PHYSICS
HANDBOOK

For [JEE Main & JEE Advanced & NEET] Examination

Motion

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IIT - JEE SYLLABUS

GENERAL PHYSICS

Units and dimensions, dimensional analysis; least count, significant figures; Methods of measurement and error analysis for physical quantities pertaining to the following experiments: Experiments based on using vernier calipers and screw gauge (micrometer), Determination of g using simple pendulum, Young's modulus by Searle's method, Specific heat of a liquid using calorimeter, focal length of a concave mirror and a convex lens using u-v method, Speed of sound using resonance column, Verification of Ohm's law using voltmeter and ammeter, and specific resistance of the material of a wire using meter bridge and post office box.

MECHANICS

Kinematics in one and two dimensions (Cartesian coordinates only), projectiles; Circular motion (uniform and non-uniform); Relative velocity. Newton's laws of motion; Inertial and uniformly accelerated frames of reference; Static and dynamic friction; Kinetic and potential energy; Work and power; Conservation of linear momentum and mechanical energy. Systems of particles; Centre of mass and its motion; Impulse; Elastic and inelastic collisions. Law of gravitation; Gravitational potential and field; Acceleration due to gravity; Motion of planets and satellites in circular orbits. Rigid body, moment of inertia, parallel and perpendicular axes theorems, moment of inertia of uniform bodies with simple geometrical shapes; Angular momentum; Torque; Conservation of angular momentum; Dynamics of rigid bodies with fixed axis of rotation; Rolling without slipping of rings, cylinders and spheres; Equilibrium of rigid bodies; Collision of point masses with rigid bodies.

SHM , WAVES

Linear and angular simple harmonic motions. Wave motion (plane waves only), longitudinal and transverse waves, Superposition of waves; progressive and stationary waves; Vibration of strings and air columns. Resonance; Beats; Speed of sound in gases; Doppler effect (in sound).

GRAVITATION , FLUID , HEAT

Pressure in a fluid; Pascal's law; Buoyancy; Surface energy and surface tension, capillary rise; Viscosity (Poiseuille's equation excluded), Stoke's law; Terminal velocity, Streamline flow, Equation of continuity, Bernoulli's theorem and its applications. Thermal expansion of solids, liquids and gases; Calorimetry, latent heat; Heat conduction in one dimension; Elementary concepts of convection and radiation; Newton's law of cooling; Ideal gas laws; Specific heats (Cv and Cp for monatomic and diatomic gases); Isothermal and adiabatic processes, bulk modulus of gases; Equivalence of heat and work; First law of thermodynamics and its applications (only for ideal gases). Blackbody radiation: absorptive and emissive powers; Kirchhoff's law, Wien's displacement law, Stefan's law. Hooke's law, Young's modulus.

OPTICS

Rectilinear propagation of light; Reflection and refraction at plane and spherical surfaces; Total internal reflection; Deviation and dispersion of light by a prism; Thin lenses; Combinations of mirrors and thin lenses; Magnification. Wave nature of light: Huygen's principle, interference limited to Young's double-slit experiment.

ELECTROMAGNETISM

Coulomb's law; Electric field and potential; Electrical Potential energy of a system of point charges and of electrical dipoles in a uniform electrostatic field, Electric field lines; Flux of electric field; Gauss's law and its application in simple cases, such as, to find field due to infinitely long straight wire, uniformly charged infinite plane sheet and uniformly charged thin spherical shell. Capacitance; Parallel plate capacitor with and without dielectrics; Capacitors in series and parallel; Energy stored in a capacitor. Electric current: Ohm's law; Series and parallel arrangements of resistances and cells; Kirchhoff's laws and simple applications; Heating effect of current. Biot-Savart law and Ampere's law, magnetic field near a current-carrying straight wire, along the axis of a circular coil and inside a long straight solenoid; Force on a moving charge and on a current-carrying wire in a uniform magnetic field. Magnetic moment of a current loop; Effect of a uniform magnetic field on a current loop; Moving coil galvanometer, voltmeter, ammeter and their conversions. Electromagnetic induction: Faraday's law, Lenz's law; Self and mutual inductance; RC, LR and LC circuits with d.c. and a.c. sources.

MODERN PHYSICS

Atomic nucleus; Alpha, beta and gamma radiations; Law of radioactive decay; Decay constant; Half-life and mean life; Binding energy and its calculation; Fission and fusion processes; Energy calculation in these processes. Photoelectric effect; Bohr's theory of hydrogen-like atoms; Characteristic and continuous X-rays, Moseley's law; de Broglie wavelength of matter waves.

PART-I
MECHANICS

1. PHYSICAL QUANTITY

The quantities which can be measured by an instrument and by means of which we can describe the laws of physics are called physical quantities.

Types of physical quantities :

Fundamental | Derived | Supplementry

Seven physical fundamental quantities

(1) length
(2) mass
(3) time
(4) electric current,
(5) thermodynamic temperature
(6) amount of substance
(7) luminous intensity

2. DEFINITIONS OF SOME IMPORTANT SI UNITS

(i) Metre : 1 m = 1,650, 763.73 wavelengths in vaccum, of radiation corresponding to organ-red light of krypton-86.

(ii) Second : 1 s = 9,192, 631,770 time periods of a particular form Ceasium - 133 atom.

(iii) Kilogram : 1kg = mass of 1 litre volume of water at 4°C

(iv) Ampere : It is the current which when flows through two infinitely long straight conductors of negligible cross-section placed at a distance of one metre in vacuum produces a force of 2×10^{-7} N/m between them.

(v) Kelvin : 1 K = 1/273.16 part of the thermodynamic temperature of triple point of water.

(vi) Mole : It is the amount of substance of a system which contains as many elementary particles (atoms, molecules, ions etc.) as there are atoms in 12g of carbon - 12.

(vii) Candela : It is luminous intensity in a perpendicular direction of a surface of $\left(\frac{1}{600000}\right)m^2$ of a black body at the temperature of freezing point under a pressure of 1.013×10^5 N/m^2.

(viii) Radian : It is the plane angle between two radii of a circle which cut-off on the circumference, an arc equal in length to the radius.

(ix) Steradian : The steradian is the solid angle which having its vertex at the centre of the sphere, cut-off an area of the surface of sphere equal to that of a square with sides of length equal to the radius of the sphere

3. CHARACTERISTICS OF BASE UNITS OR STANDARDS :

Some special types of units :

1. 1 Micron (1μ) = 10^{-4} cm = 10^{-6}m (length)
2. 1 Angstrom (1A) = 10^{-8} cm = 10^{-10}m (length)
3. 1 fermi (1 f) = 10^{-13} cm = 10^{-15}m (length)
4. 1 inch = 2.54 cm (length)
5. 1m = 39.37 inch = 3.281feet (length)
6. 1 mile = 5280 feet = 1.609 km (length)
7. 1 atmosphere = 10^5 N/m^2 = 76 torr
   = 76 mm of Hg pressure (pressure)
8. 1 litre = 10^{-3} m^3 = 1000 cm^3 (volume)
9. 1 carat = 0.0002 kg (weight)
10. 1 pound (lb) = 0.4536 kg (weight)

Different quantities with units. symbol and dimensional formula.

