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BASIC MATHEMATICS

EXERCISE 1 JEE MAIN/BOARDS

Q.1 Solve

(i) $\log_{16} 32$

(ii) $\log_8 16$

(iii) $\log_{1/9}(1/9)$

(iv) $\log_{2/\sqrt{3}}(1728)$

(v) $\log_2 \cos 45^\circ$

(vi) $\log_2(\log_2 4)$

(vii) $\log_3(\tan 30^\circ)$

Q.2 Prove the following

(i) $\log_5 \sqrt{5\sqrt{5\sqrt{5-\infty}}} = 1$

(ii) $\log_{0.125}(8) = 1$

(iii) $\log_{1.5}(0.\bar{6}) = -1$

(iv) $\log_{2.25}(0.\bar{4}) = -1$

(v) $\log_{10}(0.\bar{9}) = 0$

Q.3 Find the no. of digits in

(i) 2^{100} (ii) 3^{10}

Q.4 Solve

(i) $\log_{x-1} 3 = 2$

(ii) $\log_3(3^x - 8) = 2 - x$

(iii) $\log_{5-x}(x^2 - 2x + 65) = 2$

(iv) $\log_3(x+1) + \log_3(x+3) = 1$

(v) $x^{2\log x} = 10^{5+\log x}$

(vi) $x^{\frac{\log x + 5}{3}} = 10^{5+\log x}$

(vii) $x^{\log_3 x} = 9$

Q.5 $1 - \log 5 = \frac{1}{3}\left(\log\frac{1}{2} + \log x + \frac{1}{3}\log 5\right)$

Q.6

$\log x - \frac{1}{2}\log\left(x - \frac{1}{2}\right) = \log\left(x + \frac{1}{2}\right) - \frac{1}{2}\log\left(x + \frac{1}{8}\right)$

Q.7 $x^{\frac{\log_{10} x + 7}{4}} = 10^{\log x + 1}$

Q.8 $\left(\frac{\log x}{2}\right)^{\log^2 x + \log x^2 - 2} = \log\sqrt{x}$

Q.9 $\sqrt[3]{\log_2 x} - \log_2 8x + 1 = 0$

Q.10 $\log_{1/3} x - 3\sqrt{\log_{1/3} x} + 2 = 0$

Q.11 $\left(a^{\log_{10} x}\right)^2 - 5x^{\log_{10} x} + 6 = 0$

Q.12

$\log_4(x^2 - 1) - \log_4(x-1)^2 - \log_4\left(\sqrt{(4-x)^2}\right)$

Q.13 $2\log_3 \frac{x-3}{x-7} + 1 = \log_3 \frac{x-3}{x-1}$

Q.14 $\log_x(9x^2)\log_3^2 x = 4$

Q.15 $\log_{0.5x} x^2 + 14\log_{16x} x^2 + 40\log_{4x}\sqrt{x} = 0$

Q.16 $\log_3\left(\log_{1/2}^2 x - 3\log_{1/2} x + 5\right) = 2$

Q.17 $\log_3(x/4) = \frac{15}{\log_2 \frac{x}{8} - 1}$

Q.18 $\frac{1}{2}\log(5x-4) + \log\sqrt{x+1} = 2 + \log 0.18$

Q.19 $\log x^2 = \log(5x-4)$

Q.20 $\frac{1}{6}\log_2(x-2) - \frac{1}{3} = \log_{1/8}\sqrt{3x-5}$

Q.21 $\frac{\log\left(\sqrt{x+1}+1\right)}{\log\left(\sqrt[3]{x-40}\right)} = 30$

Q.22 $1 - \frac{1}{2}\log(2x-1) = \frac{1}{2}\log(x-9)$

Q.23 $\log(3x^2 + 7) - \log(3x-2) = 1$

Q.24 $\left(1 + \frac{1}{2x}\right)\log 3 + \log 2 = \log\left(27 - 3^{1/x}\right)$

Q.25 $\frac{1}{2}\log x + 3\log\sqrt{2+x} = \log\sqrt{x(x+2)} + 2$

Q.26 $\log_2(4^x + 1) = x + \log_2(2^{x+3} - 6)$

Q.27 $\log_{\sqrt{5}}(4^x - 6) - \log_{\sqrt{5}}(2^x - 2) = 2$

Q.28 $\log(3^x - 2^{4-x}) = 2 + \frac{1}{4}\log 16 - \frac{x\log 4}{2}$

Q.29 $\log(\log x) + \log(\log x^4 - 3) = 0$

Q.30 $\log_2(9^x + 9) = \log_3 3^x(28 - 2.3^x)$

EXERCISE 2 JEE MAIN

Q.1 $\frac{1}{\log_{\sqrt{a/b}} abc}+\frac{1}{\log_{\sqrt{ab}} abc}+\frac{1}{\log_{\sqrt{ab}} abc}$ has the value equal to

(A) 1/2 (B) 1 (C) 2 (D) 4

Q.2 The equation,

$\log_2(2x^2)+\log_2 x.x^{\log(\log x+1)}$

$+\frac{1}{2}\log_4^2 x^4+2^{-3\log_{3/2}(\log x)}$ has

(A) exactly one real solution

(B) two real solutions

(C) 3 real solutions

(D) no solution

Q.3 Number of zeros after decimal before a significant figure in $(75)^{-10}$ is:

(Use $\log_{10} 2 = 0.301$ & $\log_{10} 3= 0.477$)

(A) 20 (B) 19

(C) 18 (D) None

Q.4 If $5x^{\log_2 3}+3^{\log_2 x}=162$ then logarithm of x to the base 4 has the value equal to

(A) 2 (B) 1

(C) –1 (D) 3/2

Q.5 $x^{\log_{10}^2+\log_{10}x^3}=\frac{2}{\frac{1}{\sqrt{x+1}-1}\frac{1}{\sqrt{1+1}+1}}$

where $x_1 > x_2 > x_3$, then

(A) $x_1+x_3=2x_2$

(B) $x_1.x_3=x_2^2$

(C) $x_2=\frac{2x_1x_2}{x_1+x_2}$

(D) $x_1^{-1}+x_1^{-1}=x_3^{-1}$

Q.6 Let $x=2^{\log 3}$ and y = $y=3^{\log 2}$ where base of the logarithm is 10, then which one of the following holds good?

(A) 2x < y (B) 2y < x

(C) 3x = 2y (D) y = x

Q.7 Number of real solution(s) of the equation $|x-3|^{3x^2-10x+3}=1$ is-

(A) exactly four (B) exactly three

(C) exactly two (D) exactly one

Q.8 If $\left(\sqrt{5\sqrt{2}-7}\right)^x+6\left(\sqrt{5\sqrt{2}+7}\right)^x=7$, then the value of x can be equal to-

(A) 0 (B) $\log_{(5\sqrt{2}-7)}36$

(C) $\frac{-2}{\log_6(5\sqrt{2}+7)}$ (D) $\log_{\sqrt{5\sqrt{2}-7}}6$

Q.9 Consider the following statements

Statement-1: The equation $5^{\log_5(x^2+1)}-x^2=1$ has two distinct real solutions.

Because.

Statement-2: $a^{\log_a N}=N$ when a > 0, a ≠ 1 and N > 0.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false

(D) Statement-1 is false, statement-2 is true

Q.10

Column-I	Column-II
(A) The expression $x=\log_2\log_9\sqrt{6+\sqrt{6+\sqrt{6+.........\infty}}}$ simplifies to	(P) an integer
(B) The number $N=2^{(\log_2 3.\log_3 4.\log_4 5.....\log_{99}100)}$ simplifies to	(Q) a prime
(C) The expression $\frac{1}{\log_5 3}+\frac{1}{\log_6 3}-\frac{1}{\log_{10}3}$ simplifies to	(R) a natural
(D) The number N = $\sqrt{2+\sqrt{5}-\sqrt{6-3\sqrt{5}\sqrt{14-6\sqrt{5}}}}$ simplifies to	(S) a composite

Q.11 If x_1 and x_2 are the roots of the equation $\sqrt{2010}x^{\log_{2000}x}=x^2$, then find the cyphers at the end of the product (x_1x_2)

Q.12 Let x = 2 or x = 3 satisfy the equation, $\log_4(x^2+bx+c)=1$. Then find the value of |bc|.

EXERCISE 1 JEE ADVANCED

Q.1 Let A denotes the value of

$$\log_{10}\left(\frac{ab+\sqrt{(ab)^2-4(a+b)}}{2}\right)+\log_{10}\left(\frac{ab-\sqrt{(ab)^2-4(a+b)}}{2}\right)$$ when a = 43 and b = 57 and B denotes the value of the expression $\left(2^{\log_6 18}\right).\left(3^{\log_6 3}\right)$. Find the value of (A.B).

Q.2 Simplify:

(a) $\log_{10}\sqrt[4]{729\sqrt[3]{9^{-1}.27^{-4/3}}}$

(b) $a^{\frac{\log_b(\log_b N)}{\log_b a}}$

Q.3 (a) Which is smaller ? 2 or $(\log_\pi 2+\log_2 \pi)$

(b) Prove that $\log_3 5$ and $\log_2 7$ are both irrational

Q.4 Find the square of the sum of the roots of the equation

$\log_3 x \cdot \log_4 x \cdot \log_5 x = \log_3 x \cdot \log_4 x + \log_4 x \cdot \log_5 x + \log_5 x \cdot \log_3 x$.

Q.5 Find the value of the expression $\frac{2}{\log_4(2000)^6}+\frac{3}{\log_3(2000)^6}$

Q.6 Simplify:

$$\frac{81^{\frac{1}{\log_5 9}}+3^{\frac{1}{\log\sqrt{6}^3}}}{409}\left(\left(\sqrt{7}\right)^{\frac{2}{\log_{25}}}-(125)^{\log_5 6}\right)$$

Q.7 Simplify:

$$5^{\log_5 2^{(1)}}+\log_{\sqrt{2}}\frac{4}{\sqrt{7}+\sqrt{3}}+\log_{1/2}\frac{1}{10+2\sqrt{21}}$$

Q.8 Given that $\log_2 a$ = s, $\log_4 b = 5^2$ and $\log_2 (8) = \frac{2}{5^3+1}$. Write $\log_2 \frac{2}{5^3+1}$ as function of 's' (a, b, c > 0) c ≠ 1) .

Q.9 Prove that $\frac{\log_2 24}{\log_{96} 2}-\frac{\log_2 192}{\log_{12} 2}=3$

Q.10 Prove that $a^x - b^y = 0$ wher x = $\sqrt{\log_a b}$ and y = $\sqrt{\log_b a}$, a > 0, b > 0 & a, b = 1.

Q.11 (a) Solve for x, $\frac{\log_{10}(x-3)}{\log_{10}(x^2-21)}=\frac{1}{2}$

(b) log (log x) + log ($\log x^3$ –2) = 0; where base of log is 10 everywhere

(c) $\log_x 2 . \log_{2x} 2 = \log_{4x} 2$

(d) $5^{\log x} + 5x^{\log 5} = 3$ (a > 0); where base of log is a

Q.12 Solve the system of equations:

$\log_a x \log_a (xyz) = 48$

$\log_a y \log_a (xyz) = 12$

$\log_a z \log_a (xyz) = 84$

Q.13 Let 'L' denotes the antilog of 0.4 to the base 1024.

and 'M' denotes the nuber of digits in 6^{10} (Given $\log_{10}2$ =0.3010, $\log_{10}3$ = 0.4771) and 'N' denotes the number of positive integers which have the characteristic 2, when base of the logarithm is 6. Find the value of LMN.

Q.14 Prove the identity.

$\log_a N . \log_b N + \log_b N . \log_c N + \log N . \log_a N = \frac{\log_a N \log_b N \log_c N}{\log_{abc} N}$

Q.15 If x, y > 0, $\log_y x + \log_x y = \frac{10}{3}$ and xy = 144, then $\frac{x+y}{2} = \sqrt{N}$ where N is a natural number, find the vaue of N.

Q.16 If $\log_{10}2$ = 0.0310, $\log_{10}3$ = 0.4771. Find the number of integers in:

(a) 5^{200}

(b) 6^{15}

(c) the number of zeros after the decimal in 3^{-100}.

Q.17 $\log_5 120$ + (x – 3) –2 $\log_5$ (1 – 5^{x-2}) = – $\log_5$ (2 – 5^{x-4})

Q.18 $\log_{x+1}(x^2 + x - 6)^2 = 4$

Q.19 x + $\log_{10}(1 + 2^x)$ = x $\log_{10}5$ + $\log_{10}6$

Q.20 If 'x' and 'y' are real numbers such that, log (2y – 3x) = log x + log y, find $\frac{x}{y}$.

Q.21 If $a = \log_{12} 18$ & $b = \log_{24} 54$ then find the value of $ab + 5(a - b)$

Q.22 Find the value of $\log_3 x$ if following is true

$$\sqrt{\log_9(9x^3)\log_3(3x)} = \log_3 x^3$$

Q.23 Positive numbers x, y and z satisfy $xyz = 10^{11}$ and $(\log_{10} x)(\log_{10} yz) + (\log_{10} y)(\log_{10} z) = 468$. Find the value of $(\log_{10}x)^2 + (\log_{10}y)^2 + (\log_{10}z)^2$.

Q.24 Find the sum of all solutions of the equation

$$3^{(\log_9 x)^{2-2}} 2^{\log_a x+5} = 3\sqrt{3} \quad 3^{(\log_9 x)^{2-2}} 2^{\log_a x+5} = 3\sqrt{3}$$

Q.25 Let a, b, c, d are positive integers such that $\log_a b = 3/2$ and $\log_c d = 5/4$. If $(a - c) = 9$, find the value of $(b - d)$.

Q.26 Find the product of the positive roots of the equation $\sqrt{(2008)}\,(x)^{\log_{2008} x} = x^2$

Q.27 Find x satisfying the equation

$$\log^2\left(1+\frac{4}{x}\right)+\log^2\left(1-\frac{4}{x+4}\right) = 2\log^2\left(\frac{2}{x-1}-1\right)$$

Q.28 Solve: $\log_3\left(\sqrt{x}+\left|\sqrt{x}-1\right|\right)$

$= \log_9\left(4\sqrt{x}-3+4\left|\sqrt{x}-1\right|\right)$

Q.29 Prove that

$$2^{\left(\sqrt{\log_a \sqrt[4]{ab}+\log_b \sqrt[4]{ab}}-\sqrt{\log_a \sqrt{4\frac{b}{a}}+\log \sqrt{\frac{a}{b}}}\right)\sqrt{\log_a}}$$

$$= \begin{cases} 2 & \text{if } b \ge a > 1 \\ 2^{\log_a b} & \text{if} 1 < b < a \end{cases}$$

Q.30 Find the value of x satisfying the equation

$$\sqrt{\left[\log_3(3x)^{1/3}+\log_x(3x)^{1/3}\right]\log_3 x^3}$$

$$+\sqrt{\left[\log_3\left(\sqrt{\frac{x}{3}}\right)^{1/3}+\log_x\left(\frac{3}{x}\right)^{1/3}\right]\log_3 x^3} = 2$$

EXERCISE 2 JEE ADVANCED

Q.1 Number of ordered pair(s) satisfying simultaneously, the system of equations, $2^{\sqrt{x}+\sqrt{y}} = 256$ & $\log_{10}\sqrt{xy} - \log_{10} 1.5 = 1$, is:

(A) zero (B) exactly one
(C) exactly two (D) more than two

Q.2 Let ABC be a triangle right angled at C. The value of $\frac{\log_{b+c} a + \log_{c-c} a}{\log_{b+c} a.\log_{c-b} a}$

(b + c ≠, c – b ≠ 1) equals

(A) 1 (B) 2
(C) 3 (D) ½

Q.3 Let B, C, P and L be positive real number such that log (B · L) + log (B · P) = 2; log (P · L) + log (P · C) = 3; log (C · B) + log (C · L) = 4. The value of the product (BCPL) equals (base of the log is 10)

(A) 10^2 (B) 10^3
(C) 10^4 (D) 10^9

Q.4 If the equation $\frac{\log_{12}(\log_8(\log_4 x))}{\log_5(\log_4(\log_y(\log_2 x)))} = 0$ has a solution for 'x' when c < y < b, y ≠ a, where 'b' is as large as possible and 'c' is an small as possible, then the value of (a + b + c) is equal to

(A) 18 (B) 19
(C) 20 (D) 21

Q.5 The expression, $\log_p \log_p \underbrace{\sqrt{p\sqrt{p\sqrt{p\sqrt[p]{P}}}}}_{\text{n radical sign}}$

where $p \ge 2, p \in N$, when simplified is-

(A) independent of p, but dependent on n
(B) independent of n, but dependent of p
(C) dependent on both p & n
(D) negative

Q.6 The number N = $\frac{1+2\log_3 2}{(1+\log_3 2)^2} + \log_6^2 2$

when simplified reduces to-

(A) a prime number
(B) an irrational number
(C) a real number is less than $\log_3\pi$
(D) a real which is greater than $\log_7 6$

Q.7 Solution set of the inequality

$$(\log_2 x)^4 - \left(\log_{12}\frac{x^2}{8}\right)^2 + 9\log_2\left(\frac{32}{x^2}\right) < 4(\log_{12} x)^2$$

is (a, b) ∪ (c, d) then the correct statement is

(A) a = 2b and d = 2c
(B) b = 2a and d = 2c
(C) $\log_e d = \log_b a$
(D) there are 4 integers in (c, d)

Q.8 The value of x satisfying the equation $2^{2x} - 8\cdot 2^x = -12$, is

(A) $1+\frac{\log 3}{\log 2}$ (B) $\frac{1}{2}\log 6$
(C) $1+\log\frac{3}{2}$ (D) 1

Q.9 Statement-1: $\sqrt{\log_x \cos(2\pi x)}$ is a meaningful quantity only if x ∈ (0, ¼) ∪ (3/4, 1).

