CBSE Board
Class XI Mathematics
Sample Paper – 5 Solution

SECTION – A

1. We have,

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$\Rightarrow \frac{2}{\frac{2}{3}}=\frac{3}{\sin B}$$

$$\Rightarrow 3=3\sin B$$

$$\Rightarrow \sin B=1$$

$$\Rightarrow m\angle B=90^\circ$$

2.

$$\left(\frac{1}{i}\right)^{25}=\left(\frac{1}{i}\times\frac{i}{i}\right)^{25}=\left(\frac{1}{i^2}\right)^{25}(i)^{25}=(-1)(i)^{25}=(-1)(i)^{24}i=(-1)\left(i^2\right)^{12}i$$

$$=(-1)(-1)^{12}i=(-1)(1)i=-i$$

OR

$(1 + i)(1 + 2i) = 1 + 2i + i + 2i^2 = 1 + 3i - 2 = -1 + 3i$

Comparing with a + bi we get a = -1 and b = 3

3. p: I have the money;

q: I will buy an i-phone,

$p \to q$: If I have the money $(\to)$ then I will buy an i-phone

4. Relation R from P to Q is R = {(9, 3), (9, −3), (4, 2), (4, −2), (25, 5), (25, −5)}

SECTION – B

5. According to the question,

$n(P) = 40$, $n(P \cup Q) = 60$ and $n(P \cap Q) = 10$

$n(P \cup Q) = n(P) + n(Q) - n(P \cap Q)$

$60 = 40 + n(Q) - 10$

$n(Q) = 60 - 40 + 10 = 30$

6. We know that the equation of the circle described on the line segment joining (x_1, y_1) and (x_2, y_2) as a diameter is,

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

Here, $x_1 = -1$, $x_2 = 4$, $y_1 = 2$ and $y_2 = -3$

Thus, the equation of the required circle is

$(x + 1)(x - 4) + (y - 2)(y + 3) = 0$

$\Rightarrow x^2 + y^2 - 3x + y - 10 = 0$

7. One ace can be selected from 4 aces in ${}^4C_1$.

Other 4 cards which are non-aces can be selected out of 48 cards in ${}^{48}C_4$ ways.

The total number of ways = ${}^4C_1 \times {}^{48}C_4$

= 4 x 2 x 47 x 46 x 45 = 778320

OR

S = {1, 2, 3, 4, 5, 6}

n(S) = 6

1. A : Getting a prime number
   A = {2, 3, 5}
   n(A) = 3
   Required probability = 3/6 = ½
2. B : A number more tan 6
   As B is null set.
   P(B) = 0

8. Let ,

$\cos 5x = (\cos 5x + \cos x) - \cos x$

$= 2\cos\frac{5x+x}{2}\cos\frac{5x-x}{2} - \cos x$

$= 2\cos 3x \cos 2x - \cos x$

$= 2(4\cos^3 x - 3\cos x)(2\cos^2 x - 1) - \cos x$

$= 16\cos^5 x - 20\cos^3 x + 5\cos x$

9. 6 beads have to be arranged in a circular fashion which can be done in (6-1)!

But anticlockwise and clockwise arrangement of beads in a necklace are same so

$(6-1)! \times (1/2) = 60$

OR

$$\frac{1}{9!}+\frac{1}{10!}=\frac{x}{11!}$$
$$\frac{1}{9!}+\frac{1}{10\times 9!}=\frac{x}{11\times 10\times 9!}$$
$$\frac{1}{9!}\left(1+\frac{1}{10}\right)=\left(\frac{x}{11\times 10}\right)\frac{1}{9!}$$
$$\frac{11}{10}=\frac{x}{11\times 10}$$
$$x=121$$

10.

Given that the first term is 1.

Also given that each term is the sum of all the terms which follow it.

Let 1, r, r^2,... be an infinite G.P., where r is the common ratio.

Sum of terms of an infinite G.P., $S=\frac{a}{1-r}$

Here, a = r

Thus, $S=\frac{r}{1-r}$

From the given statement of the problem, we have,

$$1=\frac{r}{1-r}$$
$$\Rightarrow 1-r=r$$
$$\Rightarrow r+r=1$$
$$\Rightarrow 2r=1$$
$$\Rightarrow r=\frac{1}{2}$$

Thus the required G.P. is:

$$1,\frac{1}{2},\frac{1}{4},\frac{1}{8},\frac{1}{16},...$$

11. $-\frac{2}{7},k,-\frac{7}{2}$ are in G.P.

$$\Rightarrow k^2=\left(-\frac{2}{7}\right)\times\left(-\frac{7}{2}\right)=1$$
$$\Rightarrow k^2=1$$
$$\Rightarrow k=\pm 1$$

