MATHS

\[(a+b)^2 = a^2 + 2ab + b^2\]
Trigonometric Function

Top Concepts

1. An angle is a measure of rotation of a given ray about its initial point. The original position of the ray before rotation is called the initial side of the angle and the final position of the ray after rotation is called the terminal side of the angle. The point of rotation is called the vertex.

2. If the direction of rotation is anticlockwise, then the angle is said to be positive and if the direction of rotation is clockwise, then the angle is negative.

3. If a rotation from the initial side to the terminal side is $\left(\frac{1}{360}\right)^{th}$ of a revolution, then the angle is said to measure one degree. It is denoted by $1^\circ$.

4. A degree is divided into 60 minutes, and a minute is divided into 60 seconds. One sixtieth of a degree is called a minute and is written as $1'$ and one sixtieth of a minute is called a second and is written as $1''$.
   Thus, $1^\circ = 60'$ and $1' = 60''$
5. The angle subtended at the centre by an arc of length 1 unit in a unit circle is said to have a measure of 1 radian.

6. Basic trigonometric ratios
   Consider the following triangle:

   ![Triangle Diagram]

   i. \( \sin \theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} \)
   ii. \( \cos \theta = \frac{\text{Base}}{\text{Hypotenuse}} \)
   iii. \( \tan \theta = \frac{\text{Perpendicular}}{\text{Base}} \)
   iv. \( \csc \theta = \frac{\text{Hypotenuse}}{\text{Perpendicular}} \)
   v. \( \sec \theta = \frac{\text{Hypotenuse}}{\text{Base}} \)
   vi. \( \cot \theta = \frac{\text{Base}}{\text{Perpendicular}} \)
7. Some trigonometric identities

i. \[ \sin \theta = \frac{1}{\csc \theta} \quad \text{or} \quad \cos \theta = \frac{1}{\sec \theta} \]

ii. \[ \cos \theta = \frac{1}{\sec \theta} \quad \text{or} \quad \sec \theta = \frac{1}{\cos \theta} \]

iii. \[ \tan \theta = \frac{1}{\cot \theta} \quad \text{or} \quad \cot \theta = \frac{1}{\tan \theta} \]

iv. \[ \tan \theta = \frac{\sin \theta}{\cos \theta} \]

v. \[ \cot \theta = \frac{\cos \theta}{\sin \theta} \]

vi. \[ \sin^2 \theta + \cos^2 \theta = 1 \]

vii. \[ 1 + \tan^2 \theta = \sec^2 \theta \]

viii. \[ 1 + \cot^2 \theta = \csc^2 \theta \]

8. If a point on a unit circle is on the terminal side of an angle in the standard position, then the sine of such an angle is simply the y-coordinate of the point and the cosine of the angle is the x-coordinate of that point.

9. All the angles which are integral multiples of \( \frac{\pi}{2} \) are called quadrantal angles. Values of quadrantal angles are:

\[ \cos 0 = 1, \sin 0 = 0 \]
\[ \cos \frac{\pi}{2} = 0, \sin \frac{\pi}{2} = 1 \]
\[ \cos \pi = -1, \sin \pi = 0 \]
\[ \cos \frac{3\pi}{2} = 0, \sin \frac{3\pi}{2} = -1 \]
\[ \cos 2\pi = 1, \sin 2\pi = 0 \]

10. **Even function**: A function \( f(x) \) is said to be an even function if \( f(-x) = f(x) \), for all \( x \) in its domain.

11. **Odd function**: A function \( f(x) \) is said to be an odd function if \( f(-x) = -f(x) \), for all \( x \) in its domain.
12. Cosine is an even function and sine is an odd function
   \[ \cos(-x) = \cos x \]
   \[ \sin(-x) = -\sin x \]

13. Signs of trigonometric functions in various quadrants:
   In quadrant I, all the trigonometric functions are positive.
   In quadrant II, only sine is positive. In quadrant III, only tan is positive. In quadrant IV, only cosine function is positive. This is depicted as follows:

   ![Graph showing signs of trigonometric functions in various quadrants]

14. In quadrants, where the y-axis is positive (i.e. I and II), sine is positive, and in quadrants where the x-axis is positive (i.e. I and IV), cosine is positive.

15. A simple rule to remember the sign of the trigonometrical ratios, in all the four quadrants, is the four letter phrase—All School To College.

16. A function ‘f’ is said to be a periodic function if there exists a real number \( T > 0 \) such that \( f(x + T) = f(x) \) for all ‘x’. This ‘T’ is the period of function.