Because

Statement-2: If the number N > 0 and the base of the logarithm b (greater than zero not equal to 1) both lie on the same side of unity then $\log_b N > 0$ and if they lie on different side of unit then $\log_b N < 0$.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false
(D) Statement-1 is false, statement-2 is true

Q.10 Statement-1:

$\log_2(2\sqrt{17-2x}) = 1 - \log_2(x-1)$ has a solution.

because

Statement-2: Change of base in logarithms is possible.

Q.11 Solution set of the inequality $3^x(0.333.....)^{x-3} \le (1/27)^x$ is:

(A) [3/2, 5]
(B) (B) $(-\infty, 3/2]$
(C) $(2,\infty)$
(D) None of these

Q.12 Solution set of the inequality

(A) $(-1,\infty)$ (B) $(0,\infty)$

(C) $(2,\infty)$ (D) None of these

Q.13 Solution set of the inequality

$\left(\frac{1}{5}\right)^{\frac{2x+1}{1-x}} > \left(\frac{1}{5}\right)^{-1}$ is-

(A) $(-\infty,-2)\cup(1,\infty)$

(B) (1, 4)

(C) $(-\infty,1)\cup(2,\infty)$

(D) None of these

Paragraph for Question Nos. 14 – 16

Equations of the form (i) f ($\log_a x$) = 0, a > 0, a $\neq$ 1 and (ii) g ($\log_x A$) = 0, A > 0, then Eq. (i) is equivalent to f(t) = 0, where t = $\log_a x$. If t_1, t_2, t_3, ... t_k are the roots of f(t) = 0, then $\log_a x$ = t, $\log_a$ x = t_2, ... , x = t_k and eq. (ii) is equivalent to f(y) = 0, where y $\log_x$ A. If y_f , y_2, y_3, ..., y_k are the root of f(y) = 0, then $\log_x$ A = y_1, $\log_x$, A = y_2, ... , $\log_x$ A = y_k.

On the basis of above information, answer the following questions.

Q.14 The number of solution of the equation $\log_x^3 10 - 6\log_x^2 10 + 11\log_x 10 - 6 = 0$ is:

(A) 0 (B) 1

(C) 2 (D) 3

Q.15 The set of all x satisfying the equation $x^{\log_2 x^2 + \log_2 x^2 - 10} = \frac{1}{x^2}$ is-

(A) {1, 9} (B) $\left\{9, \frac{1}{81}\right\}$

(C) $\left\{1, 4, \frac{1}{81}\right\}$ (D) $\left\{1, 9, \frac{1}{81}\right\}$

Q.16 If $\frac{(\ln x)^2 - 3\ln x + 3}{\ln x - 1} < 1$, then x belongs to:

(A) (0, e) (B) (1, e)

(C) (1, 2e) (D) (0, 3e)

Q.17 Let

$a = (\log_3 81)(\log_{6561} 625)(\log_{125} 216)(\log_{1296} 2401)$

b denotes the sum of the roots of the equation $x^{\log_2 x} = (2x)^{\log \sqrt[n]{x}}$ and c denotes the sum of all natural solution of the equation |x + 1| + |x – 4| = 7. Find the value of (a + b) ÷ c.

Q.18

Column-I	Column-II
(A) The value of x for which the radical product $\sqrt{3\sqrt{x}\sqrt{7x+\sqrt{4x-1}}}\sqrt{2x+\sqrt{4x-1}}\sqrt{3\sqrt{x}+\sqrt{7x+\sqrt{4x-1}}}$ is equal to 13, is not greater than	(P) 4
(B) Let P(x) = $x^7 - 3x^5 + x^3 - 7x^2 + 5$ and Q (x) = x – 2. The remainder of $\frac{P(x)}{Q(x)}$ is not smaller than	(Q) 7
(C) Given a right triangle with side of length a, b and c and area equal to $a^2 + b^2 - c^2$. The ratio of the larger to the smaller leg of the triangle is	(R) 10
(D) If a, b and c $\in$ N such $\left(\sqrt[3]{4}+\sqrt{2}-2\right)\left(a\sqrt[3]{4}+b\sqrt[3]{2}+c\right)=20$ Then the value of (a + b – c), is not equal to	(S) 17

• PlancEssential Questions •

RECOMMENDED DURING REVISION

EXERCISE 1 JEE MAIN/BOARDS

Q.2 Q.3 Q.15 Q.25 Q.26 Q.30

EXERCISE 2 JEE MAIN

Q.3 Q.5 Q.9 Q.10

EXERCISE 1 JEE ADVANCED

Q.6 Q.12 Q.14 Q.16 Q.23 Q.30

EXERCISE 2 JEE ADVANCED

Q.4 Q.6 Q.11 Q.15 Q.17 Q.18

ANSWER KEY

EXERCISE 1 JEE MAIN/BOARDS

Q.1 (i) $\frac{5}{4}$ (ii) $\frac{4}{3}$ (iii) 2 (iv) 6 (v) $-\frac{1}{2}$ (vi) 1 (vii) $-\frac{1}{2}$

Q.3 (i) 31 (ii) 5

Q.4 (i) $\sqrt{3}$ (ii) 2 (iii) –5 (iv) 0 (v) $10^{\frac{\sqrt{3}+1}{2}}, 10^{\frac{1-\sqrt{3}}{2}}$ (vi) $\frac{1}{10^5}, 1000$ (vii) $\sqrt[3]{2}, \sqrt[3]{2}$

Q.5	$\frac{2^4}{5^{1/x}}$	Q.6	1
Q.7	$10^{-4}, 10^1$	Q.8	$10^{-3}, 10, 10^2$
Q.9	2, 16	Q.10	1/3, $(1/3)^4$
Q.11	$2^{\log_a b, 3\log_a b}$	Q.12	$3+\sqrt{6}$
Q.13	– 5	Q.14.	3, 1/9
Q.15	$2^{\left(-1+\sqrt{\frac{17}{5}}\right)}, 2^{\left(-1-\sqrt{\frac{17}{5}}\right)}$	Q.16	1/16, 2
Q.17	$2^7, 2^{-1}$	Q.18	$8, -\frac{41}{5}$
Q.19	4, 1	Q.20	3
Q.21	48	Q.22	13
Q.23	1, 9	Q.24	$\frac{1}{4}, \frac{1}{2}$
Q.25	98	Q.26	0
Q.27	2	Q.28	3
Q.29	$(10)^{-1/4}$, (10)	Q.30	(–1), 2

EXERCISE 2 JEE MAIN

Q.1	B	Q.2	D	Q.3	C	Q.4	D	Q.5	B	Q.6	D
Q.7	B	Q.8	A , B, C, D			Q.9	B				

Q.10 (A–P), (B–P, R, S); (C–P, R); (D–P, Q, R) Q.11 2 Q.12 50

EXERCISE 1 JEE ADVANCED

Q.1	12	Q.2	(a) –1 (b) $\log_b N$
Q.3	(a) 2	Q.4	$(61)^2$
Q.5	1/6	Q.6	1
Q.7	6	Q.8	$2s + 10s^2 - 3(s^3 + 1)$
Q.11	(a) 5 (b) 10 (c) $2^{\pm\sqrt{2}}$ (d) $2^{-\log x}$, where base of log is 5		
Q.12	(a^4, a, a^7) (a^{-4}, a^{-1}, a^{-7})	Q.13	23040
Q.15	507		
Q.16	(a) 140 (b) 12 (c) 47	Q.17	-0.410
Q.18	1	Q.19	1
Q.20	4/9	Q.21	1
Q.22	$\frac{5+3\sqrt{5}}{10}$	Q.23	5625
Q.24	2196	Q.25	93
Q.26	$(2008)^2$	Q.27	$\sqrt{2}, \sqrt{6}$
Q.28	$[0, 1] \cup \{4\}$	Q.30	$[1/3, 3] - \{1\}$

EXERCISE 2 JEE ADVANCED

Q.1	C	**Q.2**	B	**Q.3**	B	**Q.4**	B	**Q.5**	A, D	**Q.6**	C, D
Q.7	B, C	**Q.8**	A, D	**Q.9**	D	**Q.10**	B	**Q.11**	D	**Q.12**	D
Q.13	B	**Q.14**	D	**Q.15**	D	**Q.16**	A	**Q.17**	1		

Q.18 (A–Q, R, S); (B–P, Q, R, S); (C–P); (D–R)

SOLUTIONS

EXERCISE – 1 JEE MAIN

Sol.1 (i) $\log_{16}32 = \log_{2^4} 2^5$

we know $\log_{x^n} y^m = \frac{m}{n}\log_x y$

so $\Rightarrow \log_{2^4} 2^5 = \frac{5}{4}\log_2 2 = \frac{5}{4}$

(ii)$\log_8 16$

$= \log_{2^3} 2^4 = \frac{4}{3}\log_2 2 = \frac{4}{3}(1) = \frac{4}{3}$

(iii)$\log_{1/3}(1/9) = \log_{1/3}(1/3)^2$

$= 2\log_{1/3}(1/3) = 2.(1) = 2$

(iv)$\log_{2\sqrt{3}}(1728)$

$= \log_{2\sqrt{3}}(2\sqrt{3})^6 = 6\log_{2\sqrt{3}} 2\sqrt{3} = 6(1) = 6$

(v)$\log_2 \cos 45°$

$= \log_2 \frac{1}{\sqrt{2}} = \log_2 (2)^{-\frac{1}{2}} = -\frac{1}{2}\log_2 2 = -\frac{1}{2}$

Sol.3 We have to find out no. of digits in

(i)$2^{100} = x$ (assume)

$\Rightarrow \log_{10}x = \log_{10} 2^{100} = 100\log_{10} 2$

$= 100(0.3010) = 30\ 103$

$\Rightarrow x = 10^{.30\ 103} = 10^{30}(10)^{0.103}$

total no. of digit = 30 + 1 = 31

(ii)$x = 3^{10}$

$\Rightarrow \log_{10}x = \log_{10}3^{10} = 10\log_{10}3$

$= 10(0.47712) = 4.7712$

$\Rightarrow x = 10^{4.7712} = 10^4\ 10^{0.7712}$

total no. of digits = 4 + 1 = 5

Sol.4 $\log_{x-1} 3 = 2$ ($x \neq 1, 2$)

$\frac{1}{2}\log_{x-1}3 = 1 \Rightarrow \log_{x-1}3^{1/2} = 1$

$3^{1/2} = x - 1$

$\Rightarrow x = 1 + \sqrt{3}$

(ii)$\log_3(3^x - 8) = 2 - x$

$\Rightarrow (3^x - 8) = (3)^{2-x} = 3^2\ 3^{-x} = 93^{-x}$

$\Rightarrow 3^x - 93^{-x} = 8$

assume $3^x = y$

$\Rightarrow y - \frac{9}{y} = 8$

$\Rightarrow y^2 - 9 = 8y$

$\Rightarrow y^2 - 8y - 9 = 0$

$\Rightarrow y = \frac{8 \pm \sqrt{8^2 + 4(1)(9)}}{2(1)}$

$\Rightarrow y = \frac{8 \pm \sqrt{64 + 36}}{2} = \frac{8 \pm \sqrt{100}}{2}$

$\Rightarrow y = 4 \pm 5 = 9, -1$

so $3^x = 9 \Rightarrow 3^x = 3^2 \Rightarrow x = 2$

$3^x = -1 \Rightarrow$ no solution

$x = 2$

(iii)$\log_{5-x}(x^2 - 2x + 65) = 2$

$\Rightarrow x^2 - 2x + 65 = (5 - x)^2 = x^2 + 5^2 - 2(5)x$

$-2x + 65 = 25 - 10x$

$10x - 2x = 25 - 65 = -40$

$8x = -40$

$\Rightarrow x = -\frac{40}{8} = -5$

(iv) $\log_3(x + 1) + \log_3(x + 3) = 1$

$\Rightarrow \log_3[(x + 1) . (x + 3)] = 1$

$(x + 1)(x + 3) = (3)^2$

$x^2 + x + 3x + 3(1) = 3$

$x^2 + 4x = 0$

$x(x + 4) = 0$

$\Rightarrow x = 0, -4$

but at $x = -4$ equation is

$\log_3(-4 + 1) + \log_3(-4 + 3) = 1$

it can't be –ve so $x \neq -4 \Rightarrow x = 0$

(v) $x^{2\log x} = 10\, x^2$

take logarithms is both sides

$\log_{10}(x^{2\log x}) = \log_{10} 10x^2$

$2\log_{10}x\,(\log_{10}x) = \log_{10}10 + \log_{10}x^2$

$2\log_{10}x(\log_{10} x) = 1 + 2\log_{10} x$

assume $\log_{10} x = y$.... (i)

$\Rightarrow 2y(y) = 1 + 2y$

$\Rightarrow 2y^2 = 1 + 2y$

$\Rightarrow 2y^2 - 2y - 1 = 0$

$$\Rightarrow y = \frac{2 \pm \sqrt{2^2 - 4(2)(-1)}}{2(2)}$$

$$y = \frac{2 \pm \sqrt{4+8}}{4} = \frac{2 \pm 2\sqrt{3}}{4} = \frac{1 \pm \sqrt{3}}{2}$$

so from equation (i)

$$\log_{10}x = \frac{1 \pm \sqrt{3}}{2}$$

$$\Rightarrow x = 10^{\frac{(1+\sqrt{3})}{2}} \text{ and } 10^{\frac{(1-\sqrt{3})}{2}}$$