When $k=1$; G.P.: $-\frac{2}{7},1,-\frac{7}{2}$

$$r = \frac{1}{-\frac{2}{7}} = -\frac{7}{2}$$

$$\text{When } k = -1; GP : -\frac{2}{7}, -1, -\frac{7}{2}$$

$$r = \frac{-1}{-\frac{2}{7}} = \frac{7}{2}$$

OR

3, 6, 12,...

a = 3, r = 2 and n = 7

$$S_7 = a\left(\frac{r^7 - 1}{r - 1}\right) = 3\left(\frac{2^7 - 1}{2 - 1}\right) = 3 \times 127 = 381$$

12. $a_n = \frac{(1+2+3+....n)}{n} = \frac{n(n+1)}{2n}$

$$S_n = \sum_n a_n$$

$$= \frac{1}{2}\sum_{i=1}^{n}(n+1)$$

$$= \frac{1}{2}\frac{n(n+1)}{2} + \frac{n}{2}$$

$$= \frac{(n^2+n)}{4} + \frac{n}{2}$$

$$= \frac{n(n+3)}{4}$$

SECTION – C

13.

$$\frac{1+i}{1-i} - \frac{1-i}{1+i} = \frac{(1+i)^2 - (1-i)^2}{(1-i)(1+i)}$$

$$= \frac{\left(1+i^2+2i\right) - \left(1+i^2-2i\right)}{(1-i)(1+i)}$$

$$= \frac{(1-1+2i)-(1-1-2i)}{\left(1-i^2\right)} = \frac{(2i)-(-2i)}{\left(1-i^2\right)}$$

$$=\frac{2i+2i}{(1-(-1))}=\frac{4i}{(2)}=2i$$

$$\left|\frac{1+i}{1-i}-\frac{1-i}{1+i}\right|=|2i|=\sqrt{0^2+2^2}=\sqrt{4}=2$$

OR

Consider the given equation:

(a + ib) (c + id) (e + if) (g + ih) = A + iB

Let us take modulus on both sides ,

$|(a + ib)\ (c + id)\ (e + if)\ (g + ih)| = |A + iB|$

We know, $|z_1z_2|=|z_1||z_2|$

$\therefore |(a + ib)\ (c + id)\ (e + if)\ (g + ih)|=|A + iB|$

$\Rightarrow |(a + ib)||(c + id)||(e + if)||(g + ih)|=|A + iB|$

$$\Rightarrow \sqrt{a^2+b^2}.\sqrt{c^2+d^2}.\sqrt{e^2+f^2}.\sqrt{g^2+h^2}=\sqrt{A^2+B^2}$$

$$\Rightarrow \left[\sqrt{a^2+b^2}.\sqrt{c^2+d^2}.\sqrt{e^2+f^2}.\sqrt{g^2+h^2}\right]^2=\left[\sqrt{A^2+B^2}\right]^2$$

$$\Rightarrow \left(a^2+b^2\right).\left(c^2+d^2\right).\left(e^2+f^2\right).\left(g^2+h^2\right)=A^2+B^2$$

Thus, the value of $A^2+B^2=\left(a^2+b^2\right).\left(c^2+d^2\right).\left(e^2+f^2\right).\left(g^2+h^2\right)$

14. Consider the given quadratic equation:

$9x^2 - 12x + 20 = 0$

$$\Rightarrow 3x^2-4x+\frac{20}{3}=0$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$\Rightarrow x=\frac{-(-4)\pm\sqrt{(-4)^2-4\times3\times\frac{20}{3}}}{2.3}=\frac{4\pm\sqrt{16-4\times20}}{6}$$

$$=\frac{4\pm\sqrt{16-80}}{6}=\frac{4\pm\sqrt{-64}}{6}=\frac{4\pm8\sqrt{-1}}{6}=\frac{2\pm4i}{3}$$

$$\Rightarrow x=\frac{2}{3}\pm\frac{4}{3}i$$

15. To find the coefficient of x^5 in the expansion of the product,

$(1+2x)^6(1-x)^7$

Let us find the expansions of the 2 binomials.