17. Trigonometric ratios of complementary angles
   i. \( \sin\left(90^\circ - \theta\right) = \cos \theta \)
   ii. \( \cos\left(90^\circ - \theta\right) = \sin \theta \)
   iii. \( \tan\left(90^\circ - \theta\right) = \cot \theta \)
   iv. \( \csc\left(90^\circ - \theta\right) = \sec \theta \)
   v. \( \sec\left(90^\circ - \theta\right) = \csc \theta \)
   vi. \( \cot\left(90^\circ - \theta\right) = \tan \theta \)
18. Trigonometric ratios of \((90^\circ + \theta)\) in terms of \(\theta\)
   i. \(\sin(90^\circ + \theta) = \cos \theta\)
   ii. \(\cos(90^\circ + \theta) = -\sin \theta\)
   iii. \(\tan(90^\circ + \theta) = -\cot \theta\)
   iv. \(\csc(90^\circ + \theta) = \sin \theta\)
   v. \(\sec(90^\circ + \theta) = -\cos \theta\)
   vi. \(\cot(90^\circ + \theta) = -\tan \theta\)

19. Trigonometric ratios of \((180^\circ - \theta)\) in terms of \(\theta\)
   i. \(\sin(180^\circ - \theta) = \sin \theta\)
   ii. \(\cos(180^\circ - \theta) = -\cos \theta\)
   iii. \(\tan(180^\circ - \theta) = -\tan \theta\)
   iv. \(\csc(180^\circ - \theta) = \cos \theta\)
   v. \(\sec(180^\circ - \theta) = -\sec \theta\)
   vi. \(\cot(180^\circ - \theta) = -\cot \theta\)

20. Trigonometric ratios of \((180^\circ + \theta)\) in terms of \(\theta\)
   i. \(\sin(180^\circ + \theta) = -\sin \theta\)
   ii. \(\cos(180^\circ + \theta) = -\cos \theta\)
   iii. \(\tan(180^\circ + \theta) = \tan \theta\)
   iv. \(\csc(180^\circ + \theta) = -\cos \theta\)
   v. \(\sec(180^\circ + \theta) = -\sec \theta\)
   vi. \(\cot(180^\circ + \theta) = \cot \theta\)

21. Trigonometric ratios of \((360^\circ - \theta)\) in terms of \(\theta\)
   i. \(\sin(360^\circ - \theta) = -\sin \theta\)
   ii. \(\cos(360^\circ - \theta) = \cos \theta\)
   iii. \(\tan(360^\circ - \theta) = -\tan \theta\)
   iv. \(\csc(360^\circ - \theta) = -\cos \theta\)
   v. \(\sec(360^\circ - \theta) = \sec \theta\)
   vi. \(\cot(360^\circ - \theta) = -\cot \theta\)
22. Trigonometric ratios of \( (360^\circ + \theta) \) in terms of \( \theta \)
   
   i. \( \sin (360^\circ + \theta) = \sin \theta \)
   
   ii. \( \cos (360^\circ + \theta) = \cos \theta \)
   
   iii. \( \tan (360^\circ + \theta) = \tan \theta \)
   
   iv. \( \csc (360^\circ + \theta) = \csc \theta \)
   
   v. \( \sec (360^\circ + \theta) = \sec \theta \)
   
   vi. \( \cot (360^\circ + \theta) = \cot \theta \)

23. \( \sin(2\pi + x) = \sin x \), so the period of sine is \( 2\pi \). Period of its reciprocal is also \( 2\pi \).

24. \( \cos(2\pi + x) = \cos x \), so the period of cosine is \( 2\pi \). Period of its reciprocal is also \( 2\pi \).

25. \( \tan(\pi + x) = \tan x \). Period of tangent and cotangent function is \( \pi \).

26. The graph of \( \cos x \) can be obtained by shifting the sine function along the \( x \)-axis by the factor \( \frac{\pi}{2} \).

27. The \( \tan \) function differs from sine and cosine functions in two ways:
   
   (i) Function \( \tan \) is not defined at the odd multiples of \( \pi/2 \).
   
   (ii) The \( \tan \) function is not bounded.

28. **Function** | **Period**
   
   \( y = \sin x \) | \( 2\pi \)
   
   \( y = \sin (ax) \) | \( \frac{2\pi}{a} \)
   
   \( y = \cos x \) | \( 2\pi \)
   
   \( y = \cos (ax) \) | \( \frac{2\pi}{a} \)
   
   \( y = \cos 3x \) | \( \frac{2\pi}{3} \)
   
   \( y = \sin 5x \) | \( \frac{2\pi}{5} \)

29. For a function of the form \( y = kf(ax + b) \), the range will be ‘k’ times the range of function \( x \), where \( k \) is any real number.

