5. MENSURATION – CUBE, CUBOID AND CYLINDER

1. Cuboid

A rectangular solid bounded by six rectangular plane faces is called a cuboid. A match box, a brick, a book, etc., are all examples of a cuboid.

A cuboid has 6 rectangular faces, 12 edges and 8 vertices.

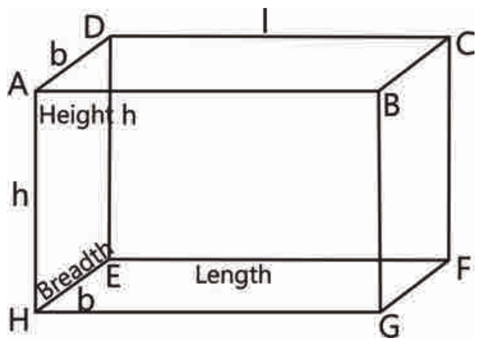


Figure 5.1

The following are some definitions of terms related to a cuboid:

(i) The space enclosed by a cuboid is called its **volume**.

(ii) The line joining opposite corners of a cuboid is called its **diagonal**.

A cuboid has four body diagonals.

A diagonal of a cuboid is the length of the longest rod that can be placed in the cuboid.

(iii) The sum of areas of all the six faces of a cuboid is known as its **total surface area**.

(iv) The four faces which meet the base of a cuboid are called the **lateral faces** of the cuboid.

(v) The sum of areas of the four walls of a cuboid is called its **lateral surface area**.

Formulae

For a cuboid of length = l units, breadth = b units and height = h units, we have:

Sum of lengths of all edge = 4 (l + b + h) units.

Diagonal of cuboid = $\sqrt{l^2 + b^2 + h^2}$ units

Total Surface Area of cuboid = 2(lb + bh + lh) sq. units

Lateral Surface Area of a cuboid = Area of four walls of a room = [2h(l + b)]] sq. units

Volume of cuboid = (l × b × h) cubic units

2. Cube

A cuboid whose length, breadth and height are all equal is called a cube.

Ice-cubes, Sugar cubes, Dice, etc. are all examples of a cube. Each edge of a cube is called its side.

Formulae

For a cube of edge = a units, we have;

Sum of lengths of all edge = 12a units.

Diagonal of cube = $(a\sqrt{3})$ units.

Total Surface Area of cube = $(6a^2)$ sq. units.

Lateral Surface Area of a cube = $(4a^2)$ sq. units.

Volume of cube = a^3 cubic units.

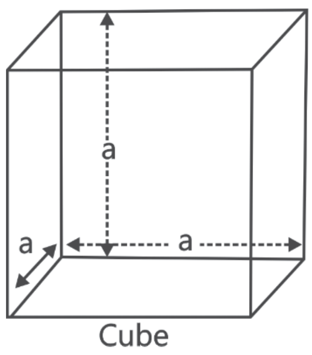


Figure 5.2

3. Cross Section

A cut which is made through a solid perpendicular to its length is called its cross section.

If the cut has the same shape and size at every point of its length, then it is called **uniform cross-section.**

Volume of a solid with uniform cross section = (Area of its cross section) × (length).

Lateral surface area of a solid with uniform cross section = (Perimeter of cross section) × (length).

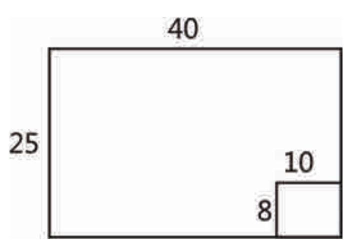
Illustration 1: A field is 40 m long and 25 m broad. In one corner of the field, a pit which is 10 m long, 8 m broad and 9 m deep has been dug out. The earth taken out of it is evenly spread over the remaining part of the field. Find the rise in the level of the field.

Solution: Area of the field = $(40 \times 25)\ m^2 = 1000\ m^2$

Area of the pit = $(10 \times 8)\ m^2 = 80\ m^2$

Area over which the earth is spread out = $(1000 - 80)\ m^2 = 920\ m^2$

Volume of earth dug out = $(10 \times 8 \times 9)\ m^3 = 720\ m^3$.



$\therefore$ Rise in level = $\left(\frac{\text{Volume}}{\text{Area}}\right) = \left(\frac{720}{920}\right)\text{m} = \left(\frac{720 \times 100}{920}\right)\text{cm} = 78.26\text{ cm}$

4. Right Circular Cylinder

Solids like circular pillars, circular pipes, circular pencils, measuring jars, road rollers and gas cylinders, etc., are said to be in cylindrical shape.

In mathematical terms, **a right circular cylinder is a solid generated by the revolution of a rectangle about its sides.**

Let the rectangle ABCD revolve about its side AB, so as to describe a right circular cylinder as shown in the figure.

You must have observed that the cross-sections of a right circular cylinder are circles congruent and parallel to each other.

The following are definitions of some terms related to a right circular cylinder:

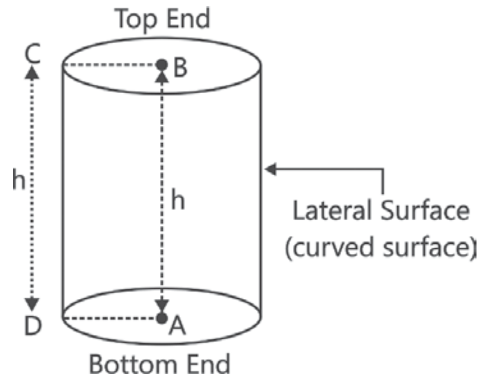


Figure 5.3

(i) The radius of any circular end is called the **radius** of the right circular cylinder. Thus, in the given figure, AD as well as BC is a radius of the cylinder.

(ii) The line joining the centres of circular ends of the cylinder, is called the **axis** of the right circular cylinder. In the above figure, the line AB is the axis of the cylinder. Clearly, the axis is perpendicular to the circular ends.

(iii) The length of the axis of the cylinder is called the **height or length** of the cylinder.

(iv) The curved surface joining the two bases of a right circular cylinder is called its **lateral surface**.

NOTE: If the line joining the centres of circular ends of a cylinder is not perpendicular to the circular ends, or if the ends of the cylinder are not proper circles then the cylinder is not a right circular cylinder.

Formulae:

For a right circular cylinder of radius = r units and height = h units, we have:

Area of each circular end $= \pi r^2$ sq. units.

Curved (Lateral) Surface Area = $(2\pi rh)$ sq. units.

Total Surface Area = Curved Surface Area + Area of two circular ends

$= (2\pi rh + 2\pi r^2)$ sq. units. $= [2\pi r(h + r)]$ sq. units.

Volume of cylinder $= \pi r^2 h$ cubic units.

The above formulae are applicable to solid cylinders only.

4.1 Hollow Right Circular Cylinders

Solids like iron pipes, rubber tubes, etc., are in the shape of hollow cylinders.

A solid bounded by two coaxial cylinders of the same height and different radii is called a hollow cylinder.