$$(1+2x)^6 = {}^6C_0(2x)^0 + {}^6C_1(2x)^1 + {}^6C_2(2x)^2 + {}^6C_3(2x)^3 + {}^6C_4(2x)^4 + {}^6C_5(2x)^5 + {}^6C_6(2x)^6$$

$$= 1\times1 + 6\times(2x) + 15\times(2x)^2 + 20\times(2x)^3 + 15\times(2x)^4 + 6\times(2x)^5 + 1\times(2x)^6$$

$$= 1 + 12x + 60x^2 + 20\times(2x)^3 + 15\times(2x)^4 + 6\times(2x)^5 + 1\times(2x)^6$$

$$(1-x)^7 = {}^7C_0(-x)^0 + {}^7C_1(-x)^1 + {}^7C_2(-x)^2 + {}^7C_3(-x)^3 + {}^7C_4(-x)^4 + {}^7C_5(-x)^5 + {}^7C_6(-x)^6 + {}^7C_7(-x)^7$$

$$= 1\times1 - 7\times(x) + 21\times(x)^2 - 35\times(x)^3 + 35\times(x)^4 - 21\times(x)^5 + 7\times(x)^6 - 1\times(x)^7$$

$$= 1 - 7x + 21x^2 - 35x^3 + 35x^4 - 21x^5 + 7x^6 - x^7$$

Product of $(1+2x)^6(1-x)^7$

$$= \left[\boxed{1} + \boxed{12}x + \boxed{60}x^2 + \boxed{160}x^3 + \boxed{240}x^4 + \boxed{192}x^5 + 64x^6\right]$$

$$\times\left[\boxed{1}\boxed{-7}x + \boxed{21}x^2\boxed{-35}x^3 + \boxed{35}x^4\boxed{-21}x^5 + 7x^6 - x^7\right]$$

$\therefore$ Coefficient of x^5 in the product

$= 1\times(-21) + 12\times35 + 60\times(-35) + 160\times21 + 240\times(-7) + 192\times1$

$= -21 + 420 - 2100 + 3360 - 1680 + 192$

$= 3972 - 3801$

$= 171$

16. Consider, $3x = 2x + x$

Rewriting $\tan 3x = \tan(2x + x)$

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\tan 3x = \frac{\tan 2x + \tan x}{1 - \tan 2x \tan x}$$

$\tan 3x - \tan 3x \tan 2x \tan x = \tan 2x + \tan x$

or $\tan 3x - \tan 2x - \tan x = \tan 3x \tan 2x \tan x$

or $\tan 3x \tan 2x \tan x = \tan 3x - \tan 2x - \tan x$

17. We need to prove that,

$$\frac{\sec 8A-1}{\sec 4A-1}=\frac{\tan 8A}{\tan 2A}$$

Consider L.H.S.

$$\frac{\frac{1\text{-}\cos 8A}{\cos 8A}}{\frac{1-\cos 4a}{\cos 4a}}=\frac{2\sin^2 4A}{\cos 8A}\times\frac{\cos 4A}{2\sin^2 2A}$$

$$=\frac{2\sin 4A\times\cos 4A\times\sin 4A}{\cos 8A\times 2\sin 2A\times\sin 2A}$$

$$=\frac{\sin 8A}{\cos 8A}\times\frac{2\sin 2A\times\cos 2A}{2\sin 2A\times\sin 2A}=\frac{\tan 8A}{\tan 2A}$$

OR

Consider the L.H.S of the given equation:

$$\text{L.H.S}=a(\sin B-\sin C)+b(\sin C-\sin A)+c(\sin A-\sin B)$$

Let us use the identity,

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$\text{Thus, L.H.S}=a\sin B-a\sin C+b\sin C-b\sin A+c\sin A-c\sin B$$

$$\Rightarrow \text{L.H.S}=b\sin A-c\sin A+c\sin B-b\sin A+c\sin A-c\sin B$$

$$\Rightarrow \text{L.H.S}=0$$

R.H.S = 0

Hence proved.

18. A – B = {*e*, *o*}, since the elements *e*, *o* belong to A but not to B and B – A = {*k*}, since the element *k* belongs to B but not to A.

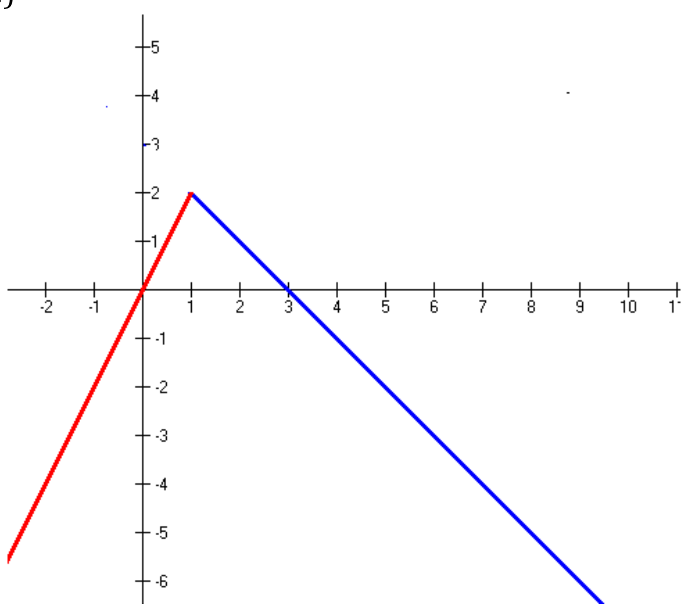
(i)We note that A – B ≠B – A.