   If \( f(x) = \) sine function in above form, the range will be equal to \([-k, k]\).

   If \( f(x) = \) cosec function in above form, the range will be equal to \( \mathbb{R} - [-k, k] \).

   If the function is of the form \( k\sec (ax + b) \) or \( k\cosec (ax + b) \), the period is equal to the period of function ‘f’ divided by ‘a’.

   The position of the graph of \( y = kf(ax + b) \) is ‘b’ units to the right or left of \( y = f(x) \) depending on whether \( b < 0 \) or \( b > 0 \).
30. The solutions of a trigonometric equation, for which $0 \leq x \leq 2\pi$, are called **principal solutions**.

31. The expression involving the integer ‘n’, which gives all the solutions of a trigonometric equation, is called the **general solution**.

32. The numerically smallest value of the angle (in degree or radian) satisfying a given trigonometric equation is called the **Principal Value**. If there are two values, one positive and the other negative, which are numerically equal, then the positive value is taken as the Principal Value.

**Top Formulae**

1. $1 \text{ radian} = \frac{180^\circ}{\pi} \approx 57^\circ16'$. approximately.

2. $1^\circ = \frac{\pi}{180^\circ} \text{ radians} = 0.01746 \text{ radians}$ approximately.

3. $s = r\theta$
   Length of arc = radius \times angle in radian.
   This relation can only be used when \( \theta \) is in radians.

4. Radian measure = $\frac{\pi}{180} \times \text{degree measure}$.

5. Degree measure = $\frac{180}{\pi} \times \text{radian measure}$.

6. Values of trigonometric ratios

<table>
<thead>
<tr>
<th></th>
<th>$0^\circ$</th>
<th>$\frac{\pi}{6}$</th>
<th>$\frac{\pi}{4}$</th>
<th>$\frac{\pi}{3}$</th>
<th>$\frac{\pi}{2}$</th>
<th>$\pi$</th>
<th>$\frac{3\pi}{2}$</th>
<th>$2\pi$</th>
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<tr>
<td><strong>sin</strong></td>
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<td>$\frac{1}{2}$</td>
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<td>$\sqrt{3}$</td>
<td>1</td>
<td>0</td>
<td>$-1$</td>
<td>0</td>
</tr>
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</table>
7. Domain and range of various trigonometric functions:

<table>
<thead>
<tr>
<th>Function</th>
<th>Domain</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = \sin x$</td>
<td>$[-\frac{\pi}{2}, \frac{\pi}{2}]$</td>
<td>$[-1, 1]$</td>
</tr>
<tr>
<td>$y = \cos x$</td>
<td>$[0, \pi]$</td>
<td>$[-1, 1]$</td>
</tr>
<tr>
<td>$y = \csc x$</td>
<td>$[-\frac{\pi}{2}, \frac{\pi}{2}]$</td>
<td>$\mathbb{R} - (-1, 1)$</td>
</tr>
<tr>
<td>$y = \sec x$</td>
<td>$[0, \pi] - {\frac{\pi}{2}}$</td>
<td>$\mathbb{R} - (-1, 1)$</td>
</tr>
<tr>
<td>$y = \tan x$</td>
<td>$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$</td>
<td>$\mathbb{R}$</td>
</tr>
<tr>
<td>$y = \cot x$</td>
<td>$(0, \pi)$</td>
<td>$\mathbb{R}$</td>
</tr>
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</table>

8. Sign convention

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
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</thead>
<tbody>
<tr>
<td>$\sin x$</td>
<td>$+$</td>
<td>$+$</td>
<td>$-$</td>
<td>$-$</td>
</tr>
<tr>
<td>$\cos x$</td>
<td>$+$</td>
<td>$-$</td>
<td>$-$</td>
<td>$+$</td>
</tr>
<tr>
<td>$\tan x$</td>
<td>$+$</td>
<td>$-$</td>
<td>$+$</td>
<td>$-$</td>
</tr>
<tr>
<td>$\csc x$</td>
<td>$+$</td>
<td>$+$</td>
<td>$-$</td>
<td>$-$</td>
</tr>
<tr>
<td>$\sec x$</td>
<td>$+$</td>
<td>$-$</td>
<td>$-$</td>
<td>$+$</td>
</tr>
<tr>
<td>$\cot x$</td>
<td>$+$</td>
<td>$-$</td>
<td>$+$</td>
<td>$-$</td>
</tr>
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</table>
9. Behavior of Trigonometric Functions in Various Quadrants