Formulae

For a hollow cylinder of height h, let the external and internal radii be R and r respectively, then

Thickness of the cylinder = (R - r) units.

Area of a cross-section $=\pi(R^2-r^2)$ sq. units

Curved (Lateral) Surface Area

= (External Curved Surface Area) + (Internal Curved Surface Area)

$=(2\pi Rh+2\pi rh)$ sq. units $=2\pi h(R+r)$ sq. units.

Total Surface Area = (Curved Surface Area) + 2 (Area of Base Ring)

$=[(2\pi Rh+2\pi rh)+2(\pi R^2-\pi r^2)]$ sq.

units $=2\pi(Rh+rh+R^2-r^2)$ sq. units.

Volume of Material $=\pi(R^2-r^2)h$ cubic units

Volume of Hollow region $=\pi r^2 h$ cubic units

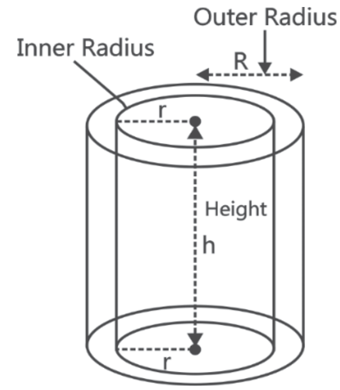


Figure 5.4

5. Prism and Pyramid

5.1 Prism

A solid having top and bottom faces identical and side faces rectangular is a prism. In a prism with a base of n sides

Number of vertices = 2n

and Number of faces = n + 2

Volume of the prism = area of base × height

Lateral surface area = perimeter of base × height

Total surface area = 2 × Base area + L.S.A. …(Where L.S.A. = Lateral surface area)

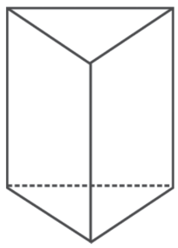


Figure 5.5

5.2 Pyramid

Volume = $\frac{1}{3}$ × base area × height

Lateral surface area = $\frac{1}{2}$ × perimeter of the base × slant height

Total surface area = lateral surface area + base

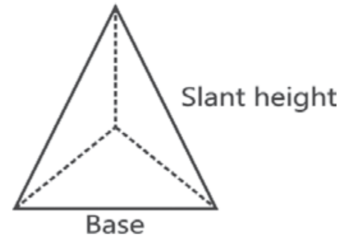


Figure 5.6(a)

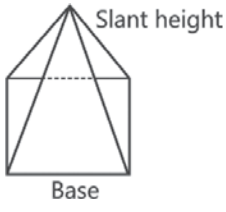


Figure 5.6(c)

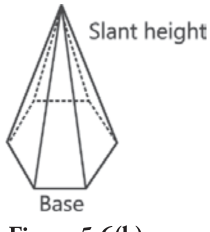


Figure 5.6(b)

Illustration 2: The volume of pyramid of base area $20cm^2$ and height 15cm is.

Solution: Volume of pyramid = $\frac{1}{3}\times$ base area × height = $\frac{1}{3}\times 20 \times 15 = 100\ cm^3$

Illustration 3: An iron of length 1 m and diameter 8 cm is melted and cast into thin wires of length 40 cm each. If the number of such wires be 4000, find the radius of each thin wire.

Solution: Let the radius of each thin wire be r cm. The, the sum of the volumes of 4000 thin wire will be equal to the volume of the iron rod. Now, the shape of the iron rod and each thin wire is cylindrical.

Hence, the volume of the iron rod of radius $\frac{8}{2}$ cm = 4 cm is $\pi\times 4^2\times 100cm^3$

Again, the volume of each thin wire = $\pi r^2 \times 40\ cm^3$

Hence, we have $\pi \times 4^3 \times 100 = \pi r^2 \times 40 \Rightarrow 160000r^2 = 1600 \qquad \Rightarrow r^2 = \frac{1}{100}$

$\Rightarrow r = \frac{1}{10}$ [Taking positive square root only]

Hence, the required radius of each thin wire is $\frac{1}{10}$ cm. of 0.1 cm.

Illustration 4: Find the total surface area of the given solid, if each edge are same and equal to 5 cm.

Solution: Total Surface = upper surface area + lower surface area + surface area of wall

$= 5\times(5)^2 + 5\times(5)^2 + 4\times 3\times(5)^2 = 125 + 125 + 300 = 550\ cm^2$

Illustration 5: A cylindrical bucket full of paint has radius 17 cm and height 40 cm. This paint with a layer of 1 mm should be painted on a wall of surface area 'A'. Find Area "A" if the paint has completely finish on the wall.

Solution: Volume of Paint in cylinder = $\frac{22}{7}\times 14\times 14\times 40 = 24640\ cm^3$

$A \times 1\ mm = 24640\ cm^3$; $A = 246400\ cm^2 = 24.64\ m^2$

Illustration 6: A box has length, breadth and height, as 80 cm 40 cm and 20 cm respectively. How many square sheets of paper of side 40 cm would be require to cover up the box.

Solution: l = 80 cm b = 40 cm h = 20 cm

Surface area of box = 2 [lb + bh + hl] = $2\,[(80\times 40) + (40\times 20) + (20\times 80)]$

= 2 [3200 + 800 + 1600] cm^2 = 11200 cm^2

Area of each sheet = $40 \times 40\ cm^2 = 1600\ cm^2$

No. of sheet required = $\left(\frac{\text{Surface area of the box}}{\text{Area of each sheet}}\right) = \frac{11200}{1600} = 7$

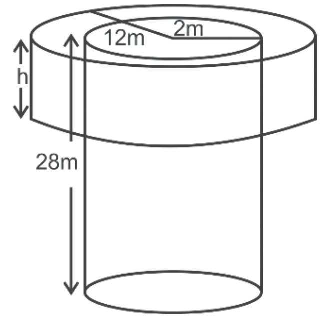


Illustration 7: A well of diameter 4 m is dug 28 m deep. The earth taken out of it and is spread evenly all around it to a width of 10 m to form an embankment. Find the height of the embankment.

Solution: Let "h" be the required height of the embankment.

The shape of the embankment will be like the shape of a cylinder of internal radius 2 m and external radius (10 + 2) m = 12 m [figure].

The volume of the embankment will be equal to the volume of the earth dug out from the well. Now, the volume of the earth= volume of the cylindrical well

$= \pi \times 2^2 \times 28\ m^3$

$= 112\ \pi\ m^3$

Also, the volume of the embankment = $\pi\ (12^2 - 2^2)\ h\ cm^3 = 140\ \pi\ h\ m^3$

Hence, we have $140\ \pi\ h = 112\ \pi \Rightarrow h = \frac{112}{140} = 0.8$ m

Hence, the required height of the embankment = 0.8 m

Illustration 8: Water in a canal, 15 dm wide and 6 dm deep, is flowing with a speed of 5 km/hr. How much area will it irrigate in 15 minutes if 4 cm of standing water is required from irrigation?