(ii)The sets (A – B) and (B – A) are mutually disjoint sets, i.e. the intersection of these two sets is a null set.

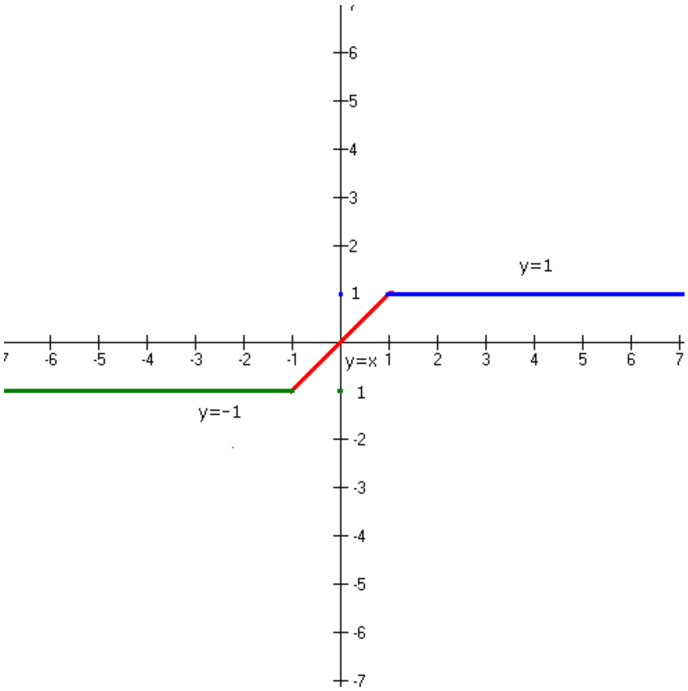
19. $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 8\theta}}}$

$$=\sqrt{2+\sqrt{2+\sqrt{2(1+\cos 8\theta)}}}$$
$$=\sqrt{2+\sqrt{2+\sqrt{2\times 2\cos^2 4\theta}}}$$
$$=\sqrt{2+\sqrt{2+2\cos 4\theta}}$$
$$=\sqrt{2+\sqrt{2(1+\cos 4\theta)}}$$
$$=\sqrt{2+\sqrt{2\times 2\cos^2 2\theta}}$$
$$=\sqrt{2+2\cos 2\theta}$$
$$=\sqrt{2(1+\cos 2\theta)}$$
$$=\sqrt{2\times 2\cos^2\theta}$$
$$=2\cos\theta$$
$$\therefore \sqrt{2+\sqrt{2+\sqrt{2+2\cos 8\theta}}}=2\cos\theta$$

20. Range of f = (-∞, 2)



OR



Range f = [-1, 1]

21. Into Function: {a, p), (b, q), (c,p),(d,p)} range must be the proper subset of set B
Onto: {a, p), (b, q) (c, r), (d, r)} range must be same as set B
No one-one function can be defined from A to B because n(B) < n(A)

22. $\cos^2 x+\cos^2(x+\frac{\pi}{3})+\cos^2(x-\frac{\pi}{3})$

$$=\cos^2 x+\cos^2(x+\frac{\pi}{3})+1-\sin^2(x-\frac{\pi}{3})$$

$$=1+\cos^2 x+\left[\cos^2(x+\frac{\pi}{3})-\sin^2(x-\frac{\pi}{3})\right]$$

$$=1+\cos^2 x+\left[\cos(x+\frac{\pi}{3}+x-\frac{\pi}{3})\cos(x+\frac{\pi}{3}-x+\frac{\pi}{3})\right] \quad \left[\because \cos^2 A-\sin^2 B=\cos(A+B)\cos(A-B)\right]$$

$$=1+\cos^2 x+\cos(2x)\cos\left(\frac{2\pi}{3}\right)$$

$$=1+\cos^2 x+\cos(2x)\left(-\frac{1}{2}\right)$$

$$=1+\cos^2 x+\left(2\cos^2 x-1\right)\left(-\frac{1}{2}\right)$$

$$=1+\cos^2 x+\left(-\cos^2 x+\frac{1}{2}\right)$$

$$=1+\frac{1}{2}=\frac{3}{2}$$

23. (i)

$$7=\frac{8+6+7+5+x+4}{6}$$
$$\Rightarrow 42=8+6+7+5+x+4$$
$$\Rightarrow 42=30+x$$
$$\Rightarrow x=12$$

(ii)If each observation is multiplied by 3, the mean will also be multiplied by 3, hence the mean is 21.