(vi) $x^{\log\frac{x+5}{3}} = 10^{5+\log x}$

take logarithm (base 10) on both side

$$\log.\left[x^{\frac{\log x+5}{3}}\right] = \log_{10} 10^{5+\log x}$$

$$\Rightarrow \left(\frac{\log x + 5}{3}\right)\log_{10}x = (5+\log x)\log_{10}10$$

$$\Rightarrow \left(\frac{5 + \log x}{3}\right)\log_{10}x = (5 + \log x).(1)$$

$\Rightarrow \log_{10}x = 1(3) = 3$

$\Rightarrow x = 10^3 = 1000$

2nd solution $\Rightarrow 5 + \log x = 0$

$\Rightarrow \log_{10}x = -5$

$\Rightarrow x = 10^{-5}$

(vii) $x^{\log_3 x} = 9$

take logarithm (base 3) in both side

$\log_3[x^{\log_3^x}] = \log_3 9 = \log_3 3^2 = 2\log_3 3$

$(\log_3 x)^2 = 2$

$\Rightarrow |\log_3 x| = 2^{1/2} \Rightarrow \log_3 x = \pm\sqrt{2}$

$\Rightarrow x = 3^{\sqrt{2}},\ 3^{-\sqrt{2}}$

Sol.5 $1 - \log 5 = \frac{1}{3}\left(\log\frac{1}{2} + \log x + \frac{1}{3}\log 5\right)$

$$3(1 - \log 5) = \log\frac{1}{2} + \log x + \frac{1}{3}\log 5$$

$$3 - 3\log 5 = \log\frac{1}{2} + \log 5^{1/3} + \log x$$

$$\Rightarrow 3 = \log 5^3 + \log\frac{1}{2} + \log 5^{1/3} + \log x$$

$$\Rightarrow 3 = \log\left[5^3 \times \frac{1}{2} \times 5^{1/3}\right] + \log x$$

$$\Rightarrow \log x = 3 - \log\left[5^{3+\frac{1}{3}} \times \left(\frac{1}{2}\right)\right]$$

$$\log x = \log_{10}10^3 - \log(5^{10/3} \times 2^{-1})$$

$$= \log\left(\frac{10^3}{5^{10/3}2^{-1}}\right) = \log_{10}\frac{5^3 \times 2^3}{5^{10/3} \times 2^{-1}}$$

$$= \log_{10}[5^{\frac{9-10}{3}}\,2^{3+1}] = \log_{10}[5^{-1/3}\,2^4]$$

$$\log x = \log_{10}\frac{2^4}{5^{1/3}} \Rightarrow x = \frac{2^4}{5^{1/3}}$$

Sol.6

$$\log x - \frac{1}{2}\log\left(x - \frac{1}{2}\right) = \log\left(x + \frac{1}{2}\right) - \frac{1}{2}\log\left(x + \frac{1}{8}\right)$$

$$2\log x - \log\left(x - \frac{1}{2}\right) = 2\log\left(x + \frac{1}{2}\right) - \log\left(x + \frac{1}{8}\right)$$

$$\log x^2 - \log\left(x - \frac{1}{2}\right) = \log\left(x + \frac{1}{2}\right)^2 - \log\left(x + \frac{1}{8}\right)$$

$$\Rightarrow \log\left(\frac{x^2}{x - \frac{1}{2}}\right) = \log\left[\frac{\left(x + \frac{1}{2}\right)^2}{\left(x + \frac{1}{8}\right)}\right]$$

$$\Rightarrow \log\left(\frac{x^2}{x - \frac{1}{2}}\right) - \log\left[\frac{\left(x + \frac{1}{2}\right)^2}{\left(x + \frac{1}{8}\right)}\right] = 0$$

$$\Rightarrow \log\left[\frac{x^2}{\left(x - \frac{1}{2}\right)} \times \frac{x + \frac{1}{8}}{\left(x + \frac{1}{2}\right)^2}\right] = 0$$

$$\Rightarrow \left(\frac{x^2}{x - \frac{1}{2}}\right)\left(\frac{x + \frac{1}{8}}{\left(x + \frac{1}{2}\right)^2}\right) = 1$$

$$\Rightarrow \frac{x^2\left(x + \frac{1}{8}\right)}{\left(x^2 - \frac{1}{4}\right)\left(x + \frac{1}{2}\right)} = 1$$

$$x^2\left(x + \frac{1}{8}\right) = \left(x^2 - \frac{1}{4}\right)\left(x + \frac{1}{2}\right)$$

$$\Rightarrow x^3 + \frac{x}{8} = x^3 + \frac{x^2}{2} - \frac{x}{4} - \frac{1}{4}\left(\frac{1}{2}\right)$$

$$\Rightarrow \frac{x}{8} = \frac{x^2}{2} - \frac{x}{4} - \frac{1}{8}$$

$$\Rightarrow x = 4x^2 - 2x - 1$$

$$\Rightarrow 4x^2 - 3x - 1 = 0$$

$$\Rightarrow x = \frac{+3 \pm \sqrt{(3)^2 - 4(4)(-1)}}{2(4)}$$

$$\Rightarrow x = \frac{+3 \pm \sqrt{9 + 16}}{8}$$

$$\Rightarrow x = \frac{+3 \pm \sqrt{25}}{8} = \frac{3 \pm 5}{8}$$

$$\Rightarrow x = \frac{3-5}{8} \text{ or } \frac{3+5}{8}$$

$$x = \frac{-2}{8} \text{ or } \frac{8}{8}$$

$$x = -\frac{1}{4} \text{ or } 1$$

at $x = -\frac{1}{4}$ equation

$$\underbrace{\log\left(-\frac{1}{4}\right)}_{\downarrow} = \frac{1}{2}\log\left(-\frac{1}{4} - \frac{1}{2}\right)$$

it is not possible

so $x \neq -\frac{1}{4}$ $\quad \Rightarrow x = 1$

Sol.7 $x^{\frac{\log_{10}x+7}{4}} = 10^{\log x+1}$

take logarithm on both side

$\log\left(x^{\frac{\log_{10}x+7}{4}}\right) = \log 10^{\log x+1}$

$\Rightarrow \left(\frac{\log_{10}x+7}{4}\right)(\log_{10}x)=(\log x+1)\log_{10}10$

$\Rightarrow$ assume $\log_{10} x = y$

$\Rightarrow \left(\frac{y+7}{4}\right)(y) = y + 1$

$\Rightarrow y^2 + 7y = 4(y + 1) = 4y + 4$

$\Rightarrow y^2 + 7y - 4y - 4 = 0$

$\Rightarrow y^2 + 3y - 4 = 0$

$\Rightarrow (y + 4)(y - 1) = 0$

$\Rightarrow y = -4$ and $+1$

$\Rightarrow \log_{10} x = -4$ or 1

$x = 10^{-4}$ or 10

Sol.8 $\left(\frac{\log x}{2}\right)^{\log^2 x+\log x^2-2} = \log\sqrt{x}$

$\Rightarrow \left(\log x^{1/2}\right)^{\log^2 x+\log x^2-2} = \log x^{1/2}$

$\Rightarrow \log^2 x + \log x^2 - 2 = 1$ or $\log x^{1/2} = 1$

$\Rightarrow \frac{1}{2}\log x = 1$

$\Rightarrow \log^2 x + 2\log x - 2 = 1$; $\log_{10} x = 2 \Rightarrow x = 10^2$

assume $\log x = y$

$\Rightarrow y^2 + 2y - 2 = 1$

$\Rightarrow y^2 + 2y - 2 - 1 = 0$

$\Rightarrow (y + 3)(y - 1) = 0$

$y = -3$ or $y = 1$

$\log_{10} x = -3$ or $\log_{10} x = 1$

$\Rightarrow x = 10^{-3}$ or 10^1

$\Rightarrow x = 10^{-3}, 10, 10^2$

Sol.9 $3\sqrt{\log_2 x} - \log_2 8x + 1 = 0$

$3\sqrt{\log_2 x} = \log_2 2^3 x - 1 = \log_2 2^3 + \log x - 1$

$3\sqrt{\log_2 x} = 2 + \log_2 x$

assume $\log_2 x = y$

$3\sqrt{y} = 2 + y$

square on both sides

$(3\sqrt{y})^2 = (2 + y)^2$

$9y = 2^2 + y^2 + 2(2)(y)$

$9y = 4 + y^2 + 4y$

$y^2 - 5y + 4 = 0$

$(y - 4)(y - 1) = 0$

$\Rightarrow y = 4$ or $y = 1$

$\log_2 x = 4$ or $\log_2 x = 1$

$\Rightarrow x = 2^4$ or $x = 2^1$

$\Rightarrow x = 16$ or 2

Sol.10 $\log_{1/3} x - 3\sqrt{\log_{1/3} x} + 2 = 0$

$\log_{1/3} x + 2 = 3\sqrt{\log_{1/3} x}$

assume $\log_{1/3} x = y$ .... (i)

$\Rightarrow y + 2 = 3\sqrt{y}$

$\Rightarrow y = 4$ or $y = 1$[Refer above solution]

$\log_{1/3} x = 4$ or $\log_{1/3} x = 1$

$\Rightarrow x = \left(\frac{1}{3}\right)^4$ or $x = \left(\frac{1}{3}\right)^1$

$\Rightarrow x = \frac{1}{81}$ or $\frac{1}{3}$

Sol.11 $(a^{\log_b x})^2 - 5x^{\log_b a} + 6 = 0$

Assume $x = b^y$

$\Rightarrow (a^y)^2 - 5(a^{\log_b x}) + 6 = 0$

$\Rightarrow a^{2y} - 5a^y + 6 = 0$

$\Rightarrow (a^y - 3)(a^y - 2) = 0$

$\Rightarrow a^y = 2, 3$

$y = \log_a 2, \log_a 3 = \log_b x$

$\therefore x = 2^{\log b/\log a}, 3^{\log b/\log a}$

Sol.12 $\log_4(x^2 - 1) - \log_4(x - 1)^2 = \log_4(\sqrt{(4-x)^2})$

$\log_4\left(\frac{x^2-1}{(x-1)^2}\right) = \log_4(\sqrt{(4-x)^2})$

$\frac{x^2-1}{(x-1)^2} = \sqrt{(4-x)^2}$

$\Rightarrow \frac{(x-1)(x+1)}{(x-1)^2} = \sqrt{(4-x)^2} \qquad x \neq -1,$

$\Rightarrow \frac{x+1}{(x-1)} = \sqrt{(4-x)^2} \qquad 4-x \neq 0$

$\Rightarrow \frac{x+1}{(x-1)} = |4 - x| = \pm(4 - x)$

(A) $|4 - x| \Rightarrow 4 - x \geq 0 \Rightarrow x \leq 4$

$\frac{x+1}{x-1} = 4 - x$

$(x + 1) = (4 - x)(x - 1)$

$x + 1 = 4x - 4 - x^2 + x$

$x^2 - 4x - x + x + 1 + 4 = 0$

$x^2 - 4x + 5 = 0$

$x = \frac{4 \pm \sqrt{4^2 - 4(5)(1)}}{2(1)}$

$x = \frac{4 \pm \sqrt{16-20}}{2} = \frac{4 \pm \sqrt{-4}}{2}$ no solution

at $4 - x < 0 \Rightarrow x \geq 4 \Rightarrow |4 - x| = x - 4$

$\frac{x+1}{x-1} = x - 4$

$x + 1 = (x - 1)(x - 4) = x^2 + 4 - x - 4x$

$x^2 - 4x - x - x + 4 - 1 = 0$

$x^2 - 6x + 3 = 0$

$x = \frac{6 \pm \sqrt{6^2 - 4(3)(1)}}{2(1)}$

$x = \frac{6 \pm \sqrt{36-12}}{2} = \frac{6 \pm \sqrt{24}}{2} = 3 \pm \sqrt{6}$

but $x > 4$

so, $x = 3 + \sqrt{6}$

Sol.13 $2\log_3 \frac{x-3}{x-7} + 1 = \log_3 \frac{x-3}{x-1}$

$\log_3\left(\frac{x-3}{x-7}\right)^2 + \log_3 3 = \log_3 \frac{x-3}{x-1}$

$\log_3\left[\frac{(x-3)^2}{(x-7)^2} \times 3\right] = \log_3 \frac{x-3}{x-1}$

$\Rightarrow \frac{3(x-3)^2}{(x-7)^2} = \frac{(x-3)}{(x-1)}$

$3(x - 3)(x - 1) = (x - 7)^2$

$\Rightarrow 3x^2 + 9 - 3x - 9x = x^2 - 14x + 49$

$\Rightarrow 2x^2 + 2x - 40 = 0$

$\Rightarrow x^2 + x - 20 = 0$

$\Rightarrow (x + 5)(x - 4) = 0$

x = – 5or x = 4

at x = 4 equation is

$2\log_3\left(\frac{4-3}{4-7}\right) + 1 = \log\frac{4-3}{4-7}$

$\frac{4-3}{4-7} = \frac{+1}{-3} \Rightarrow -ve$

its not possible so

$x \neq 4, x = -5$

Sol.14 $\log_x(9x^2)\log_3^2 x = 4$

$\Rightarrow (\log_x 3^2 x^2)(\log_3 x)^2 = 4$

$\Rightarrow 2[\log_x 3x]\left[\frac{\log_e x}{\log_e 3}\right]^2 = 4$

we know that $\log_m n = \frac{\log_e n}{\log_e m}$

$\Rightarrow 2[\log_x 3 + \log_x x]\left[\frac{\log_e x}{\log_e 3}\right]^2 = 4$

$\Rightarrow \left[\frac{\log_e 3}{\log_e x}+1\right]\left[\frac{\log_e x}{\log_e 3}\right]^2 = 2$

$\Rightarrow \frac{\log_e 3}{\log_e x} \times \frac{(\log_e x)^2}{(\log_e 3)^2} + (\log_3 x)^2 = 2$

$\Rightarrow \log_3 x + (\log_3 x)^2 = 2$

assume $\log_3 x = y$

$\Rightarrow y^2 + y = 2$

$\Rightarrow y^2 + y - 2 = 0$

$\Rightarrow (y + 2)(y - 1) = 0$

y = – 2or y = 1

$\log_2 x = -2$ or $\log_3 x = 1$

$x = 3^{-2}$or $x = 3^{+1}$

$x = \frac{1}{9}$or x = 3

Sol.15 $\log_{0.5x} x^2 + 14\log_{16x} x^2 + 40\log_{4x}\sqrt{x} = 0$

$\frac{\log_2 x^2}{\log_2(0.5x)} + \frac{14\log_2 x^2}{\log_2(16x)} + \frac{40\log_2\sqrt{x}}{\log_2(4x)} = 0$

assume $\log_2 x = y$

$\Rightarrow \frac{2y}{\log_2 .2^{-1}+y} + \frac{28y}{\log_2 2^4+y} + \frac{20y}{\log_2 2^2+y} = 0$

$\Rightarrow \frac{y}{y-1} + \frac{14y}{y+4} + \frac{10y}{y+2} = 0$

y = 0or $\left(\frac{1}{y-1}+\frac{14}{y+4}+\frac{10}{y+2}\right) = 0$

$\Rightarrow \log_2 x = y \Rightarrow x = 2^y = 2^\circ = 1$

or (y + 4)(y + 2) + 14(y – 1)(y + 2)

+ 10(y – 1)(y + 4) = 0

$\Rightarrow y^2 + 8 + 6y + 14y^2 - 28 +$

$14y + 10y^2 - 40 + 30y = 0$

$\Rightarrow 25y^2 + 50y - 60 = 0$

$\Rightarrow y^2 + 2y - \frac{60}{25} = 0$

$\Rightarrow y^2 + 2y - \frac{12}{5} = 0$

$y = -\frac{2 \pm \sqrt{(2)^2 - 4(1)\left(-\frac{12}{5}\right)}}{2(1)}$

$y = \frac{2 \pm \sqrt{2^2\left(1+\frac{12}{5}\right)}}{2}$

$y = \frac{2 \pm 2\sqrt{\frac{5+12}{5}}}{2}$

$y = -1 \pm \sqrt{\frac{17}{5}}$

$\log_2 x = y$

$\Rightarrow x = 2^{\left(-1+\sqrt{17/5}\right)}$ or $2^{\left(-1-\sqrt{17/5}\right)}$