Solution: Speed of water in the canal = 5 km/h = 10000 m/60 min = $\frac{500}{6}$ m/min.

$\therefore$ The volume of the water flowing out of the canal in 1 minute = $\left(\frac{500}{6} \times \frac{15}{5} \times \frac{6}{5}\right)$ $m^2 = 300\ m^3$

$\therefore$ In 15 min, the amount of water flowing out of the canal = $(300 \times 15)\ m^3 = 4500\ m^3$

If the required area of the irrigated land is x m^2, then the volume of water to be needed to irrigate the land = $\left(x \times \frac{4}{100}\right) m^3 = \frac{x}{25} m^3$

Hence, $\frac{x}{25} = 4500 \qquad \Rightarrow x = 4500 \times 25 = 112500$

Hence, the required area is 112500 m^2.

SUMMARY

- **Cuboid:** A rectangular solid bounded by six rectangular plane faces is called a cuboid.
  - (i) Sum of lengths of all edge = $4\,(l+b+h)$ units.
  - (ii) Diagonal of cuboid $=\sqrt{l^2+b^2+h^2}$ units
  - (iii) Total Surface Area of cuboid $=2(lb+bh+lh)$ sq. units
  - (iv) Lateral Surface Area of a cuboid = Area of four walls of a room $=[2(l+b)\times h]$ sq. units
  - (v) Volume of cuboid $=(l\times b\times h)$ cubic units
- **Cube:** A cuboid whose length, breadth and height are all equal is called a cube.
  - (i) Sum of lengths of all edge = 12 a units.
  - (ii) Diagonal of cube $=(a\sqrt{3})$ units.
  - (iii) Total Surface Area of cube $=(6a^2)$ sq. units.
  - (iv) Lateral Surface Area of a cube $=(4a^2)$ sq. units.
  - (v) Volume of cube $=a^3$ cubic units.
- **Cross section:** A cut which is made through a solid perpendicular to its length is called its cross section. If the cut has the same shape and size at every point of its length, then it is called uniform cross-section.
  - (i) Volume of a solid with uniform cross section

    = (Area of its cross section) $\times$ (length).
  - (ii) Lateral Surface Area of a solid with uniform cross section

    = (Perimeter of cross section) $\times$ (length).
- **Right circular cylinder:** a right circular cylinder is a solid generated by the revolution of a rectangle about its sides.
  - (i) Area of each circular end $=\pi r^2$ sq. units.
  - (ii) Curved (Lateral) Surface Area $=(2\pi rh)$ sq. units.
  - (iii) Total Surface Area = Curved Surface Area + Area of two circular ends $=[2\pi r(h+r)]$ sq. units.
  - (iv) Volume of cylinder $=\pi r^2 h$ cubic units.
- **Hollow right circular cylinders:** A solid bounded by two coaxial cylinders of the same height and different radii is called a hollow cylinder.
  - (i) Thickness of cylinder = (R - r) units.
  - (ii) Area of a cross-section $=\pi(R^2-r^2)$ sq. units

(iii) Curved (Lateral) Surface Area

= (External Curved Surface Area) + (Internal Curved Surface Area) $= 2\pi h(R + r)$ sq. units.

(iv) Total Surface Area = (Curved Surface Area) + 2 (Area of Base Ring) $= 2\pi(Rh + rh + R^2 - r^2)$ sq. units.

(v) Volume of Material $= \pi(R^2 - r^2)h$ cubic units

(vi) Volume of Hollow region $= \pi r^2 h$ cubic units

- **Prism:** A solid having top and bottom faces identical and side faces rectangular is a prism. In a prism with a base of n sides

  Number of vertices = 2n and Number of faces = n+2

  Volume of the prism = area of base × height

  Lateral surface area = perimeter of base × height

  Total surface area = 2 × Base area + L.S.A. (Where L.S.A. = Lateral surface area)

  Pyramid

  Volume = $\frac{1}{3}$ × base area × height

  Lateral surface area = $\frac{1}{2}$ × perimeter of the base × slant height

  Total surface area = lateral surface area + base area

SOLVED EXAMPLES

Example 1: 100 small solid metal cubes of side 5 cm are melted and form a cuboid. Area of one of the face of cuboid is 125 cm^2 and in that face one side is 5 times the other side. Find the dimension of the cuboid.

Solution: Volume of 100 cubes = $100 \times 5 \times 5 \times 5 = 12500\ cm^3$

Volume of cuboid = Volume of 100 cubes

Area of one of the face = lb = 125 cm^2

$b = 5 \times l$

$5\, l^2 = 125$

$l^2 = 25$

l = 5 cm; b = 25 cm; $h = \frac{12500}{5 \times 25} = 100\ cm$

Example 2: Is it possible for a solid cuboid of volume 500 cm^3 is melted and form an 10 cubes of side 5 cm.

Solution: Volume of cube = $10 \times 5^3 = 1250\ cm^3$

But the volume of cuboid is 500 cm^3

Therefore, Volume of 10 cubes is greater than volume of cuboid

Hence it is not possible.

Example 3: Water in a rectangular reservoir having base 40 m by 30 m is 3.25 m deep. In what time can the water be emptied by a pipe of which the cross-section is a square of side 10 cm, if the water runs through the pipe at the rate of 7.5 km/hr.

Solution: Given: Dimension of the reservoir is 40 m, 30m and 3.25 m

So its volume will be

$V = 30 \times 40 \times 3.25 = 3900\ m^3$

Now, Dimension of pipe is 10 cm, 10 cm.

So its area of cross-section $= \frac{10}{100} \times \frac{10}{100} m^2$

Rate of flow of water through the pipe $= 7.5 km\ hr^{-1} = 7500\ m\ h^{-1}$

In 1 hr volume of water that flow out through the pipe $= 7500 \times \frac{10}{100} \times \frac{10}{100} m^3$

Let after 't' hours the whole reservoir becomes emptied so the volume of water that flows out through the pipe after 't' hours

$= 7500 \times \frac{10}{100} \times \frac{10}{100} \times t\ \ m^3$

$\therefore\ \ 75 \times t = 3900$

$t = \frac{3900}{75} = 52$ hr

So the required time will be 52 hours.

Example 4: Water flows out through a circular pipe whose internal diameter is 1 cm, at the rate of 3 metres per second into a cylindrical tank. The radius of whose base is 30 cm. Find the rise in the level of water in 15 minutes?

Solution: Internal diameter of the pipe = 1 cm

So its radius $= 0.5 \text{ cm} = \frac{1}{200}\text{m}$

Water that flows out through the pipe is 3 ms^{-1}

So volume of water that flows out through the pipe in 1 sec $= \pi \times \left(\frac{1}{200}\right)^2 \times 3\ m^3$

$\therefore$ In 15 minutes, volume of water flow $= \pi \frac{1}{200 \times 200} \times 3 \times 15 \times 60\ m^3$

This must be equal to the volume of water that rises in the cylindrical tank after 15 min and height up to which it rises say h.

