NAGALAND
Class XII
Mathematics
Board Paper – 2016

Time allowed: 3 hours **Maximum Marks: 80**

General Instructions:

i. Approximately 15 minutes is allotted to read the question paper and revise the answers.
ii. The question paper consists of 26 questions. All questions are compulsory.
iii. Marks are indicated against each question.
iv. Internal choice has been provided in some questions.
v. Use of simple calculators (non-scientific and non-programmable) only is permitted.

N.B: Check that all pages of the question paper is complete as indicated on the top left side.

Section A

1. Choose the correct answer from the given alternatives.

(a) On the set of real number, which of the following binary operations is commutative? [1]
(i) a*b = a - b
(ii) $a*b = ab^2$
(iii) a*b = a + 2b
(iv) $a*b = (a + b)^2$

(b) The principle value of $\cot^{-1}(-1)$ is [1]
(i) $\frac{3\pi}{4}$
(ii) $\frac{\pi}{4}$
(iii) $\frac{-\pi}{4}$
(iv) $\frac{-3\pi}{4}$

(c) The value of x from the equation $\begin{vmatrix} x & 2 & 3 \\ 4 & x & 1 \\ x & 2 & 5 \end{vmatrix} = 0$ is [1]

(i) $\pm\sqrt{2}$
(ii) $\pm\sqrt{3}$
(iii) $\pm 2\sqrt{2}$
(iv) $\pm 2\sqrt{3}$

(d) If $\sqrt{x} + \sqrt{y} = b$, then $\frac{dy}{dx}$ is [1]

(i) $-\sqrt{\frac{x}{y}}$
(ii) $-\sqrt{\frac{y}{x}}$
(iii) $\sqrt{\frac{x}{y}}$
(iv) $\sqrt{\frac{y}{x}}$

(e) $\frac{d}{dx}\left(e^{\cot^{-1} x^2}\right)$ is equal to [1]

(i) $\frac{2}{1+x^2} e^{\cot^{-1} x^2}$
(ii) $\frac{-2x}{1+x^2} e^{\cot^{-1} x^2}$
(iii) $\frac{-2}{1+x^4} e^{\cot^{-1} x^2}$
(iv) $\frac{-2x}{1+x^4} e^{\cot^{-1} x^2}$

(f) The tangents to the curve $y = x^2 - 7x + 1$ at the points (3, 0) and (4, 0) are [1]
(i) parallel
(ii) perpendicular
(iii) coincident
(iv) intersecting

(g) $\int \log x \, dx =$ [1]
(i) x(logx + 1) + C
(ii) x(1 – logx) + C
(iii) x(logx - 1) + C
(iv) x logx + C

(h) Which of the following is wrong? [1]

(i) $\int \sqrt{x^2 - a^2}\,dx = \frac{x}{2}\sqrt{x^2 - a^2} + \frac{a^2}{2}\log\left|x + \sqrt{x^2 - a^2}\right| + C$

(ii) $\int \sqrt{x^2 + a^2}\,dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\log\left|x + \sqrt{x^2 + a^2}\right| + C$

(iii) $\int \frac{1}{\sqrt{x^2 + a^2}}\,dx = \log\left|x + \sqrt{x^2 + a^2}\right| + C$

(iv) $\int \frac{1}{\sqrt{x^2 - a^2}}\,dx = \log\left|x + \sqrt{x^2 - a^2}\right| + C$

(i) The general solution of $\frac{dy}{dx} = e^{x-y}$ is [1]

(i) $e^x + e^y = C$
(ii) $e^x - e^y = C$
(iii) $e^x + e^{-y} = C$
(iv) $e^x - e^{-y} = C$

(j) The projection of $\hat{i} + \hat{j} + k$ on $\hat{i} - \hat{j}$ is [1]

(i) 0
(ii) 1
(iii) $\sqrt{2}$
(iv) $\frac{1}{\sqrt{2}}$

Section B

2. Consider the set A = {1, 2, 4, 5, 7, 9, 10}. Define the relation R on A as "aRb" if and only if |a - b| is an even number". Then
(i) Write the elements of the set R and (ii) find the domain and range of R. [2]

3. Prove that $\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)$ [2]

4. Find the value of x such that, $\begin{bmatrix}1 & 1 & x\end{bmatrix}\begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 1 & 0\end{bmatrix}\begin{bmatrix}5\\ 5\\ 5\end{bmatrix} = 0$ [2]

5. If x = $\tan\left(\frac{1}{a}\log y\right)$, show that $(1 + x^2)\frac{dy}{dx} = ay$ [2]

6. Differentiate $\tan^{-1}\left(\frac{2^{x+1}}{1-4^x}\right)$ with respect to x. [2]

7. Using differentials, find the approximate value of $\sqrt{26}$ [2]

8. Evaluate $\int x\sqrt{x^4+1}\,dx$ [2]

9. Find the differential equation of the family of circles having centres on the x-axis and passing through the origin. [2]

10. Find the direction ratios and the direction cosines of the vector joining the points (4, 7, 2) and (5, 11, -4) [2]

11. If A and B are two events such that $P(A)=\frac{1}{4}$, $P(B)=\frac{1}{3}$ and $P(A\cup B)=\frac{1}{2}$, show that A and B are independent events.

