 + (40-1)3\right]$$

$$= 2460$$

ii) divisible by 5

5, 10, 15,...

Now,

n = 40, a = 5, d = 5

$$S_n = \frac{n}{2}\left(2\times a + (n-1)d\right)$$

$$S_{40} = \frac{40}{2}\left(2\times 5 + (40-1)5\right)$$

$$= 4100$$

iii) divisible by 6

6, 12, 18,...

Now,

n = 40, a = 3, d = 6

$$S_n = \frac{n}{2}\left(2\times a + (n-1)d\right)$$

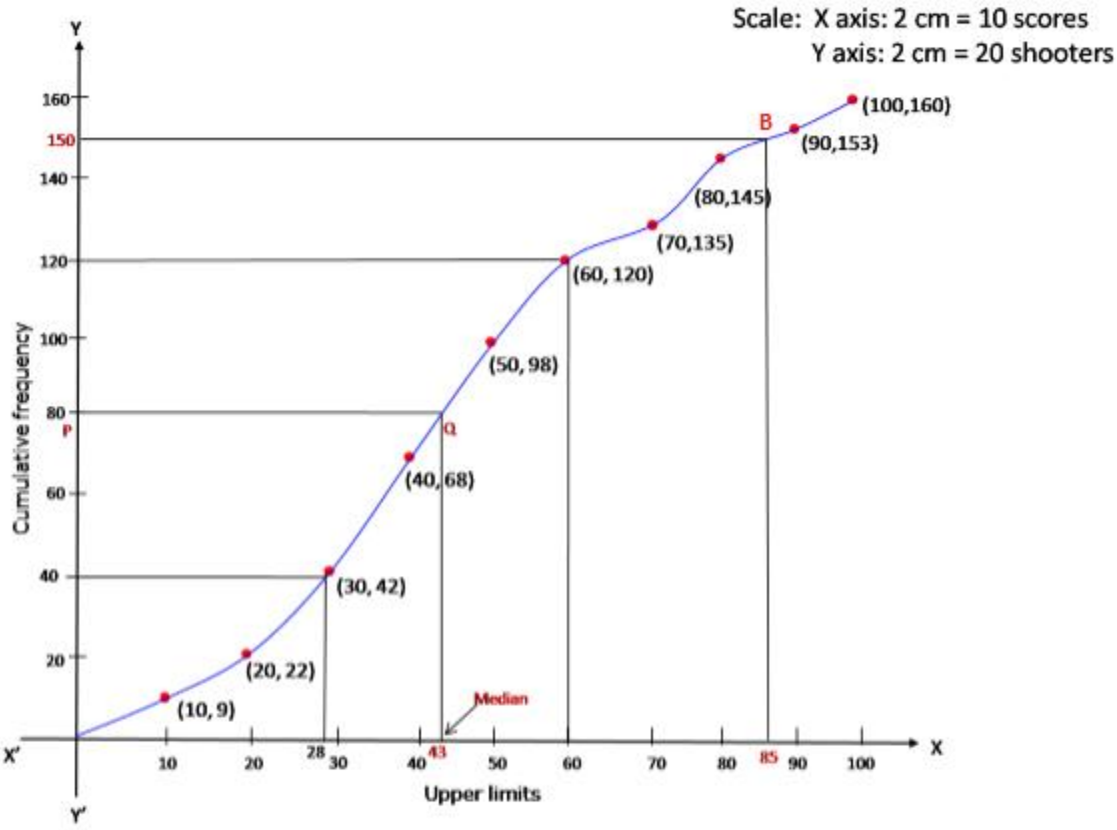
$$S_{40} = \frac{40}{2}\left(2\times 6 + (40-1)6\right)$$

$$= 4920$$

b)

Scores	F	c.f.
0–10	9	9
10–20	13	22
20–30	20	42
30–40	26	68
40–50	30	98
50–60	22	120
60–70	15	135
70–80	10	145
80–90	8	153
90–100	7	160
	n = 160	

The ogive is shown below:



(i) Median $=\left(\frac{n}{2}\right)^{th}$ term $=\left(\frac{160}{2}\right)^{th}$ term $=80^{th}$ term

Through mark 80 on y-axis, draw a horizontal line which meets the ogive drawn at point Q.

Through Q, draw a vertical line which meets the x-axis at the mark of 43.

$\Rightarrow$ Median $= 43$

(ii) Since the number of terms $= 160$

Lower quartile $(Q_1)=\left(\frac{160}{4}\right)^{th}$ term $=40^{th}$ term $=28$

Upper quartile $(Q_3)=\left(\frac{3\times160}{4}\right)^{th}$ term $=120^{th}$ term $=60$

$\therefore$ Inter-quartile range $=Q_3-Q_1=60-28=32$

(iii) Since 85% scores = 85% of 100 = 85

Through mark for 85 on x-axis, draw a vertical line which meets the ogive drawn at point B.

Through the point B, draw a horizontal line which meets the y-axis at the mark of 150.

$\Rightarrow$ Number of shooters who obtained more than 85% score $= 160 - 150 = 10$