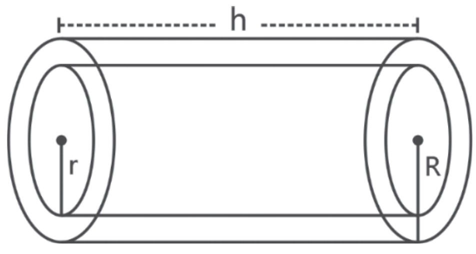
$\therefore$ Radius of tank = 30 cm $= \frac{30}{100}\text{m}$ $\qquad$ $\therefore$ Volume $= \pi\left(\frac{30}{100}\right)^2 h$

$\pi\left(\frac{30}{100}\right)^2 h = \pi \times \frac{1}{200 \times 200} \times 3 \times 15 \times 60;$ $\qquad$ $\frac{9}{100}h = 0.0675$

$\therefore\ \ \ \ h = 0.75\text{m}$

So required height will be 0.75 m.

Example 5: A cylindrical tube, open at both ends, is made of metal. The internal diameter of the tube is 10.4 cm and its length is 25 cm. The thickness of the metal is 8 mm everywhere. Calculate the volume of the metal.



Solution: Internal diameter of the tube = 10.4 cm

Hence internal radius (r) = 5.2 cm

h = length of the tube = 25 cm

Let t = thickness = 8 mm $= \frac{8}{10}\text{cm} = 0.8 \text{ cm}$

So, external radius (R) = 5.2 + 0.8 = 6 cm

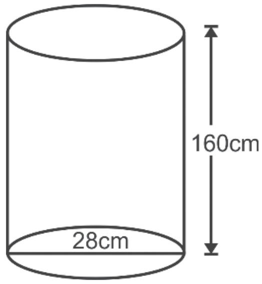
Now, volume of the metal which is the volume of the shaded portion

$= \pi R^2 h - \pi r^2 h = \pi h(R^2 - r^2)$

Putting the values, we get

$V = \frac{22}{7} \times 25 \times (6^2 - 5.2^2) = \frac{22}{7} \times 25 \times (6+5.2)(6-5.2) = \frac{22}{7} \times 25 \times 11.2 \times 0.8 = 704\ cm^3$

Example 6: The height of a cylinder is 160 cm and diameter of the base is 28 cm. Find its curved surface area, total surface area and volume.



Solution: Height (h) = 160 cm, Diameter of base = 28 cm

$\therefore$ Radius of base $(r) = \frac{28}{2} = 14cm$

$\therefore$ Curved surface area of cylinder $= 2\pi rh$

$= 2 \times \frac{22}{7} \times 14 \times 160 = 44 \times 80 = 14080$ sq. cm

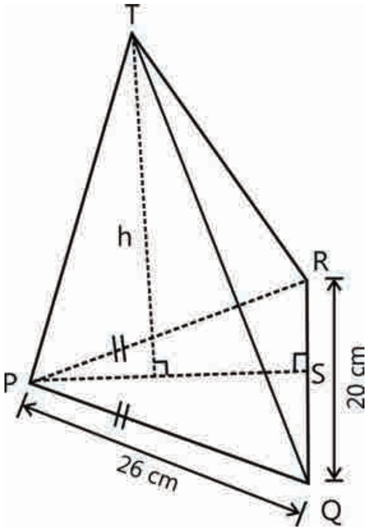
Total surface area (TSA) = C.S.A + Area of two ends

$= 2\ \pi rh + 2\pi r^2 = 2\pi r(h + r) = 2 \times \frac{22}{7} \times 14 \times (160 + 14) = 15312$

T.S.A of cylinder = 15312 sq. cm

Volume (V) of cylinder $= \pi r^2 h = \frac{22}{7} \times (14)^2 \times 160 = 98560$ cu cm.

Example 7: The diagram shows a right pyramid that has an isosceles triangular base. If the volume of the pyramid is 2640 cm^3, calculate its height, h.



Solution: $PS = \sqrt{26^2 - 10^2} = \sqrt{576} = 24\,cm$

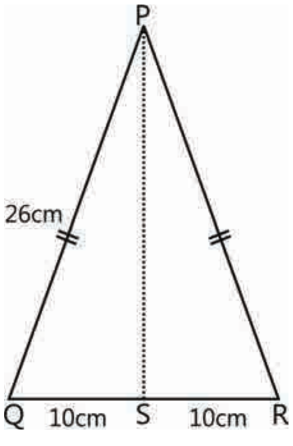
Area of base $= \frac{1}{2} \times BC \times AD = \frac{1}{2} \times 20 \times 24 = 240\,cm^2$

Volume of pyramid $= \frac{1}{3} \times$ (Area of base) $\times$ Height

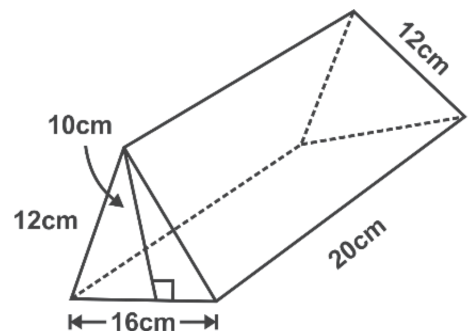
$2640 = \frac{1}{3} \times 240 \times h$

80 h = 2640

$\therefore h = \frac{2640}{80} = 33$ cm



Example 8: Calculate the surface area of the following Prism.



Solution: The given prism has 2 triangles and 3 rectangles on its surfaces.

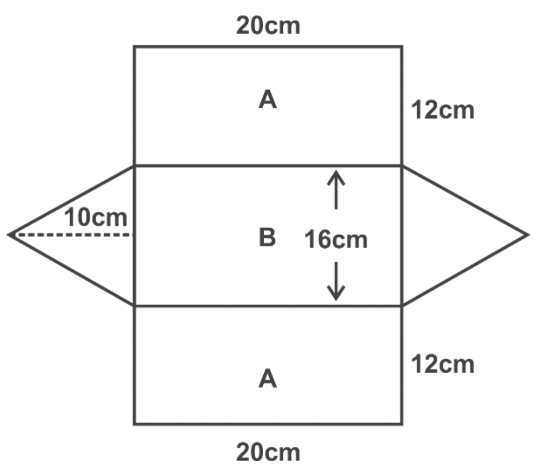
Area of the two triangles $= 2\left[\frac{1}{2} \times 16 \times 10\right]$

Area of rectangle type A = 2 (20 × 12)

Area of rectangle type B = 16 × 20

∴ Total surface area = area of two triangles + area of three rectangles

$= 2\left(\frac{1}{2} \times 16 \times 10\right) + 2(20 \times 12) + (16 \times 20) = 160 + 480 + 320 = 960 \text{ cm}^2$



EXERCISE 1 – For School Examinations

Fill in the Blanks

Directions: Complete the following statements with an appropriate word/term to be filled in the blank space(s).