Sol.16 $\log_3[\log^2_{1/2} x - 3\log_{1/2}x + 5] = 2$

assume $\log_{1/2} x = y$

$\Rightarrow \log_3[y^2 - 3y + 5] = 2$

$y^2 - 3y + 5 = 9$

$y^2 - 3y - 4 = 0$

$(y - 4)(y + 1) = 0$

$y = 4$ or $y = -1$

$\log_{1/2} x = 4$ or $\log_{1/2}x = -1$

$x = \left(\frac{1}{2}\right)^4$ or, $x = \left(\frac{1}{2}\right)^{-1}$

$x = \frac{1}{16}$ or, $x = 2.$

Sol.17 $\log_2(x/4) = \dfrac{15}{\log_2 \frac{x}{8} - 1}$

$\Rightarrow \log_2 x - \log_2 4 = \dfrac{15}{\log_2 x - \log_2 8 - 1}$

$\Rightarrow$ assume $\log_2 x = y$

$y - 2 = \dfrac{15}{y-3-1} = \dfrac{15}{y-4}$

$\Rightarrow (y - 2)(y - 4) = 15$

$\Rightarrow y^2 - 6y + 8 = 15$

$\Rightarrow y^2 - 6y - 7 = 0$

$\Rightarrow (y - 7)(y + 1) = 0$

$y = 7$ or $y = -1$

$\log_2 x = 7$ or $\log_2 x = -1$

$x = 2^7$ or $x = 2^{-1}$

Sol.18 $\frac{1}{2}\log(5x - 4) + \log\sqrt{x+1} = 2 + \log 0.18$

$\log(5x - 4) + 2\log\sqrt{x+1} = 2[2 + \log 0.18]$

$\log(5x - 4) + \log(x + 1) = 4 + 2\log 0.18$

$\log[(5x - 4)(x + 1)] = 4 + \log(0.18)^2$

$\log[(5x - 4)(x + 1)] = \log_{10}[10^4 \times (0.18)^2]$

$(5x - 4)(x + 1) = 10^4(0.0324) = 324$

$\Rightarrow 5x^2 + x - 4 = 324$

$\Rightarrow 5x^2 + x - 328 = 0$

$x = \dfrac{-1 \pm \sqrt{(1)^2 - 4(5)(-328)}}{(10)}$

$x = -\dfrac{-1 \pm \sqrt{1 + 20(328)}}{10} = \dfrac{-1 \pm \sqrt{6561}}{10}$

$x = \dfrac{-1 \pm 81}{10} = 8, -\dfrac{41}{5}$

Sol.19 $\log x^2 = \log[5x - 4)$

$\Rightarrow x^2 = 5x - 4$

$\Rightarrow x^2 - 5x + 4 = 0$

$\Rightarrow (x - 4)(x - 1) = 0$

$x - 4 = 0$ or $x - 1 = 0$

$x = 4, 1$

Sol.20 $\frac{1}{6}\log_2(x - 2) - \frac{1}{3} = \log_{1/8}\sqrt{3x-5}$

$\frac{1}{6}\log_2(x - 2) - \frac{1}{3} = \log_{2^{-3}}\sqrt{3x-5}$

$\frac{1}{6}\log_2(x - 2) - \frac{1}{3} = -\frac{1}{3}\log_2\sqrt{3x-5}$

$\frac{1}{2}\log_2(x-2) - 1 = -\log_2\sqrt{3x-5}$

$\log_2(x-2) + 2\log_2\sqrt{3x-5} = 2$

$\log_2(x-2) + \log_2(3x-5) = \log_2 2^2$

$\Rightarrow (x-2)(3x-5) = 4$

$\Rightarrow 3x^2 + 10 - 6x - 5x = 4$

$\Rightarrow 3x^2 - 11x + 6 = 0$

$x = \frac{11 \pm \sqrt{112 - 4(3)(6)}}{2(3)} = \frac{11 \pm \sqrt{121-72}}{6}$

$x = \frac{11 \pm \sqrt{49}}{6} = 3, \frac{2}{3}$

at $x = \frac{2}{3}$

eq. $\Rightarrow \frac{1}{6}\log_2\left(\frac{1}{3} - 2\right) - \frac{1}{3}$

$= \log_{1/8}\sqrt{3\frac{(2)}{3} - 5} = \sqrt{-3}$

not possible solution

so $x = 3$

Sol.21 $\frac{\log(\sqrt{x+1}+1)}{\log(x-40)^{1/3}} = 3$

$\frac{\log(\sqrt{x+1}+1)}{\frac{1}{3}\log(x-40)} = 3$

$\log(\sqrt{x+1} + 1) = \log(x-40)$

$\sqrt{x+1} + 1 = x - 40$

$\sqrt{x+1} = x - 40 - 1 = x - 41$

square both side

$x + 1 = (x-41)^2 = x^2 + 41^2 - 2(41)x$

$x^2 - 82x - x + 41^2 - 1 = 0$

$x^2 - 83x + 1680 = 0$

$x = 83 \pm \frac{\sqrt{(83)^2 - 4(1680)(1)}}{2(1)}$

$= \frac{83 \pm \sqrt{169}}{2} = \frac{83 \pm 13}{2} = 48, 35$

for $x = 35$

equations $\frac{\log\sqrt{35+1}+1}{\log 3\sqrt{35-40}} = 3\sqrt{-5}$

not possible so

$x \neq 35$ and $x = 48$

Sol.22 $1 - \frac{1}{2}\log(2x-1) = \frac{1}{2}\log(x-9)$

$2 - \log(2x-1) = \log(x-9)$

$\log(x-9) + \log(2x-1) = 2$

$\log(x-9)(2x-1) = \log_{10}10^2$

$(x-9)(2x-1) = 100$

$2x^2 - 18x - x + 9 = 100$

$2x^2 - 19x - 91 = 0$

$x = \frac{19 \pm \sqrt{19^2 - 4(2)(-91)}}{2(2)} = \frac{19 \pm \sqrt{1089}}{4} = 13, -\frac{7}{2}$

but $x = -\frac{7}{2}$ is not in the domain

so $x = 13$

Sol.23 $\log(3x^2 + 7) - \log(3x-2) = 1$

$\log_{10}\left(\frac{3x^2+7}{3x-2}\right) = 1 = \log_{10}10$

$\frac{3x^2+7}{3x-2} = 10;\quad 3x^2 + 7 = 10(3x-2)$

$3x^2 + 7 = 30x - 20$

$3x^2 - 30x + 27 = 0$

$x^2 - 10x + 9 = 0$

$(x - 1)(x - 9) = 0$

$x = 9, 1$

Sol.24 $\left(1+\frac{1}{2x}\right)\log 3 + \log 2 = \log(27 - 3^{1/x})$

$\log 3^{(1+1/2x)} + \log 2 = \log(27 - 3^{1/x})$

$\log 2\times(3)^{1+1/2x} = \log(27-3^{1/x})$

$\Rightarrow 2 \times 3^{1+1/2x} = 27 - 3^{1/x}$

assume $3^{1/x} = y$

$\Rightarrow 2\times3\times\sqrt{y} = 27 - y$

square both sides

$\Rightarrow 2^2 \times 3^2 \times y = (27 - y)^2$

$36y = 27^2 + y^2 - 2(27)y$

$y^2 - 54y - 36y + 27^2 = 0$

$y^2 - 90y + 27^2 = 0$

$(y - 81)(y - 9) = 0$

$y = 81, 9$

so $x = \frac{1}{\log_3 y} \Rightarrow x = \frac{1}{\log_3 81}$ or $\frac{1}{\log_3 9} = \frac{1}{4}, \frac{1}{2}$

Sol.25 $\frac{1}{2}\log x + 3\log\sqrt{2+x} = \log\sqrt{x(x+2)} + 2$

$\log x + 6\log\sqrt{2+x} = 2\log\sqrt{x(x+2)} + 4$

$\log x + \log(2 + x)^3 - \log[x(x + 2)] = 2$

$\log\left[\frac{x(2+x)^3}{x(x+2)}\right] = \log 10^2$

$(2 + x)^2 = 100$

$2 + x = \pm 100$

$x \begin{bmatrix} 100-2=98 \\ -100-2=-102 \end{bmatrix}$

$x = -102$ does not satisfy the equation

So $x = 98$

Sol.26 $\log_2(4^x + 1) = x + \log_2(2^{x+3} - 6)$

$\log_2(4^x + 1) = \log_2 2^x + \log_2(2^{x+3} - 6)$

$\log_2(4^x + 1) = \log_2[2^x[2^{x+3} - 6]]$

$\Rightarrow 4^x + 1 = 2^x[2^x 2^3 - 6]$

assume $2^x = y$

$y^2 + 1 = y[8y - 6]$

$y^2 + 1 = 8y^2 - 6y$

$7y^2 - 6y - 1 = 0$

$(y - 1)(7y + 1) = 0$

$y = 1 \text{ or } y = -\frac{1}{7}$

$2^x = 1 \text{ or } 2^x = -\frac{1}{7}$

no solution

Sol.27 $\log_{\sqrt{5}}(4^x - 6) - \log_{\sqrt{5}}(2^x - 2) = 2$

$\log_{\sqrt{5}}\left(\frac{4^x-6}{2^x-2}\right) = 2 \Rightarrow \frac{4^x-6}{2^x-2} = 5$

assume $2^x = y$

$\Rightarrow \frac{y^2-6}{y-2} = 5$

$y^2 - 6 = 5(y - 2) = 5y - 10$

$y^2 - 5y - 6 + 10 = y^2 - 5y + 4 = 0$

$\Rightarrow (y - 4)(y - 1) = 0$

$y = 4$ or $y = 1$

$2^x = 4$ or $2^x = 1$

$x = 2$ or $x = 0$

$x = 0$ does not satisfy the equation

so $x = 2$

Sol.28 $\log(3^x - 2^{4-x}) = 2 + \frac{1}{4}\log 16 - \frac{x\log 4}{2}$

$\log(3^x - 2^{4-x}) = \log_{10}10^2 + \frac{1}{4}\log 2^4 - \frac{x\log 2^2}{2}$

$\log(3^X - 2^{4-X}) = \log_{10}100 + \frac{4}{4}\log 2 - \frac{x \times 2\log_2}{2}$

$\log(3^x - 2^{4-x}) = \log_{10}[100 \times 2] - \log 2^x$

$\log(3^x - 2^{4-x}) = \log_{10}\frac{(200)}{2^x}$

$\Rightarrow 3^x - \frac{2^4}{2^x} = \frac{200}{2^x}$

$\Rightarrow 3^x.2^x - 2^4 = 200$

$\Rightarrow 6^x = 200 + 2^4 = 216 = 6^3$

$\Rightarrow x = 3$

Sol.29 $\log(\log x) + \log(\log x^4 - 3) = 0$

$\log[(\log x)(\log x^4 - 3)] = 0$

$\Rightarrow (\log x)(\log x^4 - 3) = 1$

$(\log x)(4\log x - 3) = 1$

assume $\log x = y$

$y(4y - 3) = 1; \quad 4y^2 - 3y = 1$

$4y^2 - 3y - 1 = 0$

$(y - 1)(4y + 1) = 0$

$y = 1$ or $y = -\frac{1}{4}$

$\log_{10} x = 1$ or $\log_{10}x = -\frac{1}{4}$

$x = 10^1$ or $x = 10^{-\frac{1}{4}}$

$x = 10$

Sol.30 $\log_3(9^x + 9) = \log_3 3^x(28 - 2.3^x)$

$\Rightarrow 9^x + 9 = 3^x(28 - 2 . 3^x)$

$\Rightarrow$ assume $3^x = y$

So $9^x = (3^2)^x = (3^x)^2 = y^2$

$\Rightarrow y^2 + 9 = y(28 - 2y)$

$\Rightarrow y^2 + 9 = 28y - 2y^2$

$\Rightarrow 3y^2 - 28y + 9 = 0$

$\Rightarrow (3y - 1)(y - 9) = 0$

$y = 9, \frac{1}{3}$

$x = 2, -1$

EXERCISE – 2 JEE MAIN

Sol.1 $\frac{1}{\log_{\sqrt{bc}} abc} + \frac{1}{\log_{\sqrt{ac}} abc} + \frac{1}{\log_{\sqrt{ab}} abc}$

$= \frac{\log\sqrt{bc}}{\log abc} + \frac{\log\sqrt{ac}}{\log abc} + \frac{\log\sqrt{ab}}{\log abc}$

$= \frac{\log\sqrt{bc} + \log\sqrt{ac} + \log\sqrt{ab}}{\log abc}$

$= \frac{\log\sqrt{bc}\sqrt{ac}\sqrt{ab}}{\log abc} = \frac{\log abc}{\log abc} = 1$

Sol.2 $\log_2(2x^2) + \log_2 x.\ x^{\log_x(\log_2 x+1)}$

$+ \frac{1}{2}\log_4 2x^4 + 2^{-3\log_{1/2}(\log_2 x)} = 1$

$\Rightarrow \log_2(2x^2) + (\log_2 x)\ (x)^{\log_x(\log_2 x+1)}$

$+ \frac{1}{2}\log_4 4^{1/2}x^4 + 2^{-3\log_{1/2}(\log_2 x)} = 1$

$\Rightarrow 1 + 2\log_2 x + (\log_2 x)\,(x)^{\log_x(\log_2 x+1)}$

$+ \frac{1}{4}\log_4 4 + \frac{4}{2}\log_4 x + 2^{3\log_2(\log_2 x)} = 1$

$\Rightarrow 1 + 2\log_2 x + (\log_2 x + 1)(\log_2 x) + \frac{1}{4}$

$+ \log_2 x + (2)^{\log_2(\log_2 x)^3} = 1$

$\Rightarrow 1 + 2\log_2 x + (\log_2 x)(\log_2 x + 1) + \frac{1}{4}$

$+ \log_2 x + (\log_2 x)^3 = 1$

assume $\log_2 x = y$

$\Rightarrow 2y + y(y + 1) + \frac{1}{4} + y + y^3 = 0$

$\Rightarrow y^3 + 4y + y^2 + \frac{1}{4} = 0$

Differential of equation is

$\frac{d}{dy}[y^3 + 4y + y^2 + \frac{1}{4}] = 0$

$\Rightarrow 3y^2 + 4 + 2y = 0$

$\Rightarrow y = -\frac{-2 \pm \sqrt{2^2 - 4(4)(3)}}{2(3)}$

$y = \frac{-2 \pm \sqrt{-48 + 4}}{6}$

no solution so there is no minima and maximum

at $y = 0 \Rightarrow f(y) = 0 + 0 + 0 + \frac{1}{4} > 0$

$y = -1$, $f(y) = (-1)^3 + 4(-1) + (-1)^2 + \frac{1}{4}$

$\Rightarrow -1 - 4 + 1 + \frac{1}{4} = -4 + \frac{1}{4} = -\frac{15}{4} < 0$

it mean f(y) is zero some where

$-1 < y < 0$

so $\log_2 x < 0$

but in equation (original) $\log_2 x$ should be positive so there is no solution

Sol.3 $x = (75)^{-10}$

$\log_{10} x = \log_{10}(75)^{-10} = -10\log_{10}75$

$= -10\log_{10} 100 \times \frac{3}{4}$

$= -10[\log_{10}10^2 + + \log_{10}3 - \log_{10}2^2]$

$= -10[2 + 0.477 - 2(0.301)] = -18\ 75$

$\Rightarrow x = 10^{-18.75} = 10^{-18} \times 10^{-0.75}$

number of zeros = 18

Sol.4 $5x^{\log_2 3} + 3^{\log_2 x} = 162$

assume $x = 2^y$

$\Rightarrow 5.2^{y\log_2 3} + 3^{\log_2 2y} = 162$

$\Rightarrow 5.2^{\log_2 3^y} + 3^{y\log_2 2} = 162$

$\Rightarrow 5.3^y + 3^y = 6.3^y = 162$

$3^y = \frac{162}{6} = 27 = 3^3$

$y = 3$

$x = 2^y = 2^3 = 8$

$\log_4 x = \log_4 8 = \log_4 (4)^{3/2} = \frac{3}{2}$

Sol.5 $(x)^{\log_{10}^2 x + \log_{10} x^3 + 3}$

$= \frac{2}{\frac{1}{\sqrt{x+1}-1} - \frac{1}{\sqrt{x+1}+1}} = B$

(assume)

$B = \frac{2}{\frac{1}{\sqrt{x+1}-1} - \frac{1}{\sqrt{x+1}+1}} = \frac{2}{\frac{\sqrt{x+1}+1-\sqrt{x+1}+1}{(\sqrt{x+1}-1)(\sqrt{x+1}+1)}}$

$B = ((\sqrt{x+1})^2 - (1)^2 = x + 1 - 1 = x$

so $(x)^{\log_{10}^2 x + 3\log_{10} x + 3} = x \Rightarrow x = 1$

or $\Rightarrow$ assume $\log_{10} x = y$

$\Rightarrow y^2 + 3y + 3 = 1$

$y^2 + 3y + 2 = 0$

$(y + 2)(y + 1) = 0$

$y = -2$ or $y = -1$

$\log_{10} x = -2$ or $\log_{10} x = -1$

$x = 10^{-2}, 10^{-1}$

$x_1, x_2, x_3 = 1, 10^{-1}, 10^{-2}$

$x_1 . x_3 = 1.10^{-2} = (10^{-1})^2 = (x_2)^2$

Sol.6 $x = 2^{\log 3}$, $y = 3^{\log 2}$

$x = 2^{\log 3} = 3^{\log 2} = y$

as $a^{\log_n m} = m^{\log_n a}$

Sol.7 $|x-3|^{3x^2-10x+3} = 1$ $\quad x \neq 3$

or if $|x - 3| = 1$

$\Rightarrow x = 2$ or 4 is solution

if $x - 3 \neq 0$

then $3x^2 - 10x + 3 = 0$ is another sol^n

$3x^2 - 10x + 3 = 0$

$(3x - 1)(x - 3) = 0$

$x = +3$ or $= \frac{+1}{3}$

but $x \neq 3$

so $x = \frac{1}{3}$

total solution $\Rightarrow x = \frac{1}{3}, 2, 4$

Sol.8 $\left(\sqrt{5\sqrt{2}-7}\right)^x + 6\left(\sqrt{5\sqrt{2}+7}\right)^x = 7$

assume $x = \log_{\sqrt{5\sqrt{2}-7}} y$

$\Rightarrow \left(\sqrt{5\sqrt{2}-7}\right)^{\log_{\sqrt{5\sqrt{2}-7}} y} + 6\left(\sqrt{5\sqrt{2}+7}\right)^{\log_{\sqrt{5\sqrt{2}-7}} y} = 7$