(iii) Arranging in the ascending order 4, 5, 6, 7, 8, 12

Median = $\frac{6+7}{2}=6.5$ ....

Forming the table, we get

x_i	4	5	6	7	8	12
$x_i-6.5$	-2.5	-1.5	-0.5	0.5	1.5	5.5
$\lvert x_i-6.5\rvert$	2.5	1.5	0.5	0.5	1.5	5.5

$$\sum_{i=1}^{n}\lvert x_i-M\rvert=\sum_{i=1}^{n}\lvert x_i-6.5\rvert=12$$

$$M.D(M)=\frac{\sum_{i=1}^{n}\lvert x_i-M\rvert}{n}=\frac{\sum_{i=1}^{n}\lvert x_i-6.5\rvert}{6}$$

$$=\frac{12}{6}=2.0$$

SECTION – D

24. Let S be the sample space. Then n(S) = 36

Let E_1 = event that a doublet appears

Let E_2 = event of getting a total of 10.

Then, $E_1=[(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)]$,

and $E_2=[(4,6),(5,5),(6,4)]$

$\therefore E_1\cap E_2=[(5,5)]$

So, $n(E_1)=6, n(E_2)=3$ and $n(E_1\cap E_2)=1$.

Thus,

$$P(E_1)=\frac{n(E_1)}{n(S)}=\frac{6}{36}=\frac{1}{6},$$

$$P(E_2)=\frac{n(E_2)}{n(S)}=\frac{3}{36}=\frac{1}{12}$$

$$P(E_1 \cap E_2) = \frac{n(E_1 \cap E_2)}{n(S)} = \frac{1}{36}$$

Therefore, the probability of getting a doublet or a total of 10

$= P(E_1 \text{ or } E_2)$

$= P(E_1 \cup E_2)$

$= P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$= \frac{1}{6} + \frac{1}{12} - \frac{1}{36}$

$= \frac{8}{36}$

$= \frac{2}{9}$

Thus P(getting neither a doublet nor a total of 10)

$= P(\overline{E_1} \text{ and } \overline{E_2})$

$= P(\overline{E_1} \cap \overline{E_2})$

$= P(\overline{E_1 \cup E_2})$

$= 1 - P(E_1 \cup E_2)$

$= 1 - \frac{2}{9}$

$= \frac{7}{9}$

OR

Let E_1, E_2, E_3 and E_4 denote respectively the events that a student will receive A, B, C and D grades.

$P(E_1) = 0.4$, $P(E_2) = 0.35$, $P(E_3) = 0.15$ and $P(E_4) = 0.10$

1. Required probability = $P(E_2 \cup E_3)$
   $= P(E_2) + P(E_3)$ $\quad\because$ Mutually exclusive events
   $= 0.35 + 0.15 = 0.5$
2. Required probability = $P(E_3 \cup E_4)$
   $= P(E_3) + P(E_4)$ $\quad\because$ Mutually exclusive events
   $= 0.15 + 0.1$
   $= 0.25$

25. Let the statement P(n) be: $\frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}+......+\frac{1}{(n)(n+1)(n+2)}=\frac{(n)(n+3)}{4(n+1)(n+2)}$

Consider P(1): $\frac{1}{1.2.3}=\frac{(1)(1+3)}{4(1+1)(1+2)}$

$\Rightarrow \frac{1}{1.2.3}=\frac{(1)(1+3)}{4(1+1)(1+2)}=\frac{1.4}{4.2.3}=\frac{1}{1.2.3}$ [P(1) is true]

Let us assume that P(k) is true

P(k): $\frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}+...+\frac{1}{(k)(k+1)(k+2)}=\frac{(k)(k+3)}{4(k+1)(k+2)}$

To prove:

P(k+1): $\frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}+...+\frac{1}{(k)(k+1)(k+2)}+\frac{1}{(k+1)(k+2)(k+3)}$

$=\frac{(k+1)(k+4)}{4(k+2)(k+3)}$

LHS $=\frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}+...+\frac{1}{(k)(k+1)(k+2)}+\frac{1}{(k+1)(k+2)(k+3)}$

$=\frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}+...+\frac{1}{k(k+1)(k+2)}+\frac{1}{(k+1)(k+2)(k+3)}$

$=\frac{k(k+3)}{4(k+1)(k+2)}+\frac{1}{(k+1)(k+2)(k+3)}$ (using P(k))

$=\frac{1}{(k+1)(k+2)}\left[\frac{k(k+3)}{4}+\frac{1}{(k+3)}\right]$

$=\frac{1}{(k+1)(k+2)}\left[\frac{k(k+3)^2+4}{4(k+3)}\right]$

$=\frac{1}{(k+1)(k+2)}\left[\frac{k(k^2+9+6k)+4}{4(k+3)}\right]$

$=\frac{1}{(k+1)(k+2)}\left[\frac{k^3+9k+6k^2+4}{4(k+3)}\right]$

$=\frac{1}{(k+1)(k+2)}\left[\frac{(k+1)^2(k+4)}{4(k+3)}\right]$

$=\frac{(k+1)(k+4)}{4(k+2)(k+3)}=$ R.H.S.