<table>
<thead>
<tr>
<th>Function</th>
<th>I quadrant</th>
<th>II quadrant</th>
<th>III quadrant</th>
<th>IV quadrant</th>
</tr>
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<tbody>
<tr>
<td>( \sin )</td>
<td>Increases from 0 to 1</td>
<td>Decreases from 1 to 0</td>
<td>Decreases from 0 to (-1)</td>
<td>Increases from (-1) to 0</td>
</tr>
<tr>
<td>( \cos )</td>
<td>Decreases from 1 to 0</td>
<td>Decreases from 0 to (-1)</td>
<td>Increases from (-1) to 0</td>
<td>Increases from 0 to 1</td>
</tr>
<tr>
<td>( \tan )</td>
<td>Increases from 0 to (\infty)</td>
<td>Increases from (-\infty) to 0</td>
<td>Increases from 0 to (\infty)</td>
<td>Increases from (-\infty) to 0</td>
</tr>
<tr>
<td>( \cot )</td>
<td>Decreases from (\infty) to 0</td>
<td>Decreases from 0 to (-\infty)</td>
<td>Decreases from (\infty) to 0</td>
<td>Decreases from 0 to (-\infty)</td>
</tr>
<tr>
<td>( \sec )</td>
<td>Increases from 1 to (\infty)</td>
<td>Increases from (-\infty) to (-1)</td>
<td>Decreases from (-1) to (\infty)</td>
<td>Decreases from (\infty) to 1</td>
</tr>
<tr>
<td>( \csc )</td>
<td>Decreases from (\infty) to 1</td>
<td>Increases from 1 to (\infty)</td>
<td>Increases from (-\infty) to (-1)</td>
<td>Decreases from (-1) to (-\infty)</td>
</tr>
</tbody>
</table>

10. Basic Formulae
   (i) \( \cos(x + y) = \cos x \cos y - \sin x \sin y \)
   (ii) \( \cos(x - y) = \cos x \cos y + \sin x \sin y \)
   (iii) \( \sin(x + y) = \sin x \cos y + \cos x \sin y \)
   (iv) \( \sin(x - y) = \sin x \cos y - \cos x \sin y \)

   If none of the angles \(x, y\) and \(x + y\) is an odd multiple of \(\frac{\pi}{2}\), then

   (v) \( \tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y} \)
   (vi) \( \tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y} \)

   If none of the angles \(x, y\) and \(x + y\) is a multiple of \(\pi\), then

   (vii) \( \cot(x + y) = \frac{\cot x \cot y - 1}{\cot y + \cot x} \)
   (viii) \( \cot(x - y) = \frac{\cot x \cot y + 1}{\cot y - \cot x} \)
11. Allied Angle Relations

\[ \cos(2\pi - x) = \cos x \quad \text{sin}(2\pi - x) = -\sin x \]

\[ \cos(2n\pi + x) = \cos x \quad \text{sin}(2n\pi + x) = \sin x \]

12. Some Important Results

i. \( \sin(x + y) \sin(x - y) = \sin^2 x - \sin^2 y = \cos^2 y - \cos^2 x \)

ii. \( \cos(x + y) \cos(x - y) = \cos^2 x - \sin^2 y = \cos^2 y - \sin^2 x \)

iii. \( \sin(x + y + z) = \sin x \cos y \cos z + \cos x \sin y \cos z + \cos x \cos y \sin z - \sin x \sin y \sin z \)

iv. \( \cos(x + y + z) = \cos x \cos y \cos z - \sin x \sin y \cos z - \sin x \cos y \sin z - \cos x \sin y \sin z \)

v. \( \tan(x + y + z) = \frac{\tan x + \tan y + \tan z - \tan x \tan y \tan z}{1 - \tan x \tan y - \tan y \tan z - \tan z \tan x} \)

13. Sum and Difference Formulae

(i) \( \cos x + \cos y = 2 \cos \frac{x + y}{2} \cos \frac{x - y}{2} \)

(ii) \( \cos x - \cos y = -2 \sin \frac{x + y}{2} \sin \frac{x - y}{2} \)

(iii) \( \sin x + \sin y = 2 \sin \frac{x + y}{2} \cos \frac{x - y}{2} \)

(iv) \( \sin x - \sin y = 2 \cos \frac{x + y}{2} \sin \frac{x - y}{2} \)