Q. 1. Total number of faces in a prism which has 12 edges is ________________ .

Q. 2. If the number of lateral surfaces of a right prism is equal to n, then the number of edges of the base of the prism is ________________ .

Q. 3. A road roller of length 3 l meters and radius $\frac{1}{3}$ meters can cover a field in 100 revolutions, moving once over. The area of the field in term of l is ________________ m^2

Q. 4. If the heights of two cylinders are equal and their radii are in the ratio of 7:5, then the ratio of their volumes is ________________ .

Q. 5. The length of the diagonal of a cube that can be inscribed in a sphere of radius 7.5 cm is ________________ .

Q. 6. W, P, H and A are whole surface area, perimeter of base, height and area of the base of a prism respectively. The relation between W, P, H and A is ________________ .

True / False

Directions: Read the following statements and write your answer as true or false.

Q. 7. If the diagonals of a quadrilateral divide it into four triangles which are equal in area, then the quadrilateral must be a parallelogram.

☐ True ☐ False

Q. 8. The three altitudes of an equilateral triangles are equal in length.

☐ True ☐ False

Q. 9. If s is the perimeter of the base of a prism, n is the number of sides of the base, S is the total length of the edges and h is the height, then S = nh + 2s.

☐ True ☐ False

Q. 10. Surface area of square pyramid is $S = s^2 + 2sl$.

☐ True ☐ False

Match the Following

Directions: Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in column I have to be matched with statements (p, q, r, s) in Column II

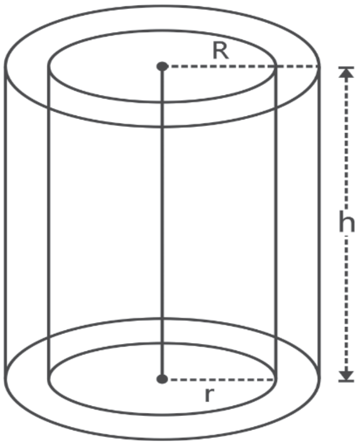
Q. 11.

	Column I		Column II
(A)	Total surface area of right circular cylinder is	(p)	$\pi r^2 h$
(B)	Volume of cuboid is	(q)	$2\pi r(r+h)$
(C)	Diagonal of cuboid is	(r)	$l \times b \times h$
(D)	Volume of cylinder is	(s)	$\sqrt{\ell^2 + b^2 + h^2}$

Very Short Answer Questions

Directions: Give answer in one word or one sentence.

Q. 12. Study the figure given below carefully and complete the statements based on it



(a) R and r are the ________ And ________ of a hollow cylinder respectively and h is its ________ .

(b) Thickness of cylinder is ________ (formula)

(c) External curved surface area is ________ (formula)

(d) Total surface area = ________ + ________ + ________ = ________ (formula)

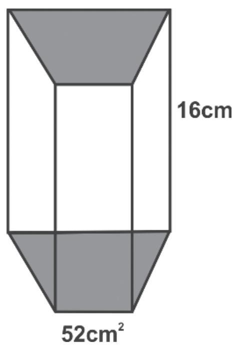
(e) Area of a cross-section is ________ (formula)

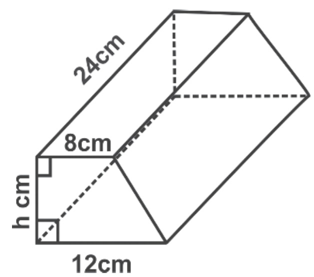
Q. 13. Find the percentage increase in the volume of a cuboid when the three edges are increased by 100%, 200% and 300% respectively.

Q. 14. A cylindrical container of radius 7 cm in height 20 cm contain oil. This oil is completely poured into a cubical box in which the best as a dimension of 11×10 cm^2. Find the height of cubical box?

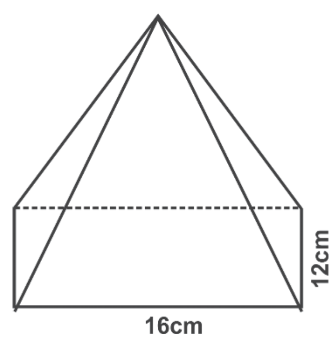
Q. 15. A cylindrical container, used for holding petrol, had a diameter of 16 m and a height of 3 m. The owner wishes to increase the volume. However, he wishes to do it such that if X m are added to either the radius or the height, the increase in volume is the same. Find the value of X.

Q. 16. Find the volumes of the prism shown. The base area is 52 sq. cm, and the height is 16 cm.





Q. 17. The cross section of a prism 12 cm long, is a trapezium with the measurements shown. If the volume of the prism is 300 cm^2, calculate the value of h.



Q. 18. The diagram shows a right rectangular pyramid. If the volume of the pyramid is 640 cm^3, calculate its height, in cm.

Short Answer Questions

Directions: Give answer in two to three sentences.

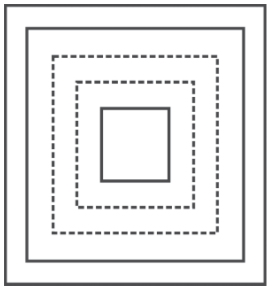
Q. 19. The total surface area of a cube is 1014 cm^2. Find its edge and lateral surface area.

Q. 20. In a hot water heating system, there is a cylindrical pipe of length 56 m and diameter 10 cm. Find the total radiating surface in the system.

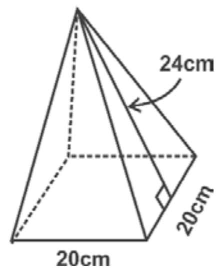
Q. 21. The total surface area of a cylinder is 924 cm^2 and its curved surface area is $\frac{2}{3}$ of the total surface area. Find the volume of the cylinder.

Q. 22 A Sheet of dimension 44×22 cm^2 is rolled such that it represent cylinder of maximum volume. Find the radius of the cylinder.

Q. 23. The figure shows a set of concentric squares. If the diagonal of the innermost square is 2 units, and if the distance between the corresponding corners of any two successive square is 1 unit, find the difference between the areas of the eighth and the seventh square, counting from the innermost square.



Q. 24. Find the surface area of the square pyramid shown below.



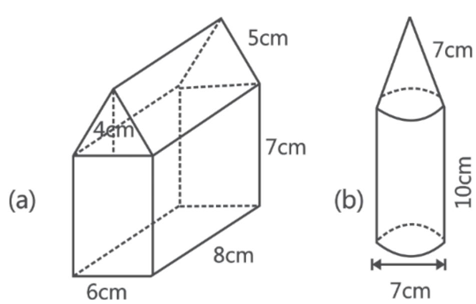
Q. 25. Find the volume of a pyramid of height 18 cm and with a rectangular base measuring 14 cm by 10 cm.

Long Answer Questions

Directions: Give answer in four to five sentences.