Quantity	Symbol	Formula	S.I. Unit	D.F.
Disp.	s	ℓ	Metre or m	M^0LT^0
Area	A	$\ell \times b$	(Metre)^2 or m^2	$M^0L^2T^0$
Volume	V	$\ell \times b \times h$	(Metre)^3 or m^3	$M^0L^3T^0$
Velocity	v	$v = \frac{\Delta s}{\Delta t}$	m/s	M^0LT^{-1}
Momentum	p	p = mv	kgm/s	MLT^{-1}
Acceleration	a	$a = \frac{\Delta v}{\Delta t}$	m/s^2	M^0LT^{-2}
Force	F	F = ma	Newton or N	MLT^{-2}
Impulse	-	F × t	N.sec	MLT^{-1}
Work	W	F . d	N . m	ML^2T^{-2}
Energy	KE	$K.E. = \frac{1}{2}mv^2$	Joule or J	ML^2T^{-2}
	or U	P.E. = mgh		

Power	P	$P = \frac{W}{t}$	watt or W	ML^2T^{-3}
Density	d	d = m/v	kg/m^3	$ML^{-3}T^0$
Coefficient of viscosity	η	$F = \eta\left(\frac{dv}{dx}\right)A$	kg/ms(poise in C.G.S.)	$ML^{-1}T^{-1}$
Gravitation	G	$F = \frac{Gm_1m_2}{r^2}$	$\frac{N-m^2}{kg^2}$	$M^{-1}L^3T^{-2}$

4. APPLICATION OF DIMENSIONS

(a) To find out the unit of a physical quantity

(b) To derive the dimensions of a physical constant

(c) To check the dimensional correctness of a given physical equation :

Every physical equation should be dimensionally balanced. This is called the 'Principle of Homogeneity'. The dimensions of each term on both sides of an equation must be the same.

(d) Conversion of units :

This is based on the fact that the numerical value (n) and its corresponding unit (u) is a constant, i.e.,

$n[u]$ = constant or $n_1[u_1] = n_2[u_2]$

(e) To establish the relation among various physical quantities :

If we know the factors on which a given physical quantity may depend, we can find a formula relating the quantity with those factors

5. LIMITATION OF DIMENSION ANALYSIS

(i) The method works only if the dependence is the product type.

e.g. $s = ut + \frac{1}{2}at^2$

(ii) The numerical constants having no dimensions cannot be deduced by the method of dimensions.

e.g. = 2π

(iii) The methods works only if there are as many equation available as there are unknown.

BASIC MATHEMATICS

1. LOGARITHMS :

(i) $e \approx 2.7183$

(ii) If $e^x = y$, then $x = \log_e y = \ln y$

(iii) If $10^x = y$, then $x = \log_{10} y$

(iv) $\log_{10} y = 0.4343 \log_e y = 2.303 \log_{10} y$

(v) $\log (ab) = \log (a) + \log (b)$

(vi) $\log\left(\frac{a}{b}\right) = \log (a) - \log (b)$

(vii) $\log a^n = n \log (a)$

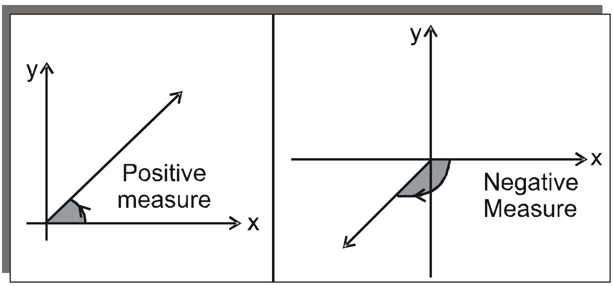
2. ANGLE CONVERSION FORMULAS

1 degree = $\frac{\pi}{180^\circ}$ (≈ 0.02) radian Degrees to radians : multiply by $\frac{\pi}{180^\circ}$

1 radian $\approx$ 57 degrees

Radians to degrees : multiply by $\frac{180^\circ}{\pi}$

Measurement of positive & Negative Angles :



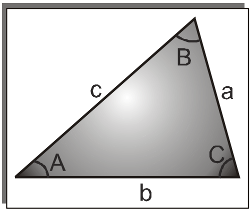
3. IMPORTANT FORMULAS

(i) $\sin^2\theta + \cos^2\theta = 1$

(ii) $1 + \tan^2\theta = \sec^2\theta$

(iii) $1 + \cot^2\theta = \text{cosec}^2\theta$

- **Sine Rule** $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$



- **Cosine rule** $a^2 = b^2 + c^2 - 2bc\cos A$

4. SMALL ANGLE APPROXIMATION

It is a useful simplification which is only approximately true for finite angles. It involves linearrization of the trigonometric functions so that, when the angle θ is measured in radians.

$$\sin\theta \simeq \theta$$

$\cos\theta \simeq 1$ or $\cos\theta \simeq 1 - \dfrac{\theta^2}{2}$

for the second - order approximation

$\tan\theta \simeq \theta$

Range 1° to 10° are small angle

5. BINOMIAL THEOREM :

$(1 \pm x)^n = 1 \pm nx + \dfrac{n(n-1)x^2}{2!}$..........

$(1 \pm x)^{-n} = 1 \mp nx + \frac{n(n+1)}{2!}x^2 ....$

If x << 1 ; then

$(1 \pm x)^n = 1 \pm nx$ (neglecting higher terms)

$(1 \pm x)^{-n} = 1 \pm (-n)x = 1 \mp nx$

$(1 + x)^2 = 1 + 2x + x^2$

$(1 + x)^3 = 1 + 3x + x^3 - 3x^2$

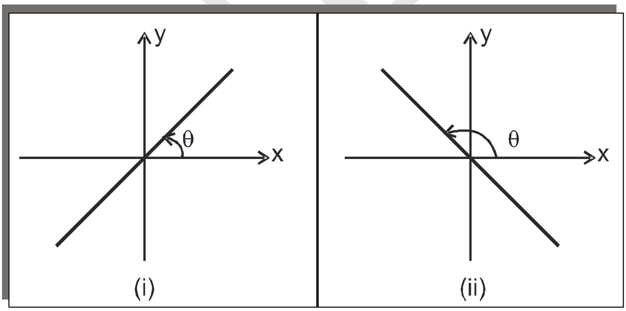
$(1 + x)^n = 1 + nx.............$

if x << 1

6. GRAPHS :

Following graphs and their corresponding equations are frequently used in Physics.

(i) y = mx, represents a straight line passing through origin. Here, $m = \tan\theta$ is also called the slope of line, where θ is the angle which the line makes with positive x-axis, when drawn in anticlockwise direction from the positive x-axis towards the line.



The two possible cases are shown in figure 1.1 (i) $\theta < 90°$. Therefore, $\tan\theta$ or slope of line is positive. In fig. 1.1 (ii), $90° < \theta < 180°$. Therefore, $\tan\theta$ or slope of line is negative.

(ii) Parabola

A general quadratic equation represents a parabola.

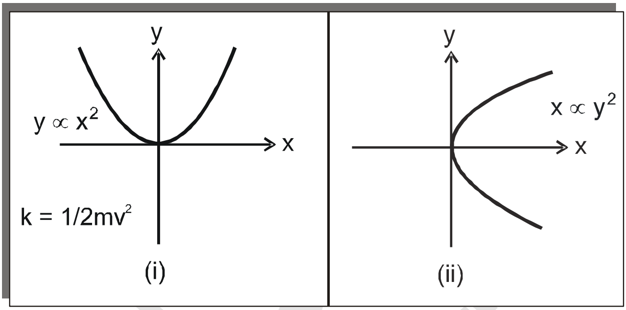
$$y = ax^2 + bx + c \qquad a \neq 0$$

if $a > 0$; It will be a opening upwards parabola.

if $a < 0$; It will be a opening downwards parabola.