26.

(i) There are a total of 60 marbles out of which 5 marbles are to be selected

Number of ways in which 5 marbles are to be selected out of 60 = ${}^{60}C_5$

(a)

Out of 20 blue marbles , 5 can be selected in= ${}^{20}C_5$

$$\text{P(all 5 blue marbles)} = \frac{{}^{20}C_5}{{}^{60}C_5} = \frac{\frac{20!}{5!15!}}{\frac{60!}{5!55!}} = \frac{20!5!55!}{5!15!60!}$$

$$= \frac{20.19.18.17.16}{60.59.58.57.56} = \frac{34}{11977}$$

(b) Number of ways in which 5 marbles are to be selected out of 60= ${}^{60}C_5$.

Out of 30 non-green marbles , 5 can be selected in ${}^{30}C_5$

$$\text{P(all 5 non-green marbles)} = \frac{{}^{30}C_5}{{}^{60}C_5} = \frac{\frac{30!}{5!25!}}{\frac{60!}{5!55!}} = \frac{30!5!55!}{5!25!60!}$$

$$= \frac{30.29.28.27.26.}{60.59.58.57.56} = \frac{117}{4484}$$

$$\text{P(atleast one green marble)} = 1 - \text{P(all 5 non} - \text{green marbles)}$$

$$= 1 - \frac{117}{4484}$$

$$= \frac{4484 - 117}{4484} = \frac{4367}{4484}$$

(ii)On the dice two faces are with number '1', three faces are with number '2' and one face is with number '3'

$$\therefore P(1) = \frac{2}{6} = \frac{1}{3}; P(2) = \frac{3}{6} = \frac{1}{2}; P(3) = \frac{1}{6}$$

$$\therefore \text{(i) } P(2) = \frac{1}{2}$$

(ii) $P(1 \text{ or } 3) = P(1) + P(3)$ [The events are mutually exclusive]

$$= \frac{1}{3} + \frac{1}{6} = \frac{1}{2}$$

$$\text{(iii) } P(\text{not}3) = 1 - P(3) = 1 - \frac{1}{6} = \frac{5}{6}$$

27. (i) sin(x+1) by the definition method

Let y=sin(x+1)

$\Rightarrow y+\delta y=\sin(x+\delta x+1)$

$\Rightarrow \delta y=(y+\delta y)-y=\sin(x+\delta x+1)\ -\ \sin(x+1)$

$=2\cos(\frac{x+\delta x+1+x+1}{2})\sin(\frac{x+\delta x+1-(x+1)}{2})$

$=2\cos(\frac{2x+\delta x+2}{2})\sin(\frac{\delta x}{2})$

$\frac{\delta y}{\delta x}=\frac{1}{\delta x}2\cos(\frac{2x+\delta x+2}{2})\sin(\frac{\delta x}{2})$

$\Rightarrow \frac{\delta y}{\delta x}=\cos(\frac{2x+\delta x+2}{2})\frac{\sin(\frac{\delta x}{2})}{\frac{\delta x}{2}}$

$\Rightarrow \lim_{\delta x\to 0}\frac{\delta y}{\delta x}=\lim_{\delta x\to 0}\cos(\frac{2x+\delta x+2}{2})\frac{\sin(\frac{\delta x}{2})}{\frac{\delta x}{2}}$

$\Rightarrow \lim_{\delta x\to 0}\frac{\delta y}{\delta x}=\lim_{\delta x\to 0}\cos(\frac{2x+\delta x+2}{2})\lim_{\delta x\to 0}\frac{\sin(\frac{\delta x}{2})}{\frac{\delta x}{2}}$

$\Rightarrow \frac{dy}{dx}=\cos(\frac{2x+2}{2}).1$

$\Rightarrow \frac{dy}{dx}=\cos(x+1)$

(ii) Let $y=\frac{x}{1+\tan x}$

$\Rightarrow \frac{dy}{dx}=\frac{(1+\tan x)\frac{d}{dx}(x)-(x)\frac{d}{dx}(1+\tan x)}{(1+\tan x)^2}$