Q. 26. Length of a class-room is two times its height and its breath is $1\frac{1}{2}$ times its height. The cost of white-washing the walls at the rate of ₹ 3.2 per m^2 is ₹ 358.4. Find the cost of tiling the floor at the rate of ₹ 13.5 per m^2

Q. 27. Calculate the total surface area of each of the given combined solids. $\left(\pi = \frac{22}{7}\right)$



EXERCISE 2 – For Competitive Examinations

Multiple Choice Questions

Directions: This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONLY ONE is correct.

Q. 1. A cuboid container has the capacity to hold 60 small boxes. If all the dimensions of the container are tripled, then it can hold small boxes of the same size.

(a) 100 (b) 200

(c) 1620 (d) 1600

Q. 2. If the perimeter of one face of a cube is 32 cm, then its surface area is

(a) 120 cm^2 (b) 384 cm^2

(c) 360 cm^2 (d) 400 cm^2

Q. 3. A hemispherical tank of radius 6 cm is full of milk. It is connected to a pipe, through which liquid is emptied at the 2/7 litre per second. The time taken to empty the tank completely?

(a) 0.302 sec (b) 1.584 sec

(c) 0.453 sec (d) 0.492 sec

Q. 4. If the radius and height of a right circular cone are 'r' and 'h' respectively, the slant height of the cone is

(a) $(h^2+r^2)^{1/3}$ (b) $(h+r)^{1/3}$

(c) $(h^2+r^2)^{1/2}$ (d) None of these

Q. 5. A solid consists of a circular cylinder with an exact fitting right circular cone placed at the top. The height of the cone is h. If the total volume of the solid is 3 times the volume of the cone, then the height of the cylinder is

(a) 2h (b) $\frac{2h}{3}$

(c) $\frac{3h}{3}$ (d) 4h

Q. 6. A cube of side 12 cm. is painted red on all the faces and then cut into smaller cubes, each of side 3 cm. What is the total number of smaller cubes having none of their faces painted?

(a) 16 (b) 8

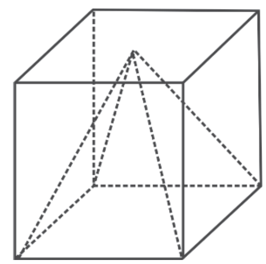
(c) 12 (d) 24

Q. 7. If length, breadth and height of a cuboid is increased by x %, y% and z% respectively then its volume is increased by –

(a) $\left[x+y+z+\frac{xy+xz+yz}{100}+\frac{xyz}{(100)^2}\right]\%$ (b) $\left[x+y+z+\frac{xy+xz+yz}{100}\right]\%$

(c) $\left[x+y+z+\frac{xyz}{(100)^2}\right]\%$ (d) None of these

Q. 8. A regular square pyramid is placed in a cube so that the base of the pyramid and that of the cube coincide. The vertex of the pyramid lies on the face of the cube opposite to the base, as shown in the below. An edge of the cube is 7 inches. How many square inches (approximately) are in the positive difference between the surface area of the cube and the surface area of the pyramid?



(a) 60.5 (b) 90.6

(c) 200.4 (d) 135.4

Q. 9. The base of a right prism is an equilateral triangle of edge 12 m. If the volume of the prism is 288 $\sqrt{3}\ m^3$, then its height is

(a) 6 m (b) 8 m

(c) 10 m (d) 12 m

Q. 10. The base of a right prism is a square of perimeter 20 cm and its height is 30 cm. The volume of the prism is

(a) 700 cm^3 (b) 750 cm^3

(c) 800 cm^3 (d) 850 cm^3

Q. 11. The base of a right pyramid is an equilateral triangle of perimeter 8 dm and the height of the pyramid is $30\sqrt{3}$ cm. The volume of the pyramid is

(a) 16000 cm^3 (b) 1600 cm^3

(c) $\frac{16000}{3}\ cm^3$ (d) $\frac{5}{4}\ cm^3$

Q. 12 A cylinder of radius r and height h and another cylinder of radius $\frac{r}{2}$ and height 2 h. Find the ratio of their curve surface area and their volumes.

(a) 1 : 2 (b) 2 : 1

(c) 1 : 3 (d) 4 : 1

Q. 13. A $30° - 60° - 90°$ triangle has the smallest side equal to 10 cm. This triangle is first rotated about the smallest side and then about the second largest side. If the volumes of the cones generated a and b respectively, then

(a) $a < b$ (b) $a = b$

(c) $a > b$ (d) $a = 2b$

More Than One Correct

Directions: This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONLY OR MORE may be correct.

Q. 14. A solid cube is cut into two cuboids of equal volumes. The ratio of the total surface area of the given cube and that of one of the cuboids is

(a) $6a^2$: $4a^2$ (b) $4a^2$: $6a^2$ (c) 3: 2 (d) 2: 3

Q. 15. A circular cylinder can be separated into

(a) Circular end at the bottom (b) Curved surface

(c) Circular end at the top (d) None of these

Q. 16. Which one of the following is correct?

(a) Volume of equilateral triangular Prism $= \frac{\sqrt{3}}{4}a^2h$

(b) Total Surface are area of equilateral triangular prism = lateral surface area + sum of areas of two ends $= 3ah + \frac{\sqrt{3}}{4}a^2$

(c) Volume of equilateral triangular prism $= \frac{\sqrt{3}}{4}$ ah

(d) Total surface area of equilateral triangular prism $= ah + ah + ah + \frac{\sqrt{3}}{4}a^2$

Q. 17. The value of each of a set coins varies as the square of its diameter, if its thickness remains constant and varies as the thickness, if the diameter remain constant. If the diameter remain constant. If the diameter of two coins are in the ratio 4:3, what should be the ratio of their thickness if the value of the first is four times that of the second?

(a) 18 : 8 (b) 9 : 4

(c) 8 : 18 (d) 4 : 9

Q. 18. Among the following, which one is / are correct?

(a) The slant height is the longest side of a pyramid.

(b) The section between the base and a plane parallel to the base of a solid is known as frustum.

(c) All the surface of a cuboid are rectangular.

(d) For a cylinder, the top, the bottom and the walls of the cylinder determine the total surface area.

Assertion and Reason

Directions: Each of these questions contains an Assertion followed by Reason. Read them carefully and answer the questions on the basis of following options. You have to select the one that best describes the two statements.

a) If both Assertion and Reason are correct and Reason is the correct explanation of Assertion.

b) If both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.

c) If Assertion is correct but Reason is incorrect.

d) If Assertion is incorrect but Reason is correct.

Q. 19. **Assertion:** If the base area and height of a prism be 25 cm^2 and 6 cm respectively, then its volume is 150 cm^3.

Reason: Volume of pyramid $= \frac{\text{Base area} \times \text{height}}{3}$

Q. 20. **Assertion:** Total surface area of the cylinder having radius of the base 14 cm and height 30 cm is 3872 cm^2

Reason: If r be the radius and h be the height of the cylinder, then total surface area $= (2\pi rh + 2\pi r^2)$

Subjective Questions

Directions: Answer the following questions.