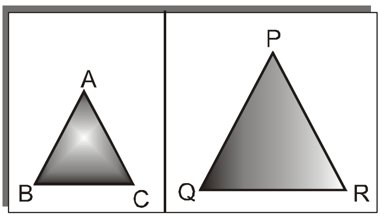
if $c = 0$; It will pass through origin.

$y \propto x^2$ or $y = 2x^2$, etc. represents a parabola passing through origin as shown in figure shown.



7. SIMILAR TRIANGLE

Two given triangle are said to be similar if



(1) All respective angle are same

or

(2) All respective side ratio are same.

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}$$

VECTORS

1. SCALAR :

In physics we deal with two type of physical quantity one is scalar and other is vector. Each scalar quantity has a magnitude and a unit.

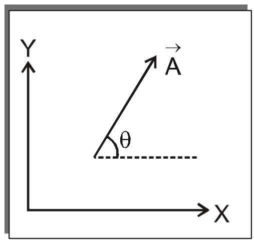
2. VECTOR :

Vector are the physical quantites having magnitude as well as specified direction.

3. GENERAL POINTS REGARDING VECTORS :

Representation of vector :

Geometrically, the vector is represented by a line with an arrow indicating the direction of vector as



Mathematically, vector is represented by $\vec{A}$.

Sometimes it is represented by bold letter **A.**

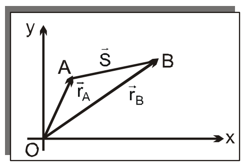
Thus, the arrow in above figure represents a vector $\vec{A}$

in xy-plane making an angle θ with x-axis.

A representation of vector will be complete if it gives us direction and magnitude.

4. POSITION VECTOR AND DISPLACEMENT VECTOR

If coordinates of points A are (x_1, y_1, z_1) and coordinates of point B are (x_2, y_2, z_2). Then



$\vec{r}_A$ = position vector of A $= x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$

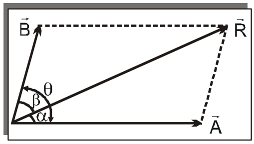
$\vec{r}_B = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$

$\vec{S} = \vec{r}_B - \vec{r}_A = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$

= displacement vector from A to B

5. VECTOR ADDITION OF TWO VECTORS

Law of parallelogram of vector addition :



$\vec{R} = \vec{A} + \vec{B} \Rightarrow R = |\vec{R}|$

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta} \text{ and } \tan\beta = \frac{A\sin\theta}{B + A\cos\theta}$$

6. VECTOR SUBTRACTION

If $\vec{S} = \vec{A} - \vec{B}$ and $S = |\vec{S}|$, then

$$S = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

7. A UNIT VECTOR IN THE DIRECTION OF $\vec{A}$

$$\hat{A} = \frac{\vec{A}}{A} \quad \text{or} \quad \vec{A} = A\hat{A}$$

A vector of zero magnitude is called a **zero or a null vector**. Its direction is arbitrary.

8. DOT PRODUCT OF TWO VECTORS

$$\vec{A}.\vec{B} = AB\cos\theta$$

8.1 Important Points in Dot Product

(a) $\vec{A}.\vec{B} = \vec{B}.\vec{A}$

(b) $\vec{A}.(-\vec{B}) = -\vec{A}.\vec{B}$

(c) $\vec{A}.(\vec{B} + \vec{C}) = \vec{A}.\vec{B} + \vec{A}.\vec{C}$

(d) If θ is acute, dot product is positive. If θ is obtuse dot product is negative and if θ is 90° dot product is zero. Hence dot product of two perpendicular vectors is zero.

(e) $\vec{A}.\vec{A} = A^2$

(f) Dot product is a scalar quantity

(g) Work done $W = \vec{F}.\vec{S} = \vec{F}.(\vec{r}_f - \vec{r}_i)$

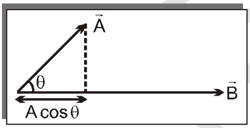
(h) $\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$, $\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0$

Angle Between Two Vectors

$$\theta = \cos^{-1}\left(\frac{\vec{A}.\vec{B}}{AB}\right)$$

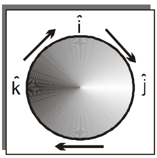
Component of $\vec{A}$ along $\vec{B}$

$$= A\cos\theta = \frac{\vec{A}.\vec{B}}{B}$$



Similarly, component of $\vec{B}$ along $\vec{A} = B\cos\theta = \dfrac{\vec{A}.\vec{B}}{A}$

9. CROSS PRODUCT OF TWO VECTORS



$\hat{i} \times \hat{j} = \hat{k}, \hat{j} \times \hat{k} = \hat{i}, \hat{k} \times \hat{i} = \hat{j}, \hat{i} \times \hat{k} = -\hat{j}, \hat{k} \times \hat{j} = -\hat{i}$

$\hat{j} \times \hat{i} = -\hat{k}, \hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} =$ a null vector

10. If Vectors are given in Terms of $\hat{i}, \hat{j}$ and $\hat{k}$

Let $\vec{A} = a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k}$ and

$\vec{B} = b_1 \hat{i} + b_2 \hat{j} + b_3 \hat{k}$, then

(a) $|\vec{A}| = A = \sqrt{a_1^2 + a_2^2 + a_3^2}$

and $|\vec{B}| = B = \sqrt{b_1^2 + b_2^2 + b_3^2}$

(b) $\vec{A} + \vec{B} = (a_1 + b_1)\hat{i} + (a_2 + b_2)\hat{j} + (a_3 + b_3)\hat{k}$

(c) $\vec{A} - \vec{B} = (a_1 - b_1)\hat{i} + (a_2 - b_2)\hat{j} + (a_3 - b_3)\hat{k}$

(d) $\vec{A}.\vec{B} = a_1 b_1 + a_2 b_2 + a_3 b_3$

(e) $|\vec{A} \times \vec{B}| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

(f) Component of $\vec{A}$ along $\vec{B}$

$$= A\cos\theta = \frac{\vec{A}.\vec{B}}{B} = \frac{a_1 b_1 + a_2 b_2 + a_3 b_3}{\sqrt{b_1^2 + b_2^2 + b_3^2}}$$

(g) Unit vector parallel to $\vec{A}$

$$= \hat{A} = \frac{\vec{A}}{A} = \frac{a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k}}{\sqrt{a_1^2 + a_2^2 + a_3^2}}$$

(h) Angle between $\vec{A}$ and $\vec{B}$, $\theta = \cos^{-1}\left(\frac{\vec{A}.\vec{B}}{AB}\right)$

$$\therefore \qquad \theta = \cos^{-1}\left(\frac{a_1b_1 + a_2b_2 + a_3b_3}{\sqrt{a_1^2 + a_2^2 + a_3^2} \times \sqrt{b_1^2 + b_2^2 + b_3^2}}\right)$$

(i) A Unit Vector Perpendicular to both $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$

Let us call it $\hat{C}$. Then $\hat{C} = \pm\frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$

11. PROCEDURE TO SOLVE THE VECTOR EQUATION

$$\vec{A} = \vec{B} + \vec{C} \qquad ...(1)$$

(a) There are 6 variables in this equation which are following :

(1) Magnitude of $\vec{A}$ and its direction

(2) Magnitude of $\vec{B}$ and its direction

(3) Magnitude of $\vec{C}$ and its direction.