$\sqrt{5\sqrt{2}-7} = \sqrt{5\sqrt{2}-7} \times \frac{\sqrt{5\sqrt{2}+7}}{\sqrt{5\sqrt{2}+7}}$

$= \frac{\sqrt{50-49}}{\sqrt{5\sqrt{2}+7}} = \left(\sqrt{5\sqrt{2}+7}\right)^{-1}$

$\Rightarrow y + 6\left(\sqrt{5\sqrt{2}+7}\right)^{-\log_{\sqrt{5\sqrt{2}+7}} y} = 7$

$\Rightarrow y + 6y^{-1} = 7$

$\Rightarrow y^2 + 6 = 7y \Rightarrow y^2 - 7y + 6 = 0$

$\Rightarrow (y - 6)(y - 1) = 0$

$y = 6$ or $y = 1$

$x = \log_{\sqrt{5\sqrt{2}-7}} 6$ or $x = \log_{\sqrt{5\sqrt{2}-7}} 1 = 0$

$\Rightarrow x = \log_{(5\sqrt{2}-7)^{1/2}} 6 = 2\log_{(5\sqrt{2}-7)} 6 = \log_{(5\sqrt{2}-7)} 36$

$x = \dfrac{2}{\log_6(5\sqrt{2}-7)} = \dfrac{-2}{\log_6(5\sqrt{2}+7)}$

Sol.9 Statement-I : $5^{\log_5(x^3+1)} - x^2 = 1$ have two distinct real solution

Statement-II : $a^{\log_a N} = N$ when $a > 0$ $a \neq 1, N > 0$

$\Rightarrow 5^{\log_5 x^3+1} - x^2 = 1$

$[5^{\log_5(x^3+1)} = x^3 + 1]$ from statement-II

$\Rightarrow x^3 + 1 - x^2 = 1$

$\Rightarrow x^3 - x^2 = 0$

$\Rightarrow x^3 = x^2 \Rightarrow x = 0$ or 1

Statement-I is true and II is true and II is not the correct explanation for statement -I

Sol.10 (A) $x = \log_2 \log_9 \sqrt{6+\sqrt{6+.....\infty}}$

assume $x = \log_2 \log_9 y$

$\Rightarrow y = \sqrt{6+\sqrt{6+\cdots\infty}} = \sqrt{6+y}$

$\Rightarrow y^2 = 6 + y$

$y^2 - 6 - y = 0$

$\Rightarrow (y - 3)(y + 2) = 0$

$\Rightarrow y = 3$ or $y = -2,\ y \neq -2$

so $y = 3$

$x = \log_2 \log_9 3 = \log_2 \log_9 (9)^{1/2}$

$x = \log_2\left(\dfrac{1}{2}\right) = \log_2 2^{-1} = -1$

$x = -1$ is an integer

(B) $N = 2^{(\log_2 3.\log_3 4.\log_4 5......\log_{99} 100)}$

$N = 2^x$ assume

$\Rightarrow x = \dfrac{\log 3}{\log 2}\cdot\dfrac{\log 4}{\log 3}\cdots\dfrac{\log 100}{\log 99} = \dfrac{\log 100}{\log 2} = \log_2 100$

$N = 2^{\log_2 100} = 100$

$N = 100$ which is a composite, integer, natural number

(C) $\dfrac{1}{\log_5 3} + \dfrac{1}{\log_6 3} - \dfrac{1}{\log_{10} 3}$

$\Rightarrow \dfrac{\log 5}{\log 3} + \dfrac{\log 6}{\log 3} - \dfrac{\log 10}{\log 3} = \left(\dfrac{\log 5 + \log 6 - \log 10}{\log 3}\right)$

$\Rightarrow \dfrac{\log(5\times 6 \div 10)}{\log 3} = \dfrac{\log 3}{\log 3} = 1$

$\Rightarrow$ 1 is natural and integer number

(D)

$N = \sqrt{2+\sqrt{5}-\sqrt{6-3\sqrt{5}+\sqrt{14-6\sqrt{5}}}}$

$N = \sqrt{2+\sqrt{5}-\sqrt{6-3\sqrt{5}+\sqrt{(3-\sqrt{5})^2}}}$

$N = \sqrt{2+\sqrt{5}-\sqrt{6-3\sqrt{5}+(-\sqrt{5}+3)}}$

$N = \sqrt{2+\sqrt{5}-\sqrt{9-4\sqrt{5}}}$

$N = \sqrt{2+\sqrt{5}-\sqrt{(\sqrt{5})^2+(2)^2-2(2)\sqrt{5}}}$

$N = \sqrt{2+\sqrt{5}-\sqrt{(\sqrt{5}-2)^2}}$

$N = \sqrt{2+\sqrt{5}-\sqrt{(\sqrt{5}-2)}}$

$= \sqrt{2+\sqrt{5}-\sqrt{5}+2} = \sqrt{4} = 2$

2 is natural prime and an integer

Sol.11 x_1 and x_2 are roots of the equation

$\sqrt{2010}\ x^{\log_{2010} x} = x^2$

assume $x = (2010)^y$

$\Rightarrow (2010)^{1/2}\ (2010)^{y \log_{2010}(2010)^y} = (2010)^{2y}$

$\Rightarrow (2010)^{1/2}\ (2010)^{y^2} = (2010)^{2y}$

$\Rightarrow y^2 + \frac{1}{2} = 2y$

$y^2 - 2y + \frac{1}{2} = 0$

$\Rightarrow y = \frac{2 \pm \sqrt{2^2 - 4(1)(1/2)}}{2} = \frac{2 \pm \sqrt{2}}{2} = 1 \pm \frac{1}{\sqrt{2}}$

$x_1x_2 = (2010)^{1-\frac{1}{\sqrt{2}}}\ (2010)^{1+\frac{1}{\sqrt{2}}}$

$= (2010)^2 = (201 \times 10)^2$

no. of zero in $x_1x_2 = 2$

Sol.12 $x = 2$ or $x = 3$ satisfy the equation

$\log_4(x^2 + bx + c) = 1 = \log_4 4$

$\Rightarrow x^2 + bx + c - 4 = 0$

$\Rightarrow -b = 2 + 3 = 5$ and $c - 4 = 2 \,.\, 3 \Rightarrow c = 10$

$bc = 10(-5) = -50$

$|bc| = 50$

EXERCISE – 1 JEE ADVANCED

Sol.1

$B = (2^{\log_6 18}).(3^{\log_6 3})$

$B = 2^{\log_6 (6\times3)} . 3^{\log_6 3}$

$B = 2^{\log_6 6 + \log_6 3} . 3^{\log_6 3}$

$B = 2^{1+\log_6 3} 3^{\log_6 3} = 2 \times 2^{\log_6 3} . 3^{\log_6 3}$

$B = 2\{6\}^{\log_6 3} = 2.3 = 6$

$A = \log_{10} \dfrac{ab + \sqrt{(ab)^2 - 4(a+b)}}{2} +$

$\log_{10} \dfrac{ab - \sqrt{(ab)^2 - 4(a+b)}}{2}$

$A = \log_{10}\left[\dfrac{ab + \sqrt{(ab)^2 - 4(a+b)}}{2} \times \dfrac{ab - \sqrt{(ab)^2 - 4(a+b)}}{2}\right]$

$= \log_{10}\left[\dfrac{(ab)^2 - ((ab)^2 - 4(a+b))^{2/2}}{4}\right]$

$= \log\left[\dfrac{(ab)^2 - (ab)^2 + 4(a+b)}{4}\right] = \log \dfrac{4(a+b)}{4}$

$= \log(a + b) = \log(43 + 57) = \log 100 = 2$

$A = 2$ and $B = 6$

so $AB = 12$

Sol.2 (a) $\log_{1/3} \sqrt[4]{729\sqrt[3]{9^{-1}}.27^{-4/3}}$

$= \log_{1/3} \sqrt[4]{729\sqrt[3]{3^{-2}.3^{-4}}}$

$= \log_{1/3} \sqrt[4]{729.3^{-2}} = \log_{1/3} \sqrt[4]{81} = \log_{1/3} 3 = -1$

(b) $a^{\frac{\log_b(\log_b N)}{\log_b a}} = a^x$ say

$x = \dfrac{\log_b(\log_b N)}{\log_b a} = \log_a(\log_b N)$

so $a^x = a^{\log_a(\log_b N)} = \log_b N$

Sol.3 (a) $\log_\pi 2 + \log_2 \pi$

$\Rightarrow \dfrac{\log 2}{\log \pi} + \dfrac{\log \pi}{\log 2}$

assume $\dfrac{\log 2}{\log \pi} = x$ +ve always

$(2 < \pi < 10)$

$\Rightarrow x + \dfrac{1}{x} = c$ (assume)

$x^2 - cx + 1 = 0$

$x = \dfrac{c \pm \sqrt{c^2 - 4}}{2}$

for x to be real

$c^2 - 4 \ge 0$

$c^2 \ge 4 \Rightarrow c \ge 2$

$c = 2 \Rightarrow x = 1 = \dfrac{\log_2}{\log_2}$

for all other value $c > 2$

so $\log_\pi 2 + \log_2 \pi$ is greater than 2

(b) $\log_3 5$ and $\log_2 7$

assume $\log_3 5$ be rational

$\therefore \log_3 5 = a$

$\therefore 5 = 3^a$

This is not possible when a is rational

$\therefore$ a is irrational

Similarly, $\log_2 7 = b$ assuming b is rational

which is not possible so b is irrational

Sol.4 $\log_3 x . \log_4 x . \log_5 x = \log_3 x.\log_4 x + \log_4 x. \log_5 x + \log_5 x \log_3 x$

assume logx = y

$$\Rightarrow \frac{\log x.\log x.\log x}{\log 3\log 4\log 5}$$

$$= \frac{\log x\log x}{\log 3\log 4} + \frac{\log x\log x}{\log 4.\log 5} + \frac{\log x.\log x}{\log 5.\log 3}$$

$\Rightarrow y^3 = (\log 5)y^2 + (\log 3)y^2 + (\log 4)y^2$

$y^3 = y^2[\log 5 + \log 3 + \log 4]$

$y^3 = y^2[\log(3.4.5)] = y^2 \log 60$

y = 0 or y = log60

logx = 0or y =logx =log60

x = 1or x = 60

sum of roots = 1 + 60 = 61

square of sum of roots = $(61)^2$

Sol.5 $\frac{2}{\log_4(2000)^6} + \frac{3}{\log_5(2000)^6}$

$$\frac{2}{6\log_4(2000)} + \frac{3}{6\log_5(2000)}$$

$$\frac{1}{6}\left[\frac{2}{\log_4(4^2\times 5^3)} + \frac{3}{\log_5(5^3\times 4^2)}\right]$$

$$\frac{1}{6}\left[\frac{2}{\log_4 4^2 + \log_4 5^3} + \frac{3}{\log_5 5^3 + \log_5 4^2}\right]$$

$$\frac{1}{6}\left[\frac{2}{2+3\log_4 5} + \frac{3}{3+2\log_5 4}\right]$$

$$\frac{1}{6}\left[\frac{2}{2+\frac{3\log 5}{\log 4}} + \frac{3}{3+\frac{2\log 4}{\log 5}}\right]$$

$$\frac{1}{6}\left[\frac{2\log 4}{2\log 4+3\log 5} + \frac{3\log 5}{3\log 5+2\log 4}\right]$$

$$\frac{1}{6}\left[\frac{2\log 4+3\log 5}{2\log 4+3\log 5}\right] = \frac{1}{6}$$

Sol.6 $\frac{81^{\frac{1}{\log_5 9}} + 3^{\frac{3}{\log_{\sqrt{6}} 3}}}{409}\left((\sqrt{7})^{\frac{2}{\log_{25} 7}} - (125)^{\log_{25} 6}\right)$

$$\frac{9^{2\log_9 5} + 3^{3\log_3\sqrt{6}}}{409}\left((\sqrt{7})^{2\log_7 25} - (25)^{\frac{3}{2}\log_{25} 6}\right)$$

$$\frac{9^{\log_9 5^2} + 3^{\log_3(\sqrt{6})^3}}{409}[7^{\log_7 25} - 25^{\log_{25} 6^{3/2}}]$$

$$\frac{5^2 + (\sqrt{6})^3}{409}[25 - 6^{3/2}] = \frac{(5^2)^2 - (6^{3/2})^2}{409}$$

$$= \frac{(25)^2 - 6^3}{409} = \frac{409}{409} = 1$$

Sol.7 $(5)^{\log_{1/5}\left(\frac{1}{2}\right)} + \log_{\sqrt{2}}\frac{4}{\sqrt{7}+\sqrt{3}}$

$$+\log_{1/2}\frac{1}{10+2\sqrt{21}}$$

$$\Rightarrow 5^{\log_5 2} + \log_{2^{\frac{1}{2}}}\left(\frac{4}{\sqrt{7}+\sqrt{3}}\right) + \log_{2^{-1}}\frac{1}{10+2\sqrt{21}}$$

$$\Rightarrow 2 + \log_2\left(\frac{4}{\sqrt{7}+\sqrt{3}}\right)^2 + \log_2 10+2\sqrt{21}$$

$$\left(\frac{4}{\sqrt{7}+\sqrt{3}}\right)^2 = \frac{16}{7+3+2\sqrt{7}\sqrt{3}} = \frac{16}{10+2\sqrt{21}}$$

$$\Rightarrow 2 + \log_2\frac{16}{10+2\sqrt{21}}\left(10+2\sqrt{21}\right)$$

$= 2+ \log_2 2^4 = 2 + 4 = 6$

Sol.8 $\log_2 a = s \Rightarrow a = 2^s$

$\log_4 b = s^2 \Rightarrow b = 4^{s^2} = (2)^{2s^2}$

and $\log_{c^2} 8 = \frac{2}{s^3+1} \Rightarrow 8^{\frac{1}{2}} = c^{\frac{2}{s^3+1}}$

$\Rightarrow c = (2^{3/2})^{\frac{s^3+1}{2}}$; $c = 2^{\frac{3(s^3+1)}{4}}$

then $\frac{a^2b^5}{c^4} = \frac{(2^s)^2(2^{2s^2})^5}{\left(2^{3\frac{(s^3+1)}{4}}\right)^4} = \frac{2^{2s}\ 2^{10s^2}}{2^{3(s^3+1)}}$