Q. 21. A pool has a uniform circular cross-section of radius 5m and uniform depth 1.4m. It is filled by a pipe which delivers water at the rate of 20 litres per second. Calculate in minutes, the time taken to fill the pool. If the pool is emptied in 42 minutes by another cylindrical pipe through which water flows at the rate of 2 m/s, calculate the radius of the pipe in cm.

Q. 22. (a) Each edge of cube is increased by 50%. Find the percentage increase in the surface area of the cube.

(b) A solid cube is cut into two cuboids of equal volumes. Find the ratio of the total surface area of the given cube and that of one of the cuboids.

Q. 23. The base of a right prism is a right angled triangle. The measure of the base of the right angled triangle is 6 m and its height 8 m . If the height of the prism is 14 m; then find

(i) the number of edges of the prism.

(ii) the volume of the prism

(iii) the total surface area of the prism.

Q.24. Base area of a cylinder and cuboid are same and height of cylinder is 2 times radius. Find the ratio of total surface area of cuboid and cylinder if, Curve surface area of cuboid and cylinder are equal.

SOLUTIONS

EXERCISE 1 – For School Examinations

Fill in the Blanks

1. 6 **2.** n **3.** $200\ \pi\ell$ **4.** 49 : 25 **5.** 15 cm **6.** W= P $\times$ H + 2 A

True / False

7. True **8.** True **9.** True **10.** True

Match the Following Columns

11. A = (q) ; B = (r) ; C = (s) ; D = (p)

Very Short Answer Questions

12. a) External, Internal, height b) R – r

c) $2\pi Rh$.

d) Total surface area = external curved area + internal curved area + area of two ends.

$= 2\pi Rh + 2\pi rh + 2\pi(R^2 - r^2) = 2\pi\ (Rh + rh + R^2 - r^2)$

e) $\pi(R^2 - r^2)$.

13. Old volume = L B H

New volume $2L\times 3B\times 4H = 24\text{ L B H}$

$\Rightarrow$ % increase in volume $= \frac{24LBH - LBH}{LBH}\times 100 = 2300\%$

14. Volume of cylinder containing oil = $\frac{22}{7}\times 7\times 7\times 20 = 22\times 7\times 20\text{ cm}^3$

Volume of cubical box = base area $\times$ height = $11\times 10\times h$

Volume of cubical box = Volume of cylindrical container

$11\times 10\times h = 22\times 7\times 20\text{ cm}^3$

h = 280 cm

$\therefore$ Height of the cubical box is 280 cm.

15. Volume = $\pi R^2 H$

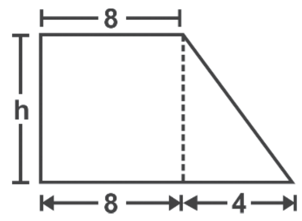
Initially H = 3, and d = 16 or R = 8

New H is 3 + x and New R is 8 + x

$\Rightarrow$ New volume = $\pi 8^2.(3 + X) \Rightarrow X = 16 / 3 = 5.33$

16. Volume of prism = Base area × Height = 52 × 16 = 832 cm^3

17. Volume of prism = Area of cross section × length



Area of cross section $= (8\times h) + \frac{1}{2}(4\times h) = 8h + 2h = 10h$

$\therefore$ Volume of prism = 10 × 24 $\Rightarrow$ 2400 = 240 h

$\therefore h = \frac{2400}{240} = 10\,cm$

18. Volume of pyramid = $\frac{1}{3}$ (Area of base) (Height)

$640 = \frac{1}{3}(12 \times 16) \times h$

$640 = 64 \times h$

$h = \frac{640}{64} = 10$ cm

Short Answer Questions

19. The total surface area of the cube = 1014 cm^2

Let each side of the cube be a cm.

Then $6a^2 = 1014\ cm^2$

$a^2 = \frac{1014}{6} cm^2 = 169\ cm^2$

$\therefore a = \sqrt{169}\,cm = 13\,cm$

Lateral surface area of the cube $= 4a^2 = 4\times(13)^2\ cm^2 = 4\times 169\ cm^2 = 676\ cm^2$

20. Diameter of the pipe d = 10 cm = 0.1 m

Radius $r = \frac{d}{2} = \frac{0.1}{2} = \frac{1}{20} m$

Length of the pipe h = 56 m

Total radiating surface = $2\pi rh = 2\times\frac{22}{7}\times\frac{1}{20}\times 56 m^2 = 17.6\ m^2$.

21. Total surface area of the cylinder = 924 cm^2

$2\pi r(h + r) = 924 \Rightarrow 2\times\frac{22}{7}r(h + r) = 924$

$r(h + r) = \frac{924\times 7}{2\times 22}$

$rh + r^2 = 147$ …(i)

Curved surface area of the cylinder $= 2\pi rh = \frac{2}{3} \times 924\ cm^2$

$2 \times \frac{22}{7} rh = 616\ cm^2 \quad \Rightarrow \quad rh = \frac{616 \times 7}{2 \times 22}$

$rh = 98$ …(ii)

Putting the value of rh in equation (i). We get

$98 + r^2 = 147$

$\Rightarrow r^2 = 147 - 98 = 49$

$\Rightarrow r = \sqrt{49} = 7\,cm$

Putting the value of r in equation (ii), we get

$7h = 98$

$\Rightarrow h = 14\ cm$

Hence, the volume of the cylinder $= \pi r^2 h = \frac{22}{7} \times (7)^2 \times 14 = 22 \times 7 \times 14 = 2156\ cm^3$

22. There are two cases possible for the cylinder

Case I: When perimeter of the base is 44 cm

$2\ \pi r = 44$ cm

r = 7 cm

Volume of cylinder = $\pi r^2 h = \frac{22}{7} \times 7 \times 7 \times 22$

$= 3388\ cm^3$

Case II: When perimeter of the base is 22 cm

$2\ \pi r = 22$ cm

$r = \frac{7}{2}$ cm

Volume of cylinder = $\frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 44 = 1694\ cm^3$

Therefore, radius of the cylinder is 7 cm as a volume of cylinder is maximum in first case.

23. Diagonal of innermost square = 2

Diagonal of next (second from innermost) square

$= 2 + 2 \times (1) = 4$

Diagonal of r^{th} square = 2 + 2 (r – 1)

Thus, diagonal of 7^{th} square = 14 and, diagonal of 8^{th} square = 16, putting r = 7 and 8

Required difference in area $= \frac{(16)^2}{2} - \frac{(14)^2}{2} = 30$ sq. unit.