(b) We can solve this equation if we know the value of 4 variables [Note : two of them must be directions]

(c) If we know the two direction of any two vectors then we will put them on the same side and other on the different side.

|Distance| ≥ |Displacement|

Displacement (+ve), (–ve), 0

Distance 0, +ve

1. ***Average speed and average velocity :***

Average speed (v_{av})

$$= \frac{\text{Total distance travelled}}{\text{Time interval}} = \frac{\Delta s}{\Delta t}$$

$$\text{Average velocity}\,(\vec{v}_{av}) = \frac{\text{Displacement}}{\text{Time interval}} = \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1}$$

2. ***Instantaneous speed and instantaneous velocity***

$$v = \lim_{\Delta s \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt} \qquad v = \lim_{\Delta t \to 0} \frac{\Delta r}{\Delta t} = \frac{dr}{dt}$$

3. ***Average and instantaneous acceleration.***

Average acceleration,

$$(\vec{a}_{av}) = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\text{Change in velocity}}{\text{Time interval}}$$

$(\vec{a})_{inst} = \lim\limits_{\Delta t \to 0} \vec{a}_{av} = \dfrac{d\vec{v}}{dt}$ = Rate of change of velocity.

* When direction of acceleration and velocity are opposite to each other, then acceleration is termed as retardation.

* $\vec{a} = \dfrac{d\vec{v}}{dt} = \dfrac{d^2\vec{r}}{dt^2} = \vec{v}\dfrac{d\vec{v}}{d\vec{r}}$

4. CONSTANT ACCELERATION FORMAT

(i) $v = u + at$

(ii) $s = ut + \frac{1}{2}at^2$

(iii) $s = s_0 + ut + \frac{1}{2}at^2$

(iv) $v^2 = u^2 + 2as$

(v) s_m = displacement (not distance) in n^{th} second $= u + \frac{a}{2}(2n - 1)$

5. ONE DIMENSIONAL MOTION WITH NON-UNIFORM ACCELERATION

(i) $s - t \rightarrow v - t \rightarrow a - t \rightarrow$ Differentiation

(ii) $a - t \rightarrow v - t \rightarrow s - t \rightarrow$ Integration

(iii) **Equations of differentiation**

(iv) **Equation of integration**

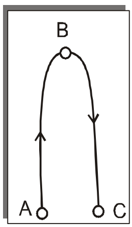
$$v = \frac{ds}{dt} \text{ and } a = \frac{dv}{dt} = v.\frac{dv}{ds}$$

$$\int ds = \int v\,dt, \int dv = \int a\,dt, \int v\,dv = \int a\,ds$$

6. REACTION TIME

When a particular situation demands our immediate action, it takes some time before we really respond. Reaction time is the time a person takes to observe, think and act. For example, if a person is driving and suddenly a boy appears on the road, then the time elapse before he applies the breaks of the car is the reaction time. Reaction time depends on complexity of the situation and on an individiual.

7. MOTION UNDER GRAVITY :



I FORMAT : (When a body is thrown vertically upward)

It includes two types of motion

(i) Deaccelerated motion from A to B because the direction of velocity and acceleration is opposite. So speed decreases

(ii) Accelerated motion from B to C because the direction of velocity and acceleration is same (downward). So speed increases

(a) Time of flight :

$$T = \frac{2u}{g}$$

(b) Maximum Height :

at maximum height v = 0, $s = H_{max} = \frac{u^2}{2g}$

(c) Final velocity

$$v_f = -u$$

i.e. the body reaches the ground with the same speed with which it was thrown vertically upwards as it thrown vertically upward.

(d) Time to reach any general height h

So, $t_1 = \frac{u - \sqrt{u^2 - 2gh}}{g}$, $t_2 = \frac{u + \sqrt{u^2 - 2gh}}{g}$ $\Rightarrow$ $t_1 + t_2 = T$ (Time of flight)

II Format (Free fall) :

A body released near the surface of the earth is accelerated downward under the influence of force of gravity.

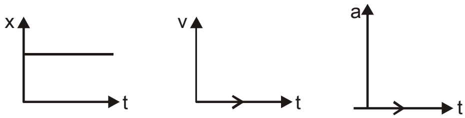
(a) Time of Flight : $T = \sqrt{\frac{2H}{g}}$

(b) Final Velocity when body reaches the ground :

$$v_f = \sqrt{2gH}$$

8. Conversion of Graphs from x-t to v - t and v - t to a - t

1.



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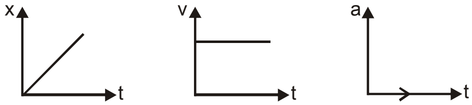

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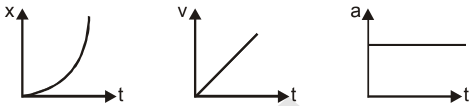

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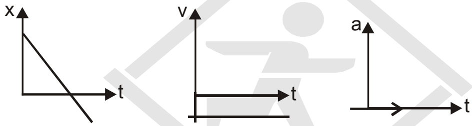

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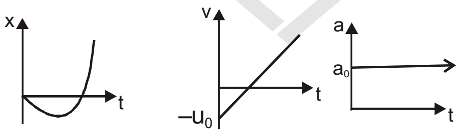

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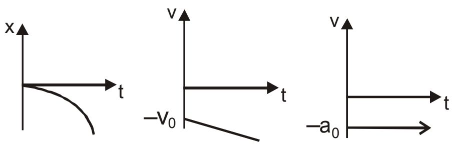


(iii) ***Slope of s-t graph = velocity, Slope of v-t graph = acceleration***

Area under v-t graph = displacement and, Area under a-t graph - change in velocity

9. RELATIVE MOTION IN 1 - D

(i) $\vec{v}_{AB}$ = velocity of A with respect to $B = \vec{v}_A - \vec{v}_B$

(ii) $\vec{a}_{AB}$ = acceleration of A with respect to

$B = \vec{a}_A - \vec{a}_B$ In one dimensional motion

take a sign convention.

(iii) $v_{AB} = v_A - v_B$

(iv) $a_{AB} = a_A - a_B$

10. MOTION IN TWO DIMENSION

2 Dimensional Motion

1-d + 1-d

$S_x = u_x t + \frac{1}{2} a_x t^2$ $\qquad$ $S_y = u_y t + \frac{1}{2} a_y t^2$

$v_x = u_x + a_x t$ $\qquad$ $v_y = u_y + a_y t$ $\qquad$ $v_x^2 = u_x^2 + 2a_x s_x$

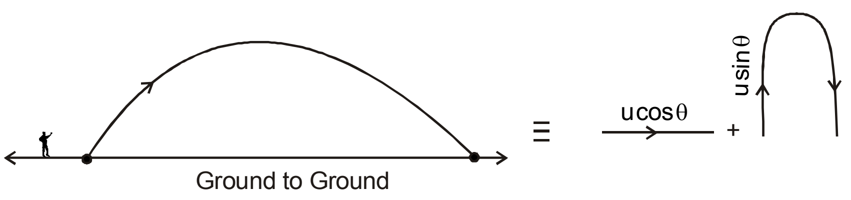
$v_y^2 = u_y^2 + 2a_y s_y$

For constant a_x $\qquad$ a_y

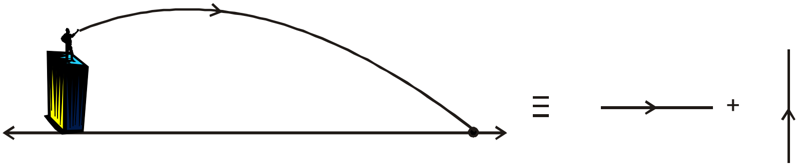
11. PROJECTILE MOTION

There are following cases which shows the projection of balls from three different situation