(v) \( 2 \cos x \cos y = \cos(x + y) + \cos(x - y) \)

(vi) \( -2 \sin x \sin y = \cos(x + y) - \cos(x - y) \)

(vii) \( 2 \sin x \cos y = \sin(x + y) + \sin(x - y) \)

(viii) \( 2 \cos x \sin y = \sin(x + y) - \sin(x - y) \)
14. Multiple Angle Formulae

(i) \( \cos 2x = \cos^2 x - \sin^2 x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x = \frac{1 - \tan^2 x}{1 + \tan^2 x} \)

(ii) \( \sin 2x = 2 \sin x \cos x = \frac{2 \tan x}{1 + \tan^2 x} \)

(iii) \( \tan 2x = \frac{2 \tan x}{1 - \tan^2 x} \)

(iv) \( \sin 3x = 3 \sin x - 4 \sin^3 x \)

(v) \( 1 + \cos 2x = 2 \cos^2 x \)

(vi) \( 1 - \cos 2x = 2 \sin^2 x \)

(vi) \( \cos 3x = 4 \cos^3 x - 3 \cos x \)

(vii) \( \tan 3x = \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x} \)

(viii) \( \cos x \cdot \cos 2x \cdot \cos 2^2 x \cdot \cos 2^3 x \cdot \ldots \cdot \cos 2^{n-1} x = \frac{\sin 2^n x}{2^n \sin x} \)

(ix) Let \( x = \frac{\pi}{2^n + 1} \) then we have \( 2^n \cos x \cdot \cos 2x \cdot \cos 2^2 x \cdot \cos 2^3 x \cdot \ldots \cdot \cos 2^{n-1} x = 1 \)

(x) \( \sin x \cdot \sin (60^\circ - x) \cdot \sin (60^\circ + x) = \frac{\sin 3x}{4} \)

(xi) \( \cos x \cdot \cos (60^\circ - x) \cdot \cos (60^\circ + x) = \frac{\cos 3x}{4} \)

(xii) \( (1 + \sec 2x)(1 + \sec 4x)(1 + \sec 8x)\ldots(1 + \sec 2^n x) = \tan 2^n x \cot x \)

15. Trigonometric ratios of angles in terms of half angle

i. \( \sin x = 2 \sin \frac{x}{2} \cos \frac{x}{2} \)

ii. \( \cos x = \cos^2 \frac{x}{2} - \sin^2 \frac{x}{2} \)

iii. \( \cos x = 2 \cos^2 \frac{x}{2} - 1 \)

iv. \( \cos x = 1 - 2 \sin^2 \frac{x}{2} \)

v. \( 1 + \cos x = 2 \cos^2 \frac{x}{2} \)

vi. \( 1 - \cos x = 2 \sin^2 \frac{x}{2} \)
vii. \[ \tan x = \frac{2 \tan \frac{x}{2}}{1 - \tan^2 \frac{x}{2}} \]

viii. \[ \sin x = \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \]

ix. \[ \cos x = \frac{1 - \tan^2 \frac{x}{2}}{2} \]

x. \[ \cos \frac{x}{2} = \pm \sqrt{\frac{1 + \cos x}{2}} \]

xi. \[ \sin \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{2}} \]

xii. \[ \tan \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{1 + \cos x}} \]

16. Trigonometrical ratios of angle in terms of \( \frac{x}{3} \) angle

i. \[ \sin x = 3 \sin \frac{x}{3} - 4 \sin^3 \frac{x}{3} \]

ii. \[ \cos x = 4 \cos^3 \frac{x}{3} - 3 \cos \frac{x}{3} \]

iii. \[ \tan x = \frac{3 \tan \frac{x}{3} - \tan^3 \frac{x}{3}}{1 - 3 \tan^2 \frac{x}{3}} \]

17. Trigonometrical ratios of important angles

i. \[ \sin 18^\circ = \frac{\sqrt{5} - 1}{4} \]

ii. \[ \cos 36^\circ = \frac{\sqrt{5} + 1}{4} \]

iii. \[ \cos 18^\circ = \frac{\sqrt{10 + 2\sqrt{5}}}{4} \]

iv. \[ \sin 36^\circ = \frac{\sqrt{10 - 2\sqrt{5}}}{4} \]
18. Trigonometric equations