24. The given pyramid has one square and four equal triangles for its surface.

Area of the 4 triangles $= 4\left(\frac{1}{2}\times 20\times 24\right) = 960$ cm^2

Area of the square = $20\times 20 = 400$ cm^2

Total surface area of the pyramid = 1360 cm^2

25. Area of base $= 14\,\text{cm}\times 10\text{ cm} = 140\text{ cm}^2$

$\therefore$ Volume of pyramid $= \frac{1}{3}(\text{Area of base}\times\text{hight})$ $= \frac{1}{3}\times 140\times 18$ cm^3 = 840 cm^3

26. Let the height of the classroom be h metres. Then, Length= 2h metres and, Breadth $= \frac{3}{2}$h metres

$\therefore$ Area of the four walls $= 2\times$ Height $\times$ (Length + Breadth) $= 7\text{ h}^2\text{ m}^2$

$\therefore$ Cost of white-washing of the four walls = ₹11.20 h^2.

But, the cost of white-washing is given as ₹179.20 $\Rightarrow$ h = 4

$\therefore$ Area of the floor of the room=(8×6) m^2=48 m^2

Cost of tiling of 1 m^2 of the floor = ₹ 6.75

$\therefore$ Cost of tilling the floor = ₹ 324

27. a) 348 cm^2

b) 335.5 cm^2

EXERCISE 2 – For Competitive Examinations

Multiple Choice Questions

1. (c) Let the dimensions of the cuboidal container be l, b and h. Then its capacity $= l \times b \times h$

When all dimensions are tripled, its capacity $= 3l \times 3b \times 3h = 27$ (previous capacity)

$\therefore$ The number of small boxes which the new container can hold $= 27 \times 60 = 1620$

2. (b) Edge of cube $= \frac{32}{4}$ cm = 8 cm, Surface area $= 6 \times 8^2\ cm^2 = 384\ cm^2$

4. (c) **5.** (b) **6.** (b)

7. (a)

% change in volume $= \frac{100^2(x+y+z)+100(xy+xz+yz)+xyz}{100^3} \times 100$

$$= \left[x+y+z+\frac{xy+xz+yz}{100}+\frac{xyz}{(100)^2}\right]\%$$

8. (d) The surface area of the pyramid is $pyr = 7^2 + 4t = 7^2 + 4 \times 27.39183 = 158.56732$

The answer is: Cube – pyr = 294 – 158.56732 = 135.43268 = 135.4

9. (b)

10. (b)

11. (c)

12. (b) CSA of first cylinder = 2 π rh

CSA of another cylinder = $2\pi \frac{r}{2} \times 2h = 2\pi rh$

Ratio = $\frac{2\pi rh}{2\pi rh} = 1$

Volume of first cylinder = $\pi r^2 h$

Volume of another cylinder = $\pi\left(\frac{r}{2}\right)^2(2h) = \frac{\pi r^2 h}{2}$

Ratio = $\frac{\pi r^2 h}{\frac{\pi r^2 h}{2}} = 2$

13. (c) The smaller sides are 10 cm. and $10\sqrt{3}$ cm. When rotated about the 10 cm side, the volume is (1/3) $\pi(10\sqrt{3})^2(10)\ cm^3$. In the second case the volume is (1/3) $\pi(10)^2(10\sqrt{3})\ cm^3$. The first volume is greater

More Than One Correct

14. (a, c) Let the edge of the solid cube = a

Since, cube is cut into two cuboids of equal volumes.

$\therefore$ Length = a, breadth = a, height $= \frac{a}{2}$

Total surface area of cube = $6a^2$ sq. unit

Total surface area of one cuboid $= 2\left(a \times a + a \times \frac{a}{2} + \frac{a}{2} \times a\right) = 4a^2$ sq. unit

Required ratio = $6a^2 : 4a^2 = 3 : 2$

15. (a, b, c)

16. (a, b, d)

17. (a, b) Let the diameters and thickness of the two coins be $d_1 t_1$ and $d_2 t_2$ respectively then

$$\frac{v_1}{v_2} = \frac{d_1^2}{d_2^2} \times \frac{t_1}{t_2} \Rightarrow \frac{4}{1} = \frac{4^2}{3^2} \times \frac{t_1}{t_2} \Rightarrow \frac{9}{4} = \frac{t_1}{t_2} \; \frac{t_1}{t_2} = \frac{9}{4} \times \frac{2}{2} = \frac{18}{8}$$

18. (a, b, c, d)

Assertion and Reason

19. (b) Assertion and reason both are correct, but reason is not the correct explanation of the assertion. Volume of a prism = Base area $\times$ height $= 25 \times 6 = 150 \text{ cm}^3$.

20. (a) Assertion and Reason both are correct and reason is the correct explanation of the assertion.

Total surface area= $= 2\pi rh + 2\pi r^2 = 2\pi r(h + r) = 2 \times \frac{22}{7} \times 14\,(30 + 14) = 88(44) = 3872 \text{ cm}^2$

Subjective Questions

21. Volume of the pool $= \pi(5)^2\,(1.4)$ cu m. ...(1)

Volume of water poured by the pipe in minute $= 20 \times 60$ litres = 1.2 cu m

Time (in min.) taken to fill the pool

$$= \frac{\text{Volume of the pool}}{\text{Volume of water pouring into the pool in one minute}} = 91\frac{2}{3} \text{ minutes}$$

Let r cm be the radius of the second pipe, i.e., $\frac{r}{100}$m is the radius of the second pipe.

Amount of water that comes out through this pipe in 2 minutes

$$= \pi\left(\frac{r}{100}\right)^2 (120) \times 42 \text{ cu m} \quad \text{....(2)}$$

Equating (1) and (2), we have $r = \frac{50}{6}$ or $8\frac{1}{3}$

Hence, the radius of the pipe is $8\frac{1}{3}$cm (or 8.33 cm)

22. **(a)** There is 125% increase in the surface area of the cube.

(b) 3: 2

23. (i) The number of the edges = The number of sides of the base $\times 3 = 3 \times 3 = 9$

(ii) The volume of the prism = Area of the base $\times$ height of the prism $= \frac{1}{2}(6\times 8)\times 14 = 336\ m^3$

(iii) TSA = LSA + 2 (area of base) = ph + 2 (area of base) where, p = perimeter of the base = sum of lengths of the sides of the given triangle,

As, hypotenuse of the triangle $\sqrt{6^2+8^2} = \sqrt{100} = 10m$

$\therefore$ Perimeter of the base = 6 + 8 + 10 = 24 m

$\Rightarrow LSA = ph = 24\times 14 = 336\ m^2$

TSA = LSA + 2 (area of base) $= 336 + 2\left(\frac{1}{2}\times 6\times 8\right) = 336 + 48 = 384 m^2$

24. Base area of cylinder = base area of cuboid

$\pi r^2 = lb$ ....(i)

Curves surface area of cylinder = curve surface area of cuboid

$2\ \pi\ r\ h' = 2\ h\ (l + b)$

$2\ \pi\ r\ (2\ r) = 2\ h\ (l + b)$

$2\ h\ (l + b) = 4\pi r^2$

Ratio of total surface area = $\left(\frac{2[lb + h(l+b)]}{2\pi r^2 + 2\pi rh'}\right) = \frac{2(\pi r^2 + 4\pi r^2)}{2\pi r^2 + 4\pi r^2} = \frac{5}{3}$ = 5: 3