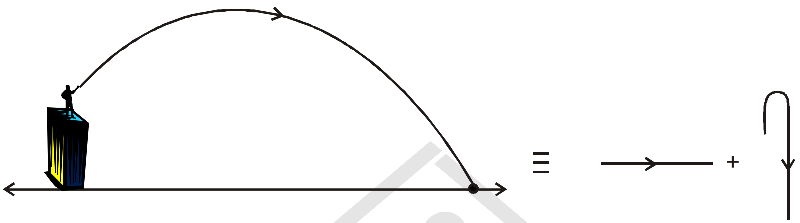
Case I



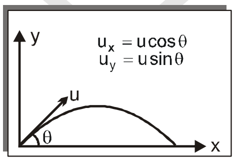
Case II :



Case III :



There are following parameters related to projectile motion :



(i) $T = \dfrac{2u\sin\theta}{g} = \dfrac{2u_y}{g}$

(ii) $H = \dfrac{u^2\sin^2\theta}{2g} = \dfrac{u_y^2}{2g}$

(iii) $R = \dfrac{u^2\sin 2\theta}{g} = u_xT = \dfrac{2u_xu_y}{g}$

(iv) $R_{max} = \dfrac{u^2}{g}$ at $\theta = 45^\circ$

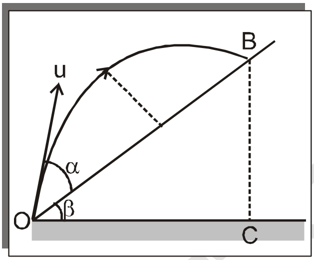
(v) $R_\theta = R_{90^\circ-\theta}$ for same value of u.

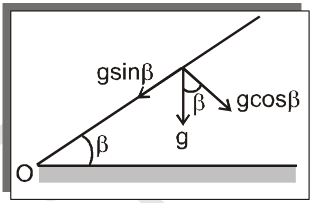
(vi) Equation of trajectory

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$\textbf{or } y = x\tan\theta\left[1-\frac{x}{R}\right]$$

12. PROJECTILE MOTION IN INCLINED PLANE





(i) Up the Plane :

Time of flight $T = \dfrac{2u\sin\alpha}{g\cos\beta}$

Range $R = u\cos\alpha\, T - \dfrac{1}{2} g\sin\beta\, T^2$

(ii) Down the inclined plane :

$$T = \frac{2u_y}{a_y} = \frac{2u\sin\alpha}{g\cos\beta}$$

Range= $u\cos\alpha\, T + \dfrac{1}{2} g\sin\beta . T^2$

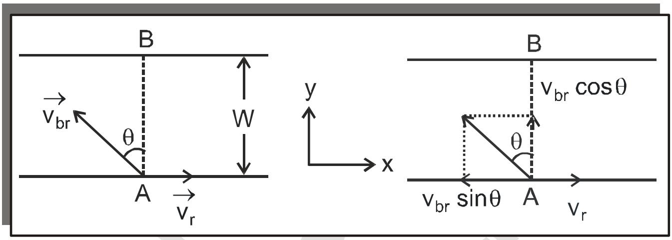
13. PROBLEMS RELATED TO 2-D RELATIVE MOTION

(a) Minimum distance between two bodies in motion

When two bodies are in motion, the questions like, the minimum distance between them or the time when one body overtakes the other can be solved easily by the principle of relative motion. In these type of problems one body is assumed to be at rest and the relative motion of the other body is considered

(b) River - Boat Problems

In river-boat problems we come across the following three terms :



$\vec{v}_r$ = absolute velocity of river

$\vec{v}_{br}$ = velocity of boatman with respect to river or velocity of boatman in still water

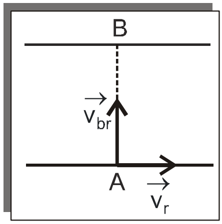
and $\vec{v}_b$ = absolute velocity of boatman.

$$t = \frac{W}{v_{br}\cos\theta} \quad \text{.........(1)}$$

$$x = (v_r - v_{br}\sin\theta)\frac{W}{v_{br}\cos\theta} \quad \text{.........(2)}$$

(i) Condition when the boatman crosses the river in shortest interval of time

From equation (1) we can see that time (t) will be minimum when $\theta = 0°$, i.e., the boatman should steer his boat perpendicular to the river current.



Also, $t_{min} = \dfrac{w}{v_{br}}$ as $\cos\theta = 1$

(ii) Condition when the boatman wants to reach point B, i.e., at a point just opposite from where he started

Hence, to reach point B the boatman should row at an angle $\theta = \sin^{-1}\left(\dfrac{v_r}{v_{br}}\right)$

upstream from AB.

(iii) Shortest path

When $v_r < v_{br}$: In this case x = 0,

when $\theta = \sin^{-1}\left(\dfrac{v_r}{v_{br}}\right)$

or $s_{min} = w$ at $\theta = \sin^{-1}\left(\dfrac{v_r}{v_{br}}\right)$

When $v_r > v_{br}$;

$s_{min} = w\left(\dfrac{v_r}{v_{br}}\right)$ at $\theta = \sin^{-1}\left(\dfrac{v_{br}}{v_r}\right)$

(c) Flag Problems :

If flag will flutter in direction of v_{wc}

then $v_{wc} = v_w - v_c$

(d) Aircraft Wind Problems

$\vec{v}_a = \vec{v}_{aw} + \vec{v}_w$

$\vec{v}_{aw}$ = velocity of aircraft with respect to wind or velocity of aircraft in still air

$\vec{v}_w$ = velocity of wind

$\vec{v}_a$ = absolute velocity of aircraft

(e) Rain Problems

In these type of problems we again come across three terms $\vec{v}_r$, $\vec{v}_m$ and $\vec{v}_{rm}$,

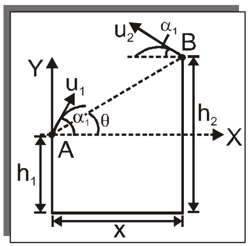
Here,

$\vec{v}_r$ = velocity of rain

$\vec{v}_m$ = velocity of man (it may be velocity of cyclist or velocity of motorist also)

and $\vec{v}_{rm}$ = velocity of rain with respect to man.

14. CONDITION FOR COLLISION OF TWO PROJECTILES :



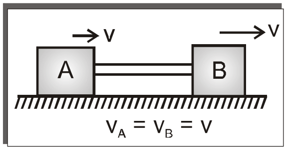
For collision to take place, in following figure

$\tan\alpha = \tan\theta = \dfrac{h_2 - h_1}{x}$

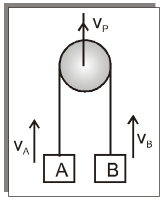
CONSTRAINED MOTION

1. STRING CONSTRAINT :

Ist format : - (when string is fixed)



II[nd] format (when pulley is also moving)



$$v_{AP} = -v_{BP}$$

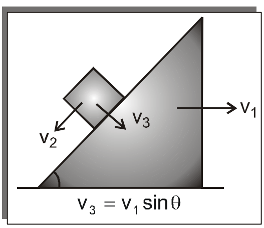
(–ve sign indicate the direction of each block is opposite with respect to Pulley)

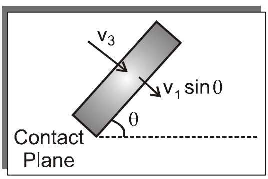
$$v_A - v_p = -v_B + v_P \quad \Rightarrow \quad v_P = \frac{v_A + v_B}{2}$$

III format :

1. First choose the longest string in the given problem which contains the point of which velocity/acceleration to be find out.
2. Now mark a point on the string wherever if comes in contact or leaves the contact of real bodies.
3. If due to motion of a point, length of the part of a string with point is related, increases then its speed will be taken +ve otherwise –ve.