Section - C

12. Prove that on the set of integers Z, the relation R defined as aRb $\Leftrightarrow$ a = $\pm$b is an equivalence relation. [4]

13.

a. Using properties of determinants, prove that:

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$$

Or [4]

b. If $A=\begin{bmatrix} 4 & 1 \\ 6 & 5 \end{bmatrix}$ and $B=\begin{bmatrix} 2 & 5 \\ 1 & 6 \end{bmatrix}$, then verify that $(AB)^{-1}=B^{-1}A^{-1}$

14.

a. Find the values of a and b so that the following function is continuous at x = 3 and x = 5, $f(x)=\begin{cases} 1, & \text{if } x\le 3 \\ ax+b, & \text{if } 3<x<5 \\ 7, & \text{if } 5\le x \end{cases}$

Or [4]

b. Verify Lagrange's Mean Value Theorem for the function f(x) = x(2-x) in [0,1]

15. Prove that $\int_0^{\pi} \log(1+\cos x)\,dx = -\pi \log 2$, given that $\int_0^{\frac{\pi}{2}} \log \sin x\,dx = \frac{-\pi}{2}\log 2$ [4]

16.

a. Prove that $\int e^{ax}\cos(bx+c)\,dx = \frac{e^{ax}}{a^2+b^2}\{a\cos(bx+c)+b\sin(bx+c)\}$

Or [4]

b. Evaluate $\int \frac{x^2+5x+3}{x^2+3x+2}\,dx$

17. Solve the differential equation $(x+y+1)\frac{dy}{dx}=1$ [4]

18.

a. Prove that $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar if $\vec{a}+\vec{b}$, $\vec{b}+\vec{c}$ and $\vec{c}+\vec{a}$ are coplanar.

Or [4]

b. Express the vector $\vec{a}=5\hat{i}-2\hat{j}+5k$ as sum of two vectors such that one is parallel to the vector $\vec{b}=3\hat{i}+k$ and the other is perpendicular to $\vec{b}$

19. Find the shortest distance between the lines

$\frac{x-1}{2}=\frac{1-y}{1}=\frac{z}{1}$ and $\frac{x-2}{3}=\frac{1-y}{5}=\frac{z+1}{2}$ [4]

20. A pair of dice is thrown 7 times. If "getting a total of 7" is considered a success, find the probability of getting: (i) no success, (ii) at least 6 successes. [4]

21.

a. Urn A contains 1 white, 2 black and 3 red balls; urn B contains 2 white, 1 black and 1 red balls; and urn C contains 4 white, 5 black and 3 red balls. One urn is chosen at random and two balls are drawn. These happen to be one white and one red. What is the probability that they come from urn A?

Or [4]

b. A coin is tossed 4 times. Let X denotes the number of heads. Find the probability distribution of X, its mean and variance.

Section-D

22.

a. Using elementary row transformations, find the inverse of the matrix

$$\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$$

Or [6]

b. Using matrix method, solve the following system of linear equations:
$2x - 3y + 5z = 11$
$3x + 2y - 4z = -5$
$x + y - 2z = -3$

23.

a. Show that the height of a cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$.

Or [6]

b. A rectangle is inscribed in a semicircle of radius r with one of its sides on the diameter of the semicircle. Find the dimensions of the rectangle so that its area is maximum. Find also the area of rectangle.

24.

a. Find the area of the region bounded by the circle $x^2 + y^2 = 16$ and the parabola $x^2 = 6y$

Or [6]

b. Find the area of the region bounded by the parabola $4y = 3x^2$ and the line $2y = 3x + 12$

25.

a. Find the direction cosines of the normal to the plane passing through the points (3, 2, 2) and (1, 0, -1) and parallel to the line $\frac{x-1}{2} = \frac{1-y}{2} = \frac{z-2}{3}$

Or [6]

b. Find the distance of the point P(1, 2, 3) from its image in the plane $x + 2y + 4z = 38$

26.

a. A shopkeeper wants to invest ₹ 5400 for two types of pens. Type A costs ₹ 180 per packet and type B costs ₹ 60 per packet. For each packet, she gets a profit of ₹ 15 on type A and ₹ 10 on type B respectively. She has space for 50 packets only. Find the number of packets for each type to get maximum profit.

Or [6]

b. If a man rides his motorcycle at 30 km/hr, he has to spend ₹ 2/km on fuel. If he rides it at 40 km/hr, the fuel cost increases to ₹ 4/km. He has ₹ 120 to spend on fuel and wishes to travel the maximum distance within 1 hour. Express this as a linear programming problem and solve it.