$= (2)^{(2s+10s^2-3(s^3+1))}$

$\Rightarrow \log_2\frac{a^2b^5}{c^4} = (2s+10s^2-3(s^3+1))$

Sol.9 $\frac{\log_2 24}{\log_{96} 2} - \frac{\log_2 192}{\log_{12} 2}$

$\Rightarrow$ we know that

$\log_m n = \frac{1}{\log_n m}$

$\Rightarrow (\log_2 96)(\log_2 24) - (\log_2 192)(\log_2 12)$

Where

$\Rightarrow \log_2 24 = \log_2 12\times 2 = \log_2 12 + \log_2 2$

$\Rightarrow \log_2 96(\log_2 12 + \log_2 2)$

$- \log_2(96\times 2)\log_2 12$

$\Rightarrow \log_2 96\log_2 12 + \log_2 96(1)$

$- \log_2 96\log_2 12 - \log_2 12\log_2 2$

$\Rightarrow \log_2 96 - \log 12(1)$

$= \log_2\frac{96}{12} = \log_2 8 = \log_2 2^3 = 3\log_2 2 = 3$

Sol.10 We have to prove that

$a^x - b^y = 0$, where $x = \sqrt{\log_a b}$

and $y = \sqrt{\log_b a} \Rightarrow x^2 = \log_a b$

$y^2 = \log_b a \Rightarrow y^2 = \frac{1}{x^2}$

$x^2y^2 = 1$

$xy = 1$ (x, y > 0)

now $a^x - b^y = (b^{y^2})^x - (a^{x^2})^y$

$\Rightarrow (b^{xy})^y - (a^{xy})^x$

$\Rightarrow b^y - a^x$

$\Rightarrow a^x - b^y = b^y - a^x = -(a^x - b^y)$

$\Rightarrow a^x - b^y + a^x - b^y = 0$

$\Rightarrow 2(a^x - b^y) = 0$

$\Rightarrow a^x - b^y = 0$

Sol.11 (a) $\frac{\log_{10}(x-3)}{\log_{10}(x^2-21)} = \frac{1}{2}$

$\Rightarrow 2\log_{10}(x-3) = \log_{10}(x^2-21)$

$\Rightarrow \log_{10}(x-3)^2 - \log_{10}(x^2-21) = 0$

$\Rightarrow \log_{10}\frac{(x-3)^2}{(x^2-21)} = 0$

$\Rightarrow \frac{(x-3)^2}{x^2-21} = 1 \Rightarrow x^2 + 3^2 - 2(3)x = x^2 - 21$

$\Rightarrow 9 - 6x = -21 \Rightarrow 6x = 9 + 21 \Rightarrow x = \frac{30}{6} = 5$

(b) $\log(\log x) + \log(\log x^3 - 2) = 0$

$\Rightarrow \log[\log x(\log x^3 - 2)] = 0$

$\Rightarrow (\log x)(\log x^3 - 2) = 1$

$\Rightarrow (\log x)(3\log x - 2) = 1$

assume $\log x = y$

$\Rightarrow y(3y-2) = 1$

$\Rightarrow 3y^2 - 2y - 1 = 0$

$\Rightarrow 3y(y-1) + 1(y-1) = 0$

$y = -\frac{1}{3}$ or $y = 1$

$\log_{10}x = -\frac{1}{3}$ or $\log_{10} x = 1$

$x = (10)^{-\frac{1}{3}}$ or $x = 10^1$

at $x = 10^{-1/3}$ equation does not satisfied so

$x = 10$

(c)$\log_x 2 . \log_{2x}2 = \log_{4x} 2$

$\Rightarrow \frac{1}{\log_2 x} . \frac{1}{\log_2 2x} = \frac{1}{\log_2 4x}$

$\Rightarrow \log_2 2^2 + \log x = (\log_2 x)(\log_2 2 + \log_2 x)$

assume $\log_2 x = y$

$\Rightarrow 2 + y = y(1 + y)$

$\Rightarrow 2 + y = y^2 + y$

$\Rightarrow y^2 = 2 \Rightarrow y = \pm\sqrt{2}$

$\log x = \pm \sqrt{2}$

$\log_2 x = +\sqrt{2}$ or $\log_2 x = -\sqrt{2}$

$x = (2)^{\sqrt{2}}$ or $x = 2^{-\sqrt{2}}$

(d) $5^{\log_a x} + 5x^{\log_a 5} = 3, (a > 0)$

assume $x = a^y$

$\Rightarrow 5^{\log_a a^y} + 5a^{y\log_a 5} = 3$

$\Rightarrow 5^y + 5a^{\log_a 5^y} = 5^y + 5.5^y = 6.5^y = 3$

$\Rightarrow 5^y = \frac{3}{6} = \frac{1}{2} = 2^{-1}$

Take logarithm (base 5) both side

$\Rightarrow \log_5 5^y = \log_5 2^{-1}$

$\Rightarrow y = \log_5 2^{-1}$

So $x = a^y = a^{\log_5 2^{-1}}$

$x = (2^{-1})^{\log_5 a} = 2^{-\log_5 a}$

Sol.12 $\log_a x \log_a(xyz) = 48$........ (1)

$\log_a y \log_a(xyz) = 12$........ (2)

$\log_a z \log_a(xyz) = 84$........ (3)

sum of all equation

$\log_a(xyz)[\log_a x + \log_a y + \log_a z]$

$= 48 + 12 + 84 = 144 = 12^2$

$(\log_a(xyz))(\log_a(xyz)) = 12^2$

$(\log_a xyz)^2 = 12^2$

$\log_a xyz = 12 (\pm 1)$

in equation

(i) $\log_a x (\pm 12) = 48$

$\log_a x = \pm 4 \Rightarrow x = a^4, a^{-4}$

(ii)$\log_a y(\pm 12) = 12$

$\log_a y = \pm 1 \Rightarrow y = a, a^{-1}$

(iii)$\log_a z(\pm 12) = 84$

$\log_a z = \pm 7 \Rightarrow z = a^7, a^{-7}$

$(x, y, z) = (a^4, a, a^7)$ or (a^{-4}, a^{-1}, a^{-7})

Sol.13 Given

L = antilog of 0.4 to the base 1024

$\Rightarrow L = (1024)^{0.4} = (2^{10})^{0.4} = 2^4 = 16$

L = 16

And M is the number of digits in 6^{10}

$\Rightarrow \log_{10} 6^{10} = 10\log_{10} 6$

$\Rightarrow 10[0.7761] = 7.761$

$\Rightarrow 6^{10} = 10^{7.761} = 10^7\ 10^{0.761}$

no. of digits = 7 + 1 = 8

M = 8

$\Rightarrow \log_6 6^2 = 2$ (characteristic 2)

$\Rightarrow \log_6 6^3 = 3$ (characteristic 3)

Total no. of positive integers which have the characteristic 2(between 6^2 and 6^3) $= 6^3 - 6^2$

$= 216 - 36 = 180$

LMN = 16 × 8 × 180 = 23040

Sol.14 $\log_a N \cdot \log_b N + \log_b N \cdot \log_c N$

$+ \log_c N \cdot \log_a N$........ (1)

$= \dfrac{\log_a N.\log_b N.\log_c N}{\log_{abc} N}$

we know that $\log_x y = \dfrac{\log y}{\log x}$

so is equation (1) R.H.S.

$= \dfrac{\frac{\log N}{\log a}\cdot\frac{\log N}{\log b}\cdot\frac{\log N}{\log c}}{\frac{\log N}{\log abc}}$

$= \dfrac{(\log N)^2 \log abc}{(\log a)\log b(\log c)}$

$= \dfrac{\log N^2(\log a + \log b + \log c)}{\log a \log b \log c}$

$= \dfrac{(\log N)(\log N)}{\log b \log c} + \dfrac{\log N \log N}{\log a \log c}$

$+ \dfrac{\log N \log N}{\log a \log b}$

$= \log_a N \log_b N + \log_a N \log_c N$

$+ \log_b N \log_c N$

R.H.S. = L.H.S.

Sol.15 x, y> 0 and

$\log_y x + \log_x y = \dfrac{10}{3}$

$\Rightarrow \dfrac{\log x}{\log y} + \dfrac{\log y}{\log x} = \dfrac{10}{3}$

assume $\dfrac{\log x}{\log y} = a$

$\Rightarrow a + \dfrac{1}{a} = \dfrac{10}{3}$

$\Rightarrow 3a^2 - 10a + 3 = 0$

$\Rightarrow (3a - 1)(a - 3) = 0$

$\Rightarrow a = 3, \left(\dfrac{1}{3}\right)$

So $\dfrac{\log x}{\log y} = 3 \Rightarrow$ add + 1 both side

$\dfrac{\log x}{\log y} + 1 = 3 + 1 = 4$

$\Rightarrow \dfrac{\log x + \log y}{\log y} = 4$

$\Rightarrow \dfrac{\log(xy)}{\log y} = \dfrac{\log_{12} 12^2}{\log y} = 4$

$\Rightarrow \dfrac{2}{\log_{12} y} = 4$

$\log_{12} y = \dfrac{2}{4} = \dfrac{1}{2} \Rightarrow y = 12^{1/2}$

so $x = \dfrac{144}{y} = 144 \times 12^{-\frac{1}{2}} = 12^{2-\frac{1}{2}} = 12^{\frac{3}{2}}$

$\dfrac{x+y}{2} = \sqrt{N}$

$\Rightarrow \dfrac{(x+y)^2}{2^2} = N$

$\Rightarrow x^2 + y^2 + 2xy = 4N$

$\Rightarrow (12^{3/2})^2 + (12^{1/2}) + 2(144) = 4\,N$

$\Rightarrow 12^3 + 12 + 2 \times 144 = 4\,N$

$4N = 2028 \Rightarrow N = \dfrac{2028}{4}$

$\Rightarrow N = 507$

Sol.16 (a)$\log_{10}2 = 0.3010$, $\log_{10}3 = 0.4771$

$\Rightarrow 5^{200} = x$ (assume)

$\log_{10} x = \log_{10} 5^{200} = 200\log_{10}5$

$=200\log_{10}\frac{10}{2}=200(\log_{10}10-\log_{10}2)$

$= 200(1 - 0.3010) = 200(0.699)$

$= 139.8$

$\Rightarrow x = 10^{139} \times 10^{0.8}$

no. of digits in x = 139 + 1 = 140

(b)$x = 6^{15}$

$\Rightarrow \log_{10}x = \log_{10} 6^{15} = 15 \log_{10} 6$

$= 15(\log 2 + \log 3) = 15 \times (0.778)$

$=11\ 67$

$\therefore x = 10^{11\ 67} = 10^{11}\ 10^{0.67}$

no. of digits in x = 11 + 1 = 12

(c)number of zeros after the decimal in

$3^{-100} = (x)$ (assume)

$\log x = \log 3^{-100} = -100 \log_{10}3$

$= -100(0.4771) = -47.71$

so $x = 10^{-47.71} = 10^{-47} \times 10^{-0.71}$

no of zeros = 47

Sol.17 $\log_5 120 + (x-3) - 2\log_5(1-5^{x-3})$

$= -\log_5(2-5^{x-4})$

$\log_5 120+(x-3)-\log_5(1-5^{x-3})^2 + \log_5(2-5^{x-4}) = 0$

$\log_5 \frac{120\times 5^{x-3}\times(2-5^{x-4})}{(1-5^{x-3})^2} = 0$

$\Rightarrow \frac{120\times 5^{x-3}\times(2-5^{x-4})}{(1-5^{x-3})^2} = 1$

$\Rightarrow \frac{120}{5^3}\ 5^x\left[2-\frac{5^x}{5^4}\right] = 1^2 + 5^{2(x-3)} - 2(5^{x-3})$

assume $5^x = y$

$\Rightarrow \frac{120}{5\times5\times5}\ y\left[2-\frac{y}{25\times25}\right] = 1 + y^2\, 5^{-6} - \frac{2\times y}{5^3}$

multiply by 5^6

$\Rightarrow 5^3 \times 120y[2 - y\, 5^{-4}] = 5^6 + y^2 - 2x5^3y$

$5^3 \times 240y - \frac{120y^2}{5} = 5^6 + y^2 - 2 \times 5^3y$

$5^3 \times 240y - 24y^2 = 5^6 + y^2 - 2 \times 5^3y$

$5^4 \times 48y - 25y^2 = 5^6 - 10 \times 5^2y$

Divide by 5^2

$5^2 \times 48y - y^2 = 5^4 - 10y$

$\Rightarrow y^2 - y(10 + 5^2 \times 48) + 5^4 = 0$

$\Rightarrow y^2 - 1210y + 625 = 0$

$\Rightarrow y = \frac{1210 \pm \sqrt{(1210)^2 - 4(1)(625)}}{2}$

$\Rightarrow y = \frac{1210 \pm 1208.96}{2}$

y = 0.51675 or y = 1209. 48

$5^x = y = 0.51675$

$x = \log_5 y$

$x = -0.410$

Sol.18 $\log_{x+1} (x^2 + x - 6)^2 = 4$

$\Rightarrow (x^2 + x - 6)^2 = (x + 1)^4$

$\Rightarrow (x^2 + x - 6) = \pm (x + 1)^2$

$+ve \rightarrow x^2 + x - 6 = (x + 1)^2$

(and $x^2 + x - 6 \geq 0$)

$x^2 + x - 6 = x^2 + 1 + 2x$

$x = -6 - 1 = -7$

in the equation base is $x + 1 = -7 + 1 = -6$ which is negative so $x \neq -7$

$-ve \rightarrow x^2 + x - 6 < 0$

$x^2 + x - 6 = -(x + 1)^2$

$x^2 + x - 6 = -x^2 - 1 - 2x$

$2x^2 + 3x - 5 = 0$

$(2x + 5)(x - 1) = 0$

$x = -\frac{5}{2}$ or $x = 1$

$x = -\frac{5}{2}$ also does not satisfy equation

so $x = 1$

Sol.19 $x + \log_{10}(1 + 2^x) = x\log_{10}5 + \log_{10}6$

$\Rightarrow \log_{10}10^x + \log_{10}(1+2^x) = \log_{10}5^x + \log_{10}6$

$\Rightarrow \log_{10}[10^x(1 + 2^x)] = \log_{10}[5^x\ 6]$

$\Rightarrow 10^x\ (1 + 2^x) = 6\ 5^x$

$\Rightarrow 10^x + 20^x = 5^x\ 6$

divide by 5^x

$\Rightarrow \frac{10^x}{5^x} + \frac{20^x}{5^x} = \frac{6 \times 5^x}{5^x} = 6$

$\Rightarrow 2^x + 4^x = 6$

$\Rightarrow 2^x + 2^{2x} = 6$

assume $2^x = y$

$y + y^2 = 6$

$y^2 + y - 6 = 0$

$\Rightarrow (y - 2)(y + 3) = 0$

$y = -3$ or $y = 2$

$2^x = -3$ or $2^x = 2$

Not possible $2^x = 2 = 2^1$

real solution $\Rightarrow x = 1$

Sol.20 $2\log(2y - 3x) = \log x + \log y$

we have to find $\left(\frac{x}{y}\right)$

$\Rightarrow \log(2y - 3x)^2 = \log(xy)$

$\Rightarrow 4y^2 - 12xy + 9x^2 = xy$

Let $x = ky$

$\Rightarrow 4y^2 - 12ky^2 + 9k^2 y^2 = ky^2$

$\Rightarrow 9k^2 - 13k + 4 = 0$

$\Rightarrow (9k - 4)(k - 1) = 0$

$\Rightarrow k = 1, \frac{4}{9}$

If $k = 1 \Rightarrow x = y \Rightarrow 2y - 3x$ is $-ve$

$\therefore \frac{x}{y} = \frac{4}{9}$

Sol.21 $a = \log_{12}18$ & $b = \log_{24}54$

$a = \frac{\log_2 18}{\log_2 12} = \frac{2\log_2 3 + 1}{2 + \log_2 3}$

$(a - 2)\log_2 3 = 1 - 2a$

$b = \frac{\log_2 54}{\log_2 24} = \frac{3\log_2 3 + 1}{3 + \log_2 3}$

$(b - 3)\log_2 3 = 1 - 3b$

$(a - 2)(1 - 3b) = (1 - 2a)(b - 3)$

$2a(b - 3) + (a - 2)(1 - 3b) = b - 3$

$2ab - 6a + a - 3ab - 2 + 6b = b - 3$

$-ab - 5a + 5b + 1 = 0$

$5(b - a) - ab + 1 = 0$

$\Rightarrow 5(a - b) + ab = 1$

Sol.22 $\sqrt{\log_9(9x^8)\log_3(3x)} = \log_3 x^3$

$\Rightarrow \sqrt{(1 + 4\log_3 x)[1 + \log_3 x]} = 3\log x$

assume $\log_3 x = y$

$\Rightarrow (1 + 4y)(1 + y) = (3y)^2 = 9y^2$

$\Rightarrow 1 + 4y^2 + 4y + y = 9y^2$

$\Rightarrow 5y^2 - 5y - 1 = 0$

$\Rightarrow y = \frac{5 \pm \sqrt{5^2 - 4(-1)(5)}}{2(5)} = \frac{5 \pm \sqrt{25+20}}{10}$