$\Rightarrow \frac{dy}{dx}=\frac{(1+\tan x).1-(x)(\sec^2 x)}{(1+\tan x)^2}$

$\Rightarrow \frac{dy}{dx}=\frac{1+\tan x-x\sec^2 x}{(1+\tan x)^2}$

OR

$$\left[\frac{x-2}{x^2-x}-\frac{1}{x^3-3x^2+2x}\right]=\left[\frac{x-2}{x(x-1)}-\frac{1}{x(x^2-3x+2)}\right]$$

$$=\left[\frac{x-2}{x(x-1)}-\frac{1}{x(x-1)(x-2)}\right]$$

$$=\left[\frac{x^2-4x+4-1}{x(x-1)(x-2)}\right]$$

$$=\frac{x^2-4x+3}{x(x-1)(x-2)}$$

$$\lim_{x\to1}\left[\frac{x^2-2}{x^2-x}-\frac{1}{x^3-3x^2+2x}\right]=\lim_{x\to1}\frac{x^2-4x+3}{x(x-1)(X-2)}$$

$$=\lim_{x\to1}\frac{(x-3)(x-1)}{x(x-1)(x-2)}$$

$$=\lim_{x\to1}\frac{x-3}{x(x-2)}=\frac{1-3}{1(1-2)}=2$$

(ii) $$\lim_{x\to0}\frac{\sin 4x}{\sin 2x}=\lim_{x\to0}\left[\frac{\sin 4x}{4x}\times\frac{2x}{\sin 2x}\times2\right]$$

$$=2\times\lim_{x\to0}\left[\frac{\sin 4x}{4x}\right]\div\left[\frac{\sin 2x}{2x}\right]$$

$$=2\times\lim_{4x\to0}\left[\frac{\sin 4x}{4x}\right]\div\lim_{2x\to0}\left[\frac{\sin 2x}{2x}\right]$$

$$=2\times1\times1=2 \qquad (\text{as } x\to0, 4x\to0 \text{ and } 2x\to0)$$

28. Perpendicular distance of the point (m,n) from the line ax+by+c=0, is given by

$$\left|\frac{a(m)+b(n)+c}{\sqrt{a^2+b^2}}\right|$$

$\therefore$ Perpendicular distance of the point $(\sqrt{a^2-b^2},0)$ from the line

$\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta-1=0$, is given by

$$\left|\frac{\frac{\cos\theta}{a}(\sqrt{a^2-b^2})+\frac{\sin\theta}{b}(0)-1}{\sqrt{\left(\frac{\cos\theta}{a}\right)^2+\left(\frac{\sin\theta}{b}\right)^2}}\right|=\left|\frac{\frac{\cos\theta}{a}(\sqrt{a^2-b^2})-1}{\sqrt{\left(\frac{\cos\theta}{a}\right)^2+\left(\frac{\sin\theta}{b}\right)^2}}\right|$$

Perpendicular distance of the point $(-\sqrt{a^2-b^2},0)$ from the line

$\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta-1=0$, is given by

$$\left|\frac{-\frac{\cos\theta}{a}(\sqrt{a^2-b^2})+\frac{\sin\theta}{b}(0)-1}{\sqrt{\left(\frac{\cos\theta}{a}\right)^2+\left(\frac{\sin\theta}{b}\right)^2}}\right|=\left|\frac{-\frac{\cos\theta}{a}(\sqrt{a^2-b^2})-1}{\sqrt{\left(\frac{\cos\theta}{a}\right)^2+\left(\frac{\sin\theta}{b}\right)^2}}\right|$$

Product of the perpendicular distances

$$=\left|\frac{\frac{\cos\theta}{a}(\sqrt{a^2-b^2})-1}{\sqrt{\left(\frac{\cos\theta}{a}\right)^2+\left(\frac{\sin\theta}{b}\right)^2}}\right|\times\left|\frac{-\frac{\cos\theta}{a}(\sqrt{a^2-b^2})-1}{\sqrt{\left(\frac{\cos\theta}{a}\right)^2+\left(\frac{\sin\theta}{b}\right)^2}}\right|$$

$$=\left|\frac{\left[\frac{\cos\theta}{a}(\sqrt{a^2-b^2})-1\right]\left[\frac{\cos\theta}{a}(\sqrt{a^2-b^2})+1\right]}{\sqrt{\left(\frac{\cos\theta}{a}\right)^2+\left(\frac{\sin\theta}{b}\right)^2}\sqrt{\left(\frac{\cos\theta}{a}\right)^2+\left(\frac{\sin\theta}{b}\right)^2}}\right|$$