2. WEDGE CONSTRAINED :





Components of velocity and acceleration perpendicular to the contact surface of the two objects is always equal if there is no deformation and they remain in contact.

1. FORCE

A pull or push which changes or tends to change the state of rest or of uniform motion or direction of motion of any object is called force. Force is the interaction between the object and the source (providing the pull or push). It is a vector quantity.

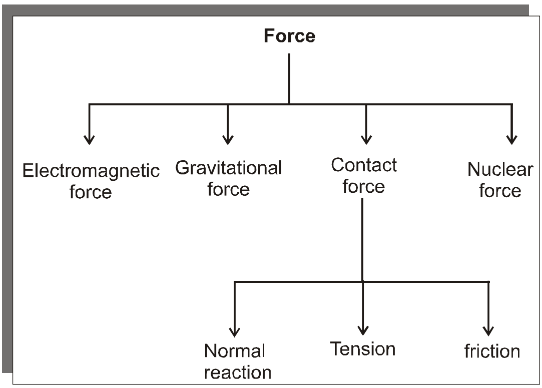
Effect of resultant force :

- may change only speed
- may change only direction of motion.
- may change both the speed and direction of motion.
- may change size and shape of a body

unit of force : newton and $\frac{kg.m}{s^2}$ (MKS System)

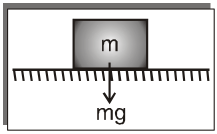
dyne and $\frac{g.cm}{s^2}$ (CGS System)

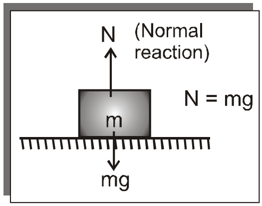
1 newton = 10^5 dyne



2. NORMAL REACTION

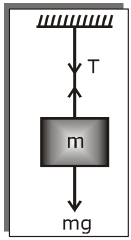
Normal reaction always act perpendicular to contact surface in such a direction that it always tries to compress the body.





3. TENSILE FORCE

in following figure T = mg



(i) Tension always acts along the string and in such a direction that it tries to reduce the length of string

(ii) If the string is massless then the tension will be same along the string but if the string have some mass then the tension will continuously change along the string.

4. NEWTON'S FIRST LAW OF MOTION :

According to this law "A system will remain in its state of rest or of uniform motion unless a net external force act on it.

1^{st} law can also be stated as "If the net external force acting on a body is zero, only then the body remains at rest."

5. NEWTON'S SECOND LAW OF MOTION :

Newton's second law states, "The rate of change of a momentum of a body is directly proportional to the applied force and takes place in the direction in which the force acts"

i.e., $\vec{F} \propto \frac{d\vec{p}}{dt}$ or $\vec{F} = k\frac{d\vec{p}}{dt}$

where k is a constant of proportionality.

$\vec{p} = m\vec{v}$, So $\vec{F} = k\frac{(dm\vec{v})}{dt}$

For a body having constant mass, $\Rightarrow$ $\vec{F} = km\frac{d\vec{v}}{dt} = km\vec{a}$

From experiments, the value of k is found to be 1.

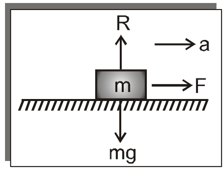
So, $\vec{F}_{net} = m\vec{a}$

6. APPLICATIONS OF NEWTON'S LAW :

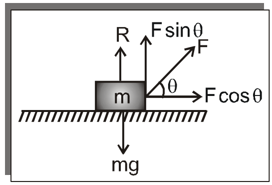
6.1 Motion of a Block on a Horizontal Smooth Surface.

Case (i) : When subjected to a horizontal pull:

F = ma or a = F/m



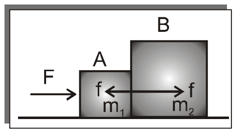
Case (ii) : When subjected to a pull acting at an angle (θ) to the horizontal :



6.2 MOTION OF BODIES IN CONTACT.

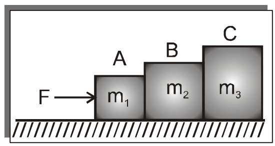
Case (i) : Two body system :

$$\Rightarrow a = \frac{F}{m_1 + m_2} \text{ and } f = \frac{m_2 F}{m_1 + m_2}$$



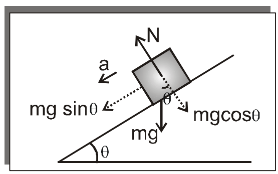
Case (ii) : Three body system :

$$\Rightarrow a = \frac{F}{m_1 + m_2 + m_3}$$



6.3 Motion of a body on a smooth inclined plane :

a = g sin θ, N = mg cosθ

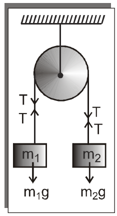


6.4 PULLEY BLOCK SYSTEM

If $m_1 > m_2$

$$a = \frac{(m_1 - m_2)g}{(m_1 + m_2)}$$

$$T = m_2(a + g)$$

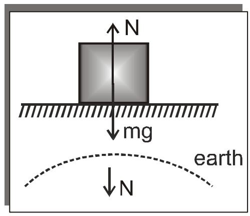


7. NEWTONS' 3RD LAW OF MOTION :

Statement : "To every action there is equal and opposite reaction".

According to Newton third law two forces are equal in magnitude and opposite in direction

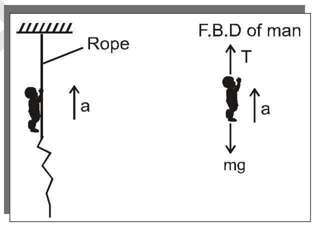
$$\vec{F}_{AB} = -\vec{F}_{BA}$$



7.1 Climbing on the Rope :

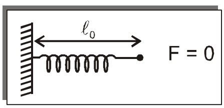
T > mg man accelerates in upward direction,

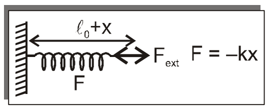
T < mg man accelerates in downward direction

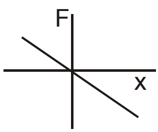


7.2 SPRING FORCE :

1. When spring is in its natural length, spring force is zero.



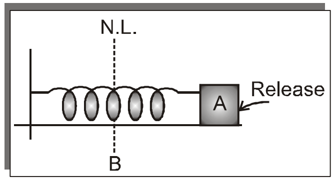




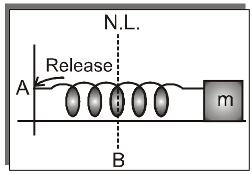
Graph between spring force v/s x

2. Cutting of the spring

(i) when the block A is released then it take some finite time to reach at B. i.e., spring force doesn't change instantaneously.