$y = \frac{5 \pm \sqrt{45}}{10} = \frac{5 \pm \sqrt{3^2 \times 5}}{10} = \frac{5 \pm 3\sqrt{5}}{10}$

in equation (i) $\log_3 x > 0$

so $y = \frac{5 + 3\sqrt{5}}{10}$

Sol.23 $xyz = 10^{81}$

$(\log_{10}x)(\log_{10}yz) + (\log_{10}y)(\log_{10}z) = 468$

we know that $(a + b + c)^2$

$= a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$= a^2 + b^2 + c^2 + 2a(b + c) + 2bc$ ....(i)

$\Rightarrow \log_{10}x(\log_{10} y + \log z) + (\log_{10} y)\ (\log_{10} z)$

$= 468$

assume $\log x = a$

$\log y = b$

$\log z = c$

$\Rightarrow a(b + c) + bc = 468$

from equation (i)

$2a(b+c) + 2bc = (a+b+c)^2 - (a^2+b^2+c^2)$

$\Rightarrow 2a(b + c) + 2bc = 2 \times 468 = 936$

$\Rightarrow (a + b + c)^2 - (a^2 + b^2 + c^2) = 936$

$\Rightarrow a + b + c = \log x + \log y + \log z$

$= \log xyz = \log 10^{81} = 81$

$\Rightarrow 81^2 - (a^2 + b^2 + c^2) = 936$

$a^2 + b^2 + c^2 = 81^2 - 936 = 5625$

which is $\rightarrow (\log x)^2 + (\log y)^2 + (\log z)^2 = 5625$

Sol.24 sum of all solution of equation

$\Rightarrow [3]^{(\log_9 x)^2 - \frac{9}{2}\log_9 x + 5} = 3\sqrt{3}$

$\Rightarrow (3)^{(\log_9 x)^2 - \frac{9}{2}\log_9 x + 5} = (3)^{3/2}$

$\Rightarrow (\log_9 x)^2 - \frac{9}{2}\log_9 x + 5 = \frac{3}{2}$

assume $\log_9 x = y$

$\Rightarrow y^2 - \frac{9}{2}y + 5 = \frac{3}{2}$

$\Rightarrow y^2 - \frac{9}{2}y + 5 - \frac{3}{2} = y^2 - \frac{9}{2}y + \frac{7}{2} = 0$

$\Rightarrow 2y^2 - 9y + 7 = 0$

$\Rightarrow (2y - 7)(y - 1) = 0$

$y = \frac{7}{2}$;$y = 1$

$\log_9 x = \frac{7}{2}\ \log_9 x = 1$

$x = (9)^{7/2} = 3^7$; $x = 9$

Sum of solution $= 3^7 + 9 = 2196$

Sol.25 a, b, c, d > 0

$\because \log_a b = \frac{3}{2}$ and $\log_c d = \frac{5}{4}$, $a - c = 9$

$\frac{\log b}{\log a} = \frac{3}{2}$; $\frac{\log d}{\log c} = \frac{5}{4}$

$2\log b = 3 \log a$

$4 \log d = 5 \log c$

$b = a^{\frac{3}{2}}$, $d = c^{\frac{5}{4}}$

$\therefore$ a should be perfect square '

& c should be perfect power of 4

Let a = 25

c = 16

$\therefore$ b = $(5)^3$ = 125

d = $(16)^{5/4}$ = 32

$\therefore$ b – d = 93

Sol.26 Refer Sol 11 of Ex 2 JEE Main

Sol.27

$\log^2\left[1+\frac{4}{x}\right] + \log^2\left[1-\frac{4}{x+4}\right] = 2\log^2\left[\frac{2}{x-1}-1\right]$

$\log^2\left[\frac{x+4}{x}\right] + \log^2\left[\frac{x+4-4}{x+4}\right] = 2\log^2\left[\frac{2-(x-1)}{x-1}\right]$

$\log^2\left(\frac{x+4}{x}\right) + \log^2\left(\frac{x}{x+4}\right) = 2\log^2\left(\frac{2-x+1}{x-1}\right)$

we know $\log\frac{1}{x} = -\log x$. So $\left(\log\frac{1}{x}\right)^2 = (\log x)^2$

$\Rightarrow \log^2\left(\frac{x+4}{x}\right) + \log^2\left(\frac{x+4}{x}\right) = 2\log^2\left(\frac{3-x}{x-1}\right)$

$\log^2\left(\frac{x+4}{x}\right) = \log^2\left(\frac{3-x}{x-1}\right)$

So $\frac{x+4}{x} = \frac{3-x}{x-1}$ or $\frac{x}{x+4} = \left(\frac{3-x}{x-1}\right)$

$x^2 + 4x - x - 4 = 3x - x^2$ or

$x^2 - x = 3x + 12 - x^2 - 4x$

$2x^2 - 4 = 0$ or $2x^2 = 12$

$x^2 = 2$ or $x^2 = 6$

$x = \pm\sqrt{2}$ or $x = \pm\sqrt{6}$

$x = -\sqrt{2}$ and $-\sqrt{6}$ don't satisfied equation

so $x = \sqrt{2}, \sqrt{6}$

Sol.28

$\log_3(\sqrt{x} + |\sqrt{x}-1|) = \log_9(4\sqrt{x} - 3 + 4|\sqrt{x}-1|)$

$\log_3\left(\sqrt{x} + |\sqrt{x}-1|\right) = \frac{1}{2}\log_3\left(4\sqrt{x} + -3 + 4|\sqrt{x}-1|\right)$

$\Rightarrow 2\log_3(\sqrt{x} + |\sqrt{x}-1|) = \log_3(4\sqrt{x} - 3 + 4|\sqrt{x}-1|)$

$\Rightarrow \log_3(\sqrt{x} + |\sqrt{x}-1|)^2 = \log_3(4\sqrt{x} - 3 + 4|(\sqrt{x}-1|)$

$\Rightarrow (\sqrt{x} + |\sqrt{x}-1|)^2 = (4\sqrt{x} - 3 + 4|\sqrt{x}-1|)$

$x + (\sqrt{x}-1)^2 + 2\sqrt{x}|\sqrt{x}-1| = 4\sqrt{x} - 3 + 4|\sqrt{x}-1|$

(i) assume $(\sqrt{x}-1) < 0$

$\Rightarrow |\sqrt{x}-1| = 1 - \sqrt{x}$

$\Rightarrow x + x + 1 - 2\sqrt{x} + 2\sqrt{x}(1-\sqrt{x})$

$= 4\sqrt{x} - 3 + 4(1-\sqrt{x})$

$\Rightarrow 1 + 2x - 2\sqrt{x} + 2\sqrt{x} - 2x = 4\sqrt{x} - 3 + 4 - 4\sqrt{x}$

1 = 1 always correct

so $\sqrt{x} - 1 < 0$ and $x > 0$

$\sqrt{x} < 0$

$\Rightarrow x \in [0, 1]$

and if $\sqrt{x} - 1 \ge 0$, $\sqrt{x} > 0$

$x + x + 1 - 2\sqrt{x} + 2\sqrt{x}(\sqrt{x}-1)$

$= 4\sqrt{x} - 3 + 4(\sqrt{x}-1)$

$2x + 1 - 2\sqrt{x} + 2x - 2\sqrt{x} = 4\sqrt{x} - 3 - 4 + 4\sqrt{x}$

$4x + 1 + 7 - 4\sqrt{x} = 8\sqrt{x}$

$4x - 12\sqrt{x} + 8 = 0$

$x - 3\sqrt{x} + 2 = 0$

$(\sqrt{x}-2)(\sqrt{x}-1) = 0$

$\Rightarrow \sqrt{x} - 2 = 0$ or $\sqrt{x} - 1 = 0$

x = 4 or x = 1

Put condition was ⇒ $\sqrt{x} - 1 \geq 0$

so x = [0, 1] & {4}

Sol.29

$$2^{\left(\sqrt{\log_a \sqrt[4]{ab}+\log_b \sqrt[4]{ab}}-\sqrt{\log_a \sqrt[4]{b/a}+\log_b \sqrt[4]{a/b}}\right)\cdot\sqrt{\log_a b}}$$

assume ⇒ 2^x

$$\Rightarrow x = \left(\sqrt{\frac{1}{4}(\log_a(a\times b)+\log_b(a\times b))} - \sqrt{(\log_a ba^{-1}+\log_b ab^{-1})\frac{1}{4}}\right)\sqrt{\log_a b}$$

$$x = \frac{1}{2}\left[\sqrt{1+\log_a b+1+\log_b a} - \sqrt{-1+\log_a b-1+\log_b a}\right]\sqrt{\log_a b}$$

$$x = \frac{1}{2}\left[\sqrt{2\log_a b+1+(\log_a b)^2} - \sqrt{-2\log_a b+(\log_a b)^2+1}\right]$$

we know $\log_a b = \frac{1}{\log_b a}$

$$x = \frac{1}{2}\left(\sqrt{(1+\log_a b)^2} - \sqrt{(\log_a b-1)^2}\right)$$

$$x = \frac{1}{2}(|1+\log_a b| - |\log_a b - 1|)$$

when $\log_a b \geq 1 \Rightarrow b \geq a > 1$

$$x = \frac{1}{2}(1+\log_a b - \log_a b + 1) = \frac{1}{2}\times 2 = 1$$

so ⇒ $2^x = 2^1 = 2$ (when $b \geq a > 1$)

when $\log_a b < 1$

⇒ b < a, a, b > 1

$$\Rightarrow x = \frac{1}{2}[1 + \log_a b - (1 - \log_a b)]$$

$$x = \frac{1}{2}[1 + \log_a b + \log_a b] = \frac{1}{2}2\log_a b$$

$x = \log_a b$

$2^x = 2^{\log_a b}$ (if 1 < b < a)

Sol.30 $\sqrt{[\log_3(3x)^{1/3}+\log_x(3x)^{1/3}]\log_3 x^3}$ +

$$\left[\sqrt{\log_3\left(\frac{x}{3}\right)^{\frac{1}{3}}+\log_x\left(\frac{3}{x}\right)^{\frac{1}{3}}}\right]\log_3 x^3$$

assume

$$A = \sqrt{\left[\frac{1}{3}\log_3(3x)+\frac{1}{3}\log_x(3x)\right]\log_3 x^3}$$

$$A = \sqrt{\frac{3}{3}[(\log_3 x+1)+(\log_x 3+1)]\log_3 x}$$

$$A = \sqrt{(2\log_3 x+(\log_3 x)^2+1)}$$

We know $\log_a b = \frac{1}{\log_b a}$

A = $|\log_3 x + 1|$

and

$$B = \sqrt{\left(\left(\log_3\frac{x}{3}\right)\frac{1}{3}+\frac{1}{3}\left(\log_x\frac{3}{x}\right)\right)\log_3 x^3}$$

$$B = \sqrt{\frac{3}{3}[\log_3 x-1+\log_x 3-1]\log_3 x}$$

$$B = \sqrt{((\log_3 x)^2-2\log_3 x+1)}$$

$$B = \sqrt{(\log_3 x-1)^2} = |\log_3 x - 1|$$

A + B = 2 ⇒ $|\log_3 x + 1| + |\log_3 x - 1| = 2$

$\log_3 x \geq 1 \Rightarrow x \geq 3$

A + B ⇒ $\log_3 x + 1 + \log_3 x - 1$

$= 2\log_3 x = 2$

$\log_3 x = 1 \Rightarrow x = 3$

$x \geq 3$ and $x = 3 \Rightarrow x = 3$

if $\log_3 x < 1$ and $\log_3 x + 1 > 0$

$\Rightarrow x < 3$ and $x > \frac{1}{3}$

$A + B \Rightarrow \log_3 x + 1 - (\log_3 x - 1)$

$= \log_3 x + 1 - \log_3 x + 1 = 2 = 2$(always)

so $x \in \left(\frac{1}{3}, 3\right)$

$\log_3 x \leq -1 \ x \leq \frac{1}{3}$

$A + B = -(\log_3 x + 1) - (\log_3 x - 1)$

$= -\log_3 x - 1 - \log_3 x + 1 = -2\log_3 x = 2$

$\Rightarrow \log_3 x = -1 \Rightarrow x = 3^{-1} = \frac{1}{3}$

$x \geq \frac{1}{3}$ and $x = \frac{1}{3} \Rightarrow x = \frac{1}{3}$

so $x = \left[\frac{1}{3}, 3\right] - \{1\}$

$x \neq 1$ because base can't be 1

EXERCISE – II JEE ADVANCED

Sol.1 $2^{\sqrt{x}+\sqrt{y}}$ =256 & $\log_{10}\sqrt{xy} - \log_{10}1.5 = 1$

$\Rightarrow 2^{\sqrt{x}+\sqrt{y}} = 256 = 2^8$

$\Rightarrow \sqrt{x} + \sqrt{y} = 8$ .... (i)

and $\log_{10}\sqrt{xy} = 1 + \log_{10}1.5$

$= \log_{10}10 + \log_{10}1.5$

$\log_{10}\sqrt{xy} = \log_{10}(10 \times 1.5) = \log_{10}15$

$\Rightarrow \sqrt{xy} = 15 \Rightarrow xy = 15^2 = 225$

$|\sqrt{x} - \sqrt{y}| = \sqrt{(\sqrt{x}+\sqrt{y})^2 - 4\sqrt{xy}}$

$= \sqrt{8^2 - 4\times 15} = \sqrt{64-60}$

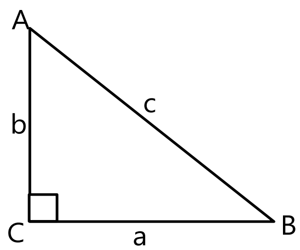
$|\sqrt{x} - \sqrt{y}| = \sqrt{4} = 2$

$\sqrt{x} + \sqrt{y} = 8$

$\Rightarrow$ if $\sqrt{x} > \sqrt{y} \Rightarrow (x, y) = (25, 9)$

$\Rightarrow$ if $\sqrt{x} < \sqrt{y} \Rightarrow (x, y) = (9, 25)$

Sol.2



$\Rightarrow c^2 = a^2 + b^2$

$\Rightarrow c^2 - b^2 = a^2$

$$\frac{\log_{b+c} a + \log_{c-b} a}{\log_{b+c} a . \log_{c-b} a} = \frac{\frac{\log_a a}{\log_a b+c} + \frac{\log_a a}{\log_a c-b}}{\frac{\log_a a}{\log_a b+c}\frac{\log_a a}{\log_a c-b}}$$

$= (\log_a(c-b) + \log_a(b+c)) = \log_a(c^2-b^2) = 2$

Sol.3 B, C, P, and L are positive number

$\therefore$ log(B.L) + log(B.P) = 2;

log(D.L) + log(P.C) = 3

and log(C.B) + log(C.L) = 4

add all the equations $\Rightarrow$

log[B.L.B.P.P.L.P.C.C.B.C.L] = 2+3+4=9

$\log(BCPL)^3 = 9$

3logBCPL = 9

$\log BCPL = \frac{9}{3} = 3$

$BCPL = 10^3$

Sol.4 $\frac{\log_{12}(\log_8(\log_4 x))}{\log_5(\log_4(\log_y(\log_2 x)))} = 0$

$c < y < b, y \neq a$

where 'b' is as large as possible and 'c' is as small as possible.