$$=\left|\frac{\left[\frac{(a^2-b^2)\cos^2\theta}{a^2}-1\right]}{\left(\frac{\cos\theta}{a}\right)^2+\left(\frac{\sin\theta}{b}\right)^2}\right|$$

$$=\frac{\left|\left[\frac{\left(a^2-b^2\right)\cos^2\theta}{a^2}-1\right]\right|}{\frac{b^2\cos^2\theta+a^2\sin^2\theta}{a^2b^2}}$$

$$=\frac{a^2b^2\left|\left[\frac{a^2\cos^2\theta-b^2\cos^2\theta-a^2}{a^2}\right]\right|}{b^2\cos^2\theta+a^2\sin^2\theta}$$

$$=\frac{b^2\left|a^2\cos^2\theta-b^2\cos^2\theta-a^2\right|}{b^2\cos^2\theta+a^2\sin^2\theta}$$

$$=\frac{b^2\left|-a^2(1-\cos^2\theta)-b^2\cos^2\theta\right|}{b^2\cos^2\theta+a^2\sin^2\theta}$$

$$=\frac{b^2\left|-a^2(\sin^2\theta)-b^2\cos^2\theta\right|}{b^2\cos^2\theta+a^2\sin^2\theta}$$

$$=\frac{b^2\left|-\left(a^2(\sin^2\theta)+b^2\cos^2\theta\right)\right|}{b^2\cos^2\theta+a^2\sin^2\theta}$$

$$=\frac{b^2\left(a^2(\sin^2\theta)+b^2\cos^2\theta\right)}{b^2\cos^2\theta+a^2\sin^2\theta}$$

$$=b^2$$

29. $5(2x-7)-3(2x+3) \leq 0$;

$2x+19 \leq 6x+47$

and

$7 \leq \frac{(3x+11)}{2} \leq 11$

Let us solve the inequalities one by one and then work out the common solution.

Inequality 1:

$5(2x-7)-3(2x+3) \leq 0$

$\Rightarrow 10x-35-6x-9 \leq 0$

$\Rightarrow 4x-44 \leq 0$

$\Rightarrow 4x \leq 44$

$\Rightarrow x \leq 11$

Inequality 2:

$2x+19 \leq 6x+47$

$\Rightarrow 2x-6x \leq -19+47$

$\Rightarrow -4x \leq 28$

$\Rightarrow -x \leq 7$

$\Rightarrow x \geq -7$

Inequality 3:

$7 \leq \frac{(3x+11)}{2} \leq 11$

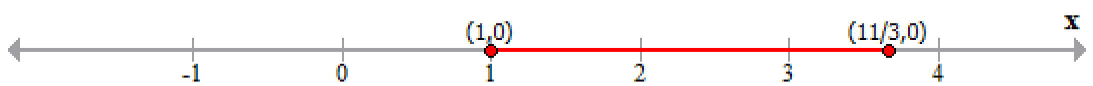
$\Rightarrow 14 \leq 3x+11 \leq 22$

$\Rightarrow 14-11 \leq 3x \leq 22-11$

$\Rightarrow 3 \leq 3x \leq 11$

$\Rightarrow 1 \leq x \leq \frac{11}{3}$

$x \leq 11, x \geq -7, 1 \leq x \leq \frac{11}{3}$ together $\Rightarrow 1 \leq x \leq \frac{11}{3}$



OR

The amount of acid in 1125lt of the 45% solution=45% of 1125=$\frac{45\times1125}{100}$

Let x lt of the water be added to it to obtain a solution between 25% and 30% solution

$$\Rightarrow 25\% < \frac{1125\times\frac{45}{100}}{1125+x} < 30\%$$

$$\Rightarrow \frac{25}{100} < \frac{1125\times\frac{45}{100}}{1125+x} < \frac{30}{100}$$

$$\Rightarrow \frac{25}{100} < \frac{1125\times45}{(1125+x)\times100} < \frac{30}{100}$$

$$\Rightarrow 25 < \frac{1125\times45}{(1125+x)} < 30$$

$$\Rightarrow \frac{1}{25} > \frac{(1125+x)}{1125\times45} > \frac{1}{30}$$

$$\Rightarrow \frac{1125\times45}{25} > (1125+x) > \frac{1125\times45}{30}$$

$$\Rightarrow \frac{50625}{25} > (1125+x) > \frac{50625}{30}$$

$$\Rightarrow 2025 > (1125+x) > 1687.5$$

$$\Rightarrow 2025 > (1125+x) > 1687.5$$

$$\Rightarrow 2025-1125 > x > 1687.5-1125$$

$$\Rightarrow 900 > x > 562.5$$

$$\Rightarrow 562.5 < x < 900$$

So the amount of water to be added must be between 562.5 to 900 lt