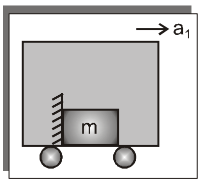
(ii) When point A of the spring is released in the above situation then the spring forces. changes instantaneously become zero because one end of the spring is free.

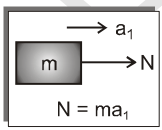


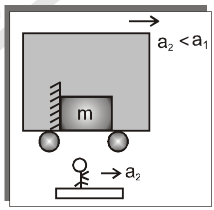
(iii) In string tension may change instantaneously.

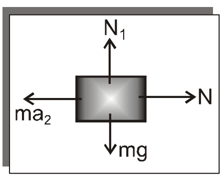
8. PSEUDO FORCE :

To apply newton's first law and second law in non inertial frame we introduce a new type of force that is pseudo force.









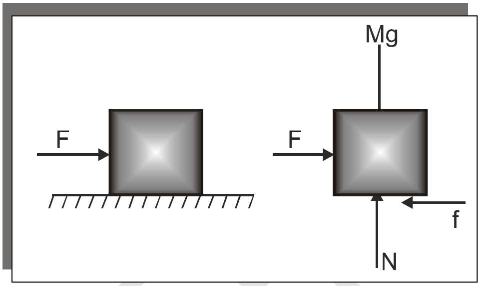
Pseudo force = ma_2

$N - ma_2 = m(a_1 - a_2)$

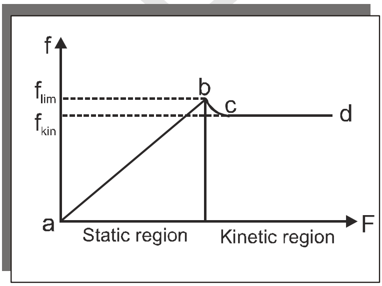
$N = ma_1$

1. FRICTION

Friction is a contact force that opposes the relative motion or tendency of relative motion of two bodies.



1.1 Static frictional force



When there is no relative motion between the contact surfaces, frictional force is called static frictional force. It is a self-adjusting force

The direction of static friction on a body is such that the total force acting on it keeps it at rest with respect to the body in contact.

1.2 Limiting Frictional Force

This frictional force acts when body is about to move. This is the maximum frictional force that can exist at the contact surface.

(i) The magnitude of limiting frictional force is proportional to the normal force at the contact surface.

$$f_{lim} \propto N \Rightarrow f_{lim} = \mu_s N$$

Here μ_s is a constant the value of which depends on nature of surfaces in contact and is called as 'coefficient of static friction'.

1.3 Kinetic Frictional Force

Once relative motion starts between the surface in contact, the frictional force is called as kinetic frictional force. The magnitude of kinetic frictional force is also proportional to normal force.

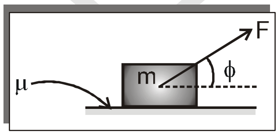
$$f_k = \mu_k N$$

From the previous observation we can say that $\mu_k < \mu_s$

Although the coefficient of kinetic friction varies with speed, we shall neglect any variation

i.e., when relative motion starts a constant frictional force starts opposing its motion.

2. MINIMUM FORCE REQUIRED TO MOVE THE PARTICLE :

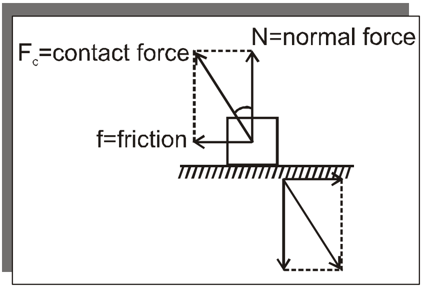


$F = \mu mg / (\cos\phi + \mu \sin\phi)$ $\qquad F_{min} = \dfrac{\mu mg}{\sqrt{1+\mu^2}}$

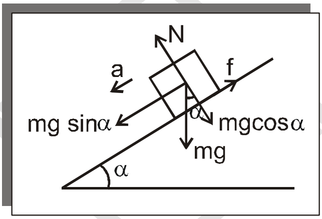
3. FRICTION AS THE COMPONENT OF CONTACT FORCE :

The perpendicular component is called the normal contact force or normal force and parallel component is called friction.

$F_{c\,max} = \sqrt{\mu^2 N^2 + N^2}$ $\qquad$ {when $f_{max} = \mu N$}



4. MOTION ON A ROUGH INCLINED PLANE



$$a = g \sin \alpha - \mu g \cos \alpha$$

ANGLE OF REPOSE :

when, $mg \sin \alpha_c = \mu\, mg \cos\alpha_c$

$\Rightarrow \tan \alpha_c = \mu$

where α_C is called angle of repose

CIRCULAR MOTION

1. Angular Velocity ω

(i) Average Angular Velocity

$$\omega_{av} = \frac{\text{Total Angle of Rotation}}{\text{Total time taken}} ;$$

$$\omega_{av} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$$

where θ_1 and θ_2 are angular position of the particle at time t_1 and t_2 respectively.

(ii) Instantaneous Angular Velocity

The rate at which the position vector of a particle with respect to the centre rotates, is called as instantaneous angular velocity with respect to the centre.

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

Relation between speed and angular velocity :

v = rω is a scalar quantity ($\vec{\omega} \neq \frac{\vec{v}}{r}$)

2. Relative Angular Velocity

$$\omega_{AB} = \frac{(V_{AB})_{\perp}}{r_{AB}}$$

here $V_{AB\perp}$ = Relative velocity $\perp$ to position vector AB

3. Angular Acceleration α :

(i) Average Angular Acceleration :

Let ω_1 and ω_2 be the instantaneous angular speeds at times t_1 and t_2 respectively, then the average angular acceleration α_{av} is defined as

$$\alpha_{av} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}$$

(ii) ***Instantaneous Angular Acceleration :***

It is the limit of average angular acceleration as Δt approaches zero, i.e.,

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt} = \omega \frac{d\omega}{d\theta}$$

Important points :

- It is also an axial vector with dimension $[T^{-2}]$ and unit rad/s^2
- If $\alpha = 0$, circular motion is said to be uniform.
- As $\omega = \frac{d\theta}{dt}$, $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$,

  i.e., second derivative of angular displacement w.r.t time gives angular acceleration.
- α is a axial vector and direction of α is along ω

  if ω increases and opposite to ω if ω decreases

4. Radial and tangential acceleration

$$a_t = \frac{dv}{dt} = \text{rate of change of speed}$$

and $$a_r = \omega^2 r = r\left(\frac{v}{r}\right)^2 = \frac{v^2}{r}$$

Following three points are important regarding the above discussion :

(i) In uniform circular motion, speed (v) of the particle is constant, i.e., $\frac{dv}{dt} = 0$.

Thus, $a_t = 0$ and $a = a_r = r\omega^2$

(ii) In accelerated circular motion, $\frac{dv}{dt}$ = positive, i.e., a_t is along $\hat{e}_t$ or tangential acceleration of particle is parallel to velocity $\vec{v}$ because $\vec{v} = r\omega\,\hat{e}_t$ and $\vec{a}_t = \frac{dv}{dt}\hat{e}_t$

(iii) In decelerated circular motion, $\frac{dv}{dt}$ = negative and hence, tangential acceleration is anti-parallel to velocity $\vec{v}$.