<table>
<thead>
<tr>
<th>No.</th>
<th>Equations</th>
<th>General solution</th>
<th>Principal value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>\sin \theta = 0</td>
<td>\theta = n\pi, n \in \mathbb{Z}</td>
<td>\theta = 0</td>
</tr>
<tr>
<td>2</td>
<td>\cos \theta = 0</td>
<td>\theta = (2n + 1) \frac{\pi}{2}, n \in \mathbb{Z}</td>
<td>\theta = \frac{\pi}{2}</td>
</tr>
<tr>
<td>3</td>
<td>\tan \theta = 0</td>
<td>\theta = n\pi</td>
<td>\theta = 0</td>
</tr>
<tr>
<td>4</td>
<td>\sin \theta = \sin \alpha</td>
<td>\theta = n\pi + (-1)^n \alpha, n \in \mathbb{Z}</td>
<td>\theta = \alpha</td>
</tr>
<tr>
<td>5</td>
<td>\cos \theta = \cos \alpha</td>
<td>\theta = 2n\pi \pm \alpha, n \in \mathbb{Z}</td>
<td>\theta = 2\alpha, \alpha &gt; 0</td>
</tr>
<tr>
<td>6</td>
<td>\tan \theta = \tan \alpha</td>
<td>\theta = n\pi + \alpha, n \in \mathbb{Z}</td>
<td>\theta = \alpha</td>
</tr>
<tr>
<td>7</td>
<td>\sin^2 \theta = \sin^2 \alpha</td>
<td>\theta = n\pi \pm \alpha, n \in \mathbb{Z}</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>\cos^2 \theta = \cos^2 \alpha</td>
<td>\theta = n\pi \pm \alpha, n \in \mathbb{Z}</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>\tan^2 \theta = \tan^2 \alpha</td>
<td>\theta = n\pi \pm \alpha, n \in \mathbb{Z}</td>
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</tr>
</tbody>
</table>

19. The equation \(a \cos \theta + b \sin \theta = c\) is solvable for \(|c| \leq \sqrt{a^2 + b^2}\).

20. (i) \sin \theta = k = \sin (n\pi + (-1)^n \alpha), n \in \mathbb{Z} \\
\quad \theta = n\pi + (-1)^n \alpha, n \in \mathbb{Z} \\
\quad \cosec \theta = \cosec \alpha \Rightarrow \sin \theta = \sin \alpha \\
\quad \theta = n\pi + (-1)^n \alpha, n \in \mathbb{Z}

(ii) \cos \theta = k = \cos (2n\pi \pm \alpha), n \in \mathbb{Z} \\
\quad \theta = 2n\pi \pm \alpha, n \in \mathbb{Z}

21. Sine Rule: The sine rule states that
\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

Or
\[
\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}
\]
22. Law of Cosine

In any \( \triangle ABC \), \( a^2 = b^2 + c^2 - 2bc \cos A \), \( \cos A = \frac{b^2 + c^2 - a^2}{2bc} \)

\( b^2 = c^2 + a^2 - 2ac \cos B \), \( \cos B = \frac{a^2 + c^2 - b^2}{2ac} \)

\( a^2 = b^2 + c^2 - 2bc \cos A \), \( \cos C = \frac{b^2 + c^2 - a^2}{2bc} \)

23. Projection Formulae

(i) \( a = b \cos C + c \cos B \)

(ii) \( b = c \cos A + a \cos C \)

(iii) \( c = a \cos B + b \cos A \)

24. Napier's Analogy (Law of Tangents)

(i) \( \tan \left( \frac{B - C}{2} \right) = \frac{b - c}{b + c} \cot \frac{A}{2} \)

(ii) \( \tan \left( \frac{A - B}{2} \right) = \frac{a - b}{a + b} \cot \frac{C}{2} \)

(iii) \( \tan \left( \frac{C - A}{2} \right) = \frac{c - a}{c + a} \cot \frac{B}{2} \)

25. Area of a \( \triangle ABC \) is given by

\( \Delta = \frac{1}{2} bc \sin A = \frac{1}{2} ca \sin B = \frac{1}{2} ab \sin C \)

Top diagrams

1. Graphs help in the visualisation of the properties of trigonometric functions. The graph of \( y = \sin \theta \) can be drawn by plotting a number of points \((\theta, \sin \theta)\) as \( \theta \) takes a series of different values. Because the sine function is continuous, these points can be joined with a smooth curve. Following similar procedures, graphs of other functions can be obtained.

i. Graph of \( \sin x \)
ii. Graph of $\cos x$

iii. Graph of $\tan x$

iv. Graph of $\sec x$

v. Graph of $\csc x$
vi. Graph of \( \cot x \)