(a + b + c) = 7

$\Rightarrow \log_{12}(\log_8(\log_4 x)) = 0$

$\Rightarrow \log_8(\log_4 x) = 1 = \log_8 8$

$\log_4 x = 8 \Rightarrow x = 4^8 = 2^{2\times 8} = 2^{16}$

and

$\log_5(\log_4(\log_y(\log_2 y))) \neq 0$

$\Rightarrow \log_5(\log_4(\log_y(\log_2 2^{16}))) \neq 0$

$\Rightarrow \log_5(\log_4(\log_y 16)) \neq 0, y \neq 1$

$\Rightarrow \log_4(\log_y 16) \neq 1$

$\log_y 16 \neq 4 \Rightarrow \log_y 16 = \frac{1}{\log_{16} y}$

$\Rightarrow \log_{2^4} y \neq \frac{1}{4}$

$\frac{1}{4}\log_2 y \neq \frac{1}{4} \Rightarrow \log_2 y \neq 1 \Rightarrow y \neq 2$

$\log_4(\log_y 16) \neq 0$

$\Rightarrow \log_y 16 \neq 1$

$\log_{16} y \neq 1 \Rightarrow y \neq 16$

$\log_4(\log_y 16) > 0$

$\log_y 16 > 0 \Rightarrow y < 16$

$y > 1$

$\log_y 16 > 0$

$a = 2, b = 16, c = 1$

$a + b + c = 2 + 16 + 1 = 19$

Sol.5 The expression, $\log_p \log_p$ n radical sign where $p \geq 2$, $P \in N$, when simplified is independent of p but dependent on n and negative

Sol.6 $N = \frac{1+2\log_3 2}{(1+\log_3 2)^2} + \log_6^2 2$

$N = \frac{1+2\log_3 2}{(1+\log_3 2)^2} + \left(\frac{\log_3 2}{\log_3 6}\right)^2$

assume $\log_3 2 = y$

$\Rightarrow N = \frac{1+2y}{(1+y)^2} + \frac{y^2}{(\log_3 2+\log_3 3)^2}$

$N = \frac{1+2y}{(1+y)^2} + \frac{y^2}{(1+y)^2} = \frac{y^2+2y+1}{(1+y)^2}$

$N = \frac{(1+y)^2}{(1+y)^2} = 1$

and $\pi = 3.147 > 3$

and $7 > 6$

so $\log_3 \pi > 1$

and $\log_7 6 < 1$

Sol.8 $2^{2x} - 8.2^x = -12$

assume $2^x = y$

$y^2 - 8y = -12$

$(y-6)(y-2) = 0$

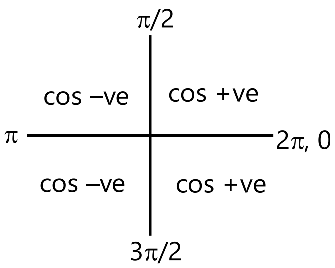
$\Rightarrow y = 6$ or $y = 2$

$2^x = 6$; $2^x = 2^1$

$x \log_{10} 2 = \log 6 = \log(2 \times 3)$

$x = \frac{\log 2+\log 3}{\log 2} = 1 + \frac{\log 3}{\log 2}$; $x = 1$

Sol.9 Statement-I



$\sqrt{\log_x \cos(2\pi x)}$ is a meaning quantity only if

$x \in \left(0,\frac{1}{4}\right) \cup \left(\frac{3}{4},1\right)$

$\cos 2\pi x > 0$

$\frac{\pi}{2} > 2\pi x > 0$

$\frac{1}{4} > x > 0$

and $x \neq 1$, $x > 0$

$\frac{3\pi}{2} < 2\pi x < 2\pi$

$\Rightarrow \frac{3}{4} < x < 1$

So $x \in \left(0, \frac{1}{4}\right) \cup \left(\frac{3}{4}, 1\right)$

But also $\log_x \cos(2\pi x) > 0 = \log_x 1$

$\cos 2\pi x > 1$ which is never possible

so statement-I is false

Statement-II If the number N > O and the base of the logarithm b(greater than zero not equal to)

Both lie on the some side of unity than $\log_b N > 0$ and if they lie on the different side of unity then $\log_b N < 0$ statement-II is true

Sol.10 Statement-I

$\log_2(2\sqrt{17-2x}) = 1 + \log_2(x-1)$ has a solution

$\Rightarrow 1 + \log_2(\sqrt{17-2x}) = 1 + \log_2(x-1)$

$\Rightarrow \sqrt{17-2x} = (x-1)$

Square both side

$\Rightarrow 17 - 2x = (x-1)^2 = x^2 - 2x + 1$

$\Rightarrow 17 = x^2 + 1 \Rightarrow x^2 = 16 \Rightarrow x = \pm 4$

$\Rightarrow x \neq -4$ is not a solution satisfied equation in statement-I

so x = 4. x has a solution

Statement-II

"change of base in logarithm is possible" which is true but not the correct explanation for statement-I

Sol.11 $3^x(0.333\ ....)^{(x-3)} \le \left(\frac{1}{27}\right)^x$

$\Rightarrow 3^x\left(\frac{1}{3}\right)^{x-3} \le \left(\frac{1}{3^3}\right)^x = \left(\frac{1}{3}\right)^{3x}$

$\Rightarrow 3^x\, 3^{-(x-3)} = 3^x \,.\, 3^{3-x} \le \left(\frac{1}{3}\right)^{3x}$

$3^3 = 27 \le \left(\frac{1}{3}\right)^{3x} = 3^{-3x}$

$3 \le -3x \Rightarrow -x \ge 1 \Rightarrow x \le -1$

$x \in [-\infty, -1]$

Sol.12 $2(25)^x - 5(10)^x + 2(4^x) \ge 0$

$2(5^2)^x - 5(5 \times 2)^x + 2(2^2)^x \ge 0$

$\Rightarrow 2(5)^{2x} - 5(5)^x(2)^x + 2(2^2)^x \ge 0$

$\Rightarrow 5^{2x} - \frac{5}{2}5^x2^x + 2^{2x} \ge 0$

when $a^2 + b^2 = (a-b)^2 + 2ab$

$\Rightarrow \left(5^x - \frac{5}{4}2^x\right)^2 - \frac{25}{16}2^{2x} + 2^{2x} \ge 0$

$\Rightarrow \left(5^x - \frac{5}{4}2^x\right)^2 > \left(\frac{25-16}{16}\right)2^{2x} = \frac{9}{16}2^{2x}$

$\Rightarrow$ root square both sides

$\Rightarrow \left|5^x - \frac{5}{4}2^x\right| \ge \frac{3}{4}2^x$

if $5^x - \frac{5}{4}2^x \ge 0$

$\Rightarrow 5^x \ge \frac{3}{4}2^x + \frac{5}{4}2^x = \frac{8}{4}2^x = 2.2^x$

$\Rightarrow 5^x \ge 2.2^x = 2^{1+x}$

$\Rightarrow 5 \ge 2^{\frac{1+x}{x}}$

assume $5 = 2^y$

$\Rightarrow y \ge \frac{1+x}{x} \Rightarrow yx \ge 1 + x$

$\Rightarrow yx - x \ge 1 \Rightarrow x \ge \frac{1}{y-1}$

Where y < 3 and y > 2

$(\because 2^3 > 5$ and $2^2 < 5)$

Sol.13 $\left(\frac{1}{5}\right)^{\frac{2x+1}{1-x}} > \left(\frac{1}{5}\right)^{-3}$

$\frac{2x+1}{1-x} < -3$

$2x + 1 < -3(1 - x) = -3 + 3x$ (if $(1 - x) > 0$)

$\Rightarrow 2x + 1 < -3 + 3x$

$\Rightarrow 3x - 2x > 1 + 3 = 4$

$\Rightarrow x > 4$

$\Rightarrow x > 4$ and $x < 1$

no solution

if $x > 1 \Rightarrow 1 - x < 0$

$\Rightarrow \frac{2x+1}{1-x} < -3$

$\Rightarrow \frac{2x+1}{1} > -3(1 - x) = 3x - 3$

$\Rightarrow 3x - 2x < 1 + 3 = 4$

$\Rightarrow x < 4$ and $x > 1 \Rightarrow x \in (1, 4)$

Sol.14 $\log_x^3 10 - 6\log_x^2 10 + 11 \log_x 10 - 6 = 0$

assume $\log_x 10 = y$

$\Rightarrow y^3 - 6y^2 + 11y - 6 = 0$

$f(y) = y^3 - 6y^2 + 11y - 6$

$\frac{df(y)}{dy} = 3y^2 - 12y + 11 \to 0$

$\Rightarrow y = \frac{12 \pm \sqrt{12^2 - 4\times 3\times 11}}{2(3)}$

$= \frac{12 \pm \sqrt{12}}{6}$

There is maxima and minima at

$y = \frac{12 \pm \sqrt{12}}{6} = 2 \pm \frac{\sqrt{6}\sqrt{2}}{6}$

$= 2 \pm \frac{\sqrt{2}}{6} = 2 \pm \frac{1}{\sqrt{3}}$

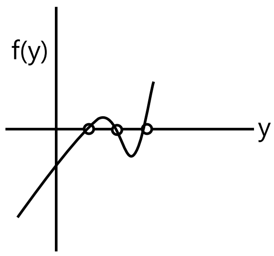
at $y = 2 + \frac{1}{\sqrt{3}}$

$y^2 - 6y^2 + 11y - 6$ is negative

and at $y = 2 - \frac{1}{\sqrt{3}}$,

equation $y^3 - 6y^2 + 11y - 6$ is positive

so there is total 3 solution for this equation



Sol.15 $x^{\log_3 x^2 + (\log_3 x)^2 - 10} = \frac{1}{x^2} = x^{-2}$

$x = 1$ $((1)^{\log_3 x^2 + (\log_3 x)^2 - 10} = 1$

or $\log_3 x^2 + (\log_3 x)^2 - 10 = -2$

assume $\log_3 x = y \to 2y + y^2 - 10 = -2$

$y^2 + 2y - 10 + 2 = y^2 + 2y - 8 = 0$

$(y + 4)(y - 2) = 0$

$y = -4$ or $y = 2$

$x = 3^{-4} = \frac{1}{81}$; $x = 9$

$x = \left\{1, 9, \frac{1}{81}\right\}$

Sol.16 $\frac{(\ell nx)^2 - 3\ell nx + 3}{\ell nx - 1} < 1$

If $\ell n\, x - 1 > 0 \Rightarrow \ell n\, x > 1 \Rightarrow x > e$

$\Rightarrow (\ell n\, x)^2 - 3\ell n\, x + 3 < 1[(\ell n(x)) - 1]$

assume $\ell n\, x = y$

$\Rightarrow y^2 - 3y + 3 < y - 1$

$\Rightarrow y^2 - 3y - y + 3 + 1 < 0$

$\Rightarrow y^2 - 4y + 4 < 0$

$\Rightarrow (y - 2)^2 < 0$always false

So if $\ell n\, x < 1 \Rightarrow x < e$ and $x > 0$

$y^2 - 3y + 3 > (y - 1)$

$y^2 - 3y - y + 3 + 1 > 0$

$y^2 - 4y + 4 > 0$

$\Rightarrow (y - 2)^2 > 0$ always true

so $x \in (0, e)$

Sol.17

$a = (\log_7 81)(\log_{6561} 625)(\log_{125} 216)(\log_{1296} 2401)$

$a = (\log_7 3^4)\ (\log_{3^8} 5^4)\ (\log_{5^3} 6^3)\ (\log_{6^4} 7^4)$

$a = 4(\log_7 3)\dfrac{4}{8}(\log_3 5)(\log_5 6)\left(\dfrac{3}{3}\right)\left(\dfrac{4}{4}\right)\log_6 7$

$a = \dfrac{2\log 3}{\log 7}\ \dfrac{\log 5}{\log 3}\ \dfrac{\log 6}{\log 5}\ \dfrac{\log 7}{\log 6} = 2$

and b = sum of roots of the equation

$x^{\log_2 x} = (2x)^{\log_2 \sqrt{x}}$

$x^{\log_2 x} = (2x)^{\log_2 x^{1/2}}$

take logarithm (base x) both sides

$\log_x x^{\log_2 x} = \log_x (2x)^{\log_2 x^{1/2}}$

$(\log_2 x)(1) = \log_2 x^{1/2}[\log_x(2x)]$

$\log_2 x = \dfrac{1}{2}\log_2 x(\log_x 2 + 1)$

$\log_2 x = 0 \Rightarrow x = 1$ or $2 = \log_x 2 + 1$

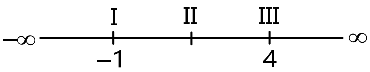
$\log_x 2 = 1 \Rightarrow x = 2$

$x_1 + x_2 = 1 + 2 = 3$

$b = 3$

and c = sum of all natural solution of equation

$|x + 1| + |x - 4| = 7$



If $x < -1 \rightarrow |x + 1| = -1 - x$

$|x - 4| = 4 - x$

$\Rightarrow$ eq.$\rightarrow -1 - x + 4 - x = 3 - 2x = 7$

$\Rightarrow 2x = 3 - 7 = -4 \Rightarrow x = -\dfrac{4}{2} = -2$

If $x > 4 \rightarrow |x + 1| = x + 1$

$|x - 4| = x - 4$

Eq. $\rightarrow x + 1 + x - 4 = 2x - 3 = 7$

$\Rightarrow 2x = \dfrac{7+3}{1} = 10 \Rightarrow 2x = 10 \Rightarrow x = \dfrac{10}{2} = 5$

if $-1 < x < 4$

$\Rightarrow |x + 1| \rightarrow 1 + x$

$|x - 4| \rightarrow 4 - x$

$\Rightarrow 1 + x + 4 - x = 5 \neq 7$ so no solution for this region $\rightarrow x = 5$ and -2

but -2 is not natural no.

so $c = 5$

$a + b = 2 + 3 = 5$

$(a + b) \div c = \dfrac{5}{5} = 1$

Sol.18 (A)

$\sqrt{3\sqrt{x} - \sqrt{7x + \sqrt{4x - 1}}}\ \sqrt{2x + \sqrt{4x - 1}}$

$$\sqrt{3\sqrt{x}+\sqrt{7x+\sqrt{4x-1}}} = 13$$

$$\sqrt{(3\sqrt{x}-\sqrt{7x+\sqrt{4x-1}})(3\sqrt{x}+\sqrt{7x+\sqrt{4x-1}})}$$

$$(\sqrt{2x+\sqrt{4x-1}})$$

$$= \sqrt{\left(3\sqrt{x}\right)^2-\left(\sqrt{7x+\sqrt{4x-1}}\right)^2\left(2x+\sqrt{4x-1}\right)}$$

$$= \sqrt{(9x-7x+\sqrt{4x-1})(2x+\sqrt{4x-1})}$$

$$= \sqrt{(2x+\sqrt{4x-1})(2x+\sqrt{4x-1})}$$

$= 2x + (\sqrt{4x-1}) = 13$

If $13 > 2x$ then

$\sqrt{4x-1} = 13 - 2x$

square

$\Rightarrow 4x - 1 = (13 - 2x)^2 = 169 + 4x^2 - 2.13\,(2x)$

$4x^2 - 52x - 4x + 169 + 1 = 0$

$\Rightarrow 4x^2 - 56x + 170 = 0$

$$\Rightarrow x = \frac{56 \pm \sqrt{56^2 - 4(170)4}}{8}$$

$$x = \frac{56 \pm \sqrt{416}}{8} = 7 \pm \frac{4\sqrt{26}}{8} = \frac{7 \pm \sqrt{26}}{2}$$

but $13 > 2x \Rightarrow x < 6.5$

so $x = 7 - \frac{\sqrt{26}}{2}$ which is less than 6.5

(b) $P(x) = x^7 - 3x^5 + x^3 - 7x^2 + 5$

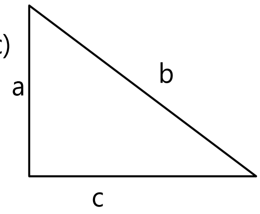
$Q(x) = x - 2$

Remainder $\frac{P(x)}{Q(x)}$

$Q(x) = 0$ at $x = 2$

So $P(2) = 2^7 - 3(2)^5 + 2^3 - 7(2)^2 + 5 = 17$

(c)



area of triangle is

Area $= a^2 + b^2 - c^2$

$b^2 = a^2 + c^2$

so area $= a^2 + (a^2 + c^2) - c^2$

$= \frac{1}{2} \times a \times c = \frac{ac}{2}$

$\Rightarrow 2a^2 = \frac{ac}{2} \Rightarrow 4 = \frac{ac}{a^2} = \frac{a}{c}$

$\Rightarrow$ ratio $= \frac{c}{a} = 4$

(D)a, b, c $\in$ N

$\therefore ((4)^{1/3} + (2)^{1/3} - 2)$

$(a(4)^{1/3}+b(2)^{1/3}+c)=20$

$= (2^{2/3}+2^{1/3}-2)(a2^{2/3} + b2^{1/3}+c)=20$

$\Rightarrow a(2^{4/3} + 2 - 2.2^{2/3}) + b[2^{3/3} + 2^{2/3} - 2.2^{1/3}]$

$+c(2^{2/3}+2^{1/3}-2^{3/3})=20$

$2^{1/3}(2a - 2b + c) + 2^{3/3}(a + b - c)$

$+ 2^{2/3}(-2a + b + c)= 20$

$a + b - c = \frac{20}{2} = 10$