Relation between angular acceleration and tangential acceleration

$$a_t = \frac{dv}{dt} = r\frac{d\omega}{dt} \quad \text{or } a_t = r\alpha$$

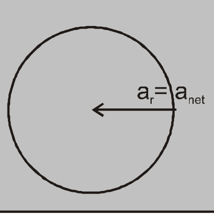
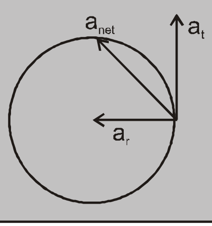
5. Relations among Angular Variables

These relations are also referred as equations of rotational motion and are -

$$\omega = \omega_0 + \alpha t \qquad ...(1)$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 \qquad ...(2)$$

$$\omega^2 = \omega_0^2 + 2\alpha\theta \qquad ...(3)$$

Uniform circular motion	Non-uniform circular motion
(1) Speed of the particle is constant i.e., ω is constant	speed of the particle is not constant i.e. ω is not constant
(ii) $a_t = \frac{d\lvert\vec{v}\rvert}{dt} = 0$	$a_t = \frac{d\lvert\vec{v}\rvert}{dt} \neq 0$
$a_r = \frac{v^2}{r} \neq 0$	$a_r \neq 0$
$\therefore\ a_{net} = a_r$	$\vec{a}_{net} = \vec{a}_r + \vec{a}_t$
	

6. Centripetal Force :

Concepts : This is necessary resultant force towards the centre called the centripetal force.

$$F = \frac{mv^2}{r} = m\omega^2 r$$

(i) A body moving with constant speed in a circle is not in equilibrium.

(ii) It should be remembered that in the absence of the centripetal force the body will move in a straight line with constant speed.

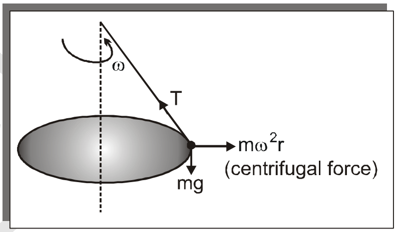
(iii) It is not a new kind of force which acts on bodies. In fact, any force which is directed towards the centre may provide the necessary centripetal force.

7. Centrifugal Force :

Centrifugal force is a fictitious force which has to be applied as a concept only in a rotating frame of reference to apply Newton's law of motion

(in that frame)

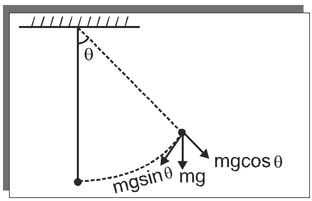
FBD of ball w.r.t non inertial frame rotating with the ball.



8. SIMPLE PENDULUM :

$$T - mg\cos\theta = mv^2/L$$

or, $$T = m(g\cos\theta + v^2/L)$$



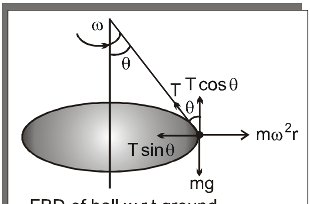
$$|\vec{F}_{net}| = \sqrt{(mg\sin\theta)^2 + \left(\frac{mv^2}{L}\right)^2}$$

$$= m\sqrt{g^2\sin^2\theta + \frac{v^4}{L^2}}$$

9. CONICAL PENDULUM :

speed $v = \dfrac{r\sqrt{g}}{(L^2 - r^2)^{1/4}}$ and

Tension $T = \dfrac{mgL}{(L^2 - r^2)^{1/2}}$



FBD of ball w.r.t ground

10. CIRCULAR TURNING ON ROADS :

Centripital force are provided by following ways.

10.1 By Friction Only:

For a safe turn without sliding safe speed

$$v \le \sqrt{\mu r g}$$

10.2 By Banking of Roads Only $v = \sqrt{rg\tan\theta}$

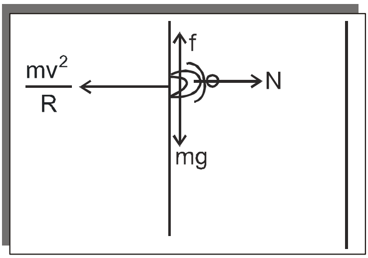
10.3 By Friction and Banking of Road Both

Maximum safe speed $v_{max} = \sqrt{\dfrac{rg(\mu + \tan\theta)}{(1 - \mu\tan\theta)}}$,

Minimum safe speed $v_{min} = \sqrt{\dfrac{rg(\mu - \tan\theta)}{(1 + \mu\tan\theta)}}$

11. DEATH WELL :

$N = \dfrac{mv^2}{R}$, $f = mg$, $f_{max} = \dfrac{\mu mv^2}{R}$



Cyclist does not drop down when

$f_{max} \ge mg \Rightarrow \dfrac{\mu mv^2}{R} \ge mg$, $v \ge \sqrt{\dfrac{gR}{\mu}}$

WORK POWER ENERGY

1. Work Done By a constant force

$$W = \vec{F}.\vec{s} = \vec{F}.(\vec{r}_f - \vec{r}_i) = Fs\cos\theta$$

= Force × displacement in the direction of force.

In cgs system, the unit of work is erg.

In mks system, the unit of work is Joule.

1 erg = 10^{-7} joule

2. WORK DONE BY A VARIABLE FORCE :

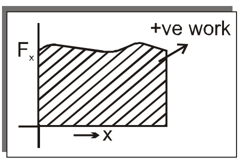
$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

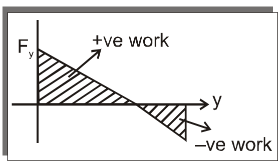
$$d\vec{s} = dx\hat{i} + dy\hat{j} + dz\hat{k}$$

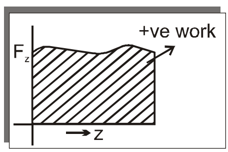
$$W = \int_{x_A}^{x_B} F_x dx + \int_{y_A}^{y_B} F_y dy + \int_{z_A}^{z_B} F_z dz$$

3. AREA UNDER FORCE DISPLACEMENT CURVE :

Graphically area under the force-displacement is the work done







The work done can be positive or negative as per the area above the x-axis or below the x-axis respectively.

4. DIFFERENCE BETWEEN CONSERVATIVE AND NON COSERVATIVE FORCES

S. No.	Conservative forces	Non-Conservative forces
1	Work done does not depend upon path	Work done depends on path.
2	Work done in a round trip is zero.	Work done in a round trip is not zero.
3	Central in nature.	Forces are velocity-dependent and retarding in nature.
4	When only a conservative force acts within a system, the kinetic energy and potential energy can change. However their sum, the mechanical energy of the system, does not change.	Work done against a non-conservative force may be disipiated as heat energy.
5	Work done is completely recoverable.	Work done in not completely recoverable.

5. WORK DONE BY CONSERVATIVE FORCES

I[st] format : (When constant force is given)

$$dw = \vec{F}.d\vec{r} \ (d\vec{r} = dx\hat{i} + dy\hat{j} + dz\hat{k})$$

II format : (When F is given as a function of x, y, z)

$$dw = (F_x\hat{i} + F_y\hat{j} + F_z\hat{k}).(dx\hat{i} + dy\hat{j} + dz\hat{k})$$

$$\Rightarrow \quad dw = F_x dx + F_y dy + F_z dz$$

IIIrd format (perfect differential format)

$$dw = (y\hat{i} + x\hat{j}).(dx\hat{i} + dy\hat{j}) \Rightarrow dw = ydx + xdy$$

6. ENERGY

A body is said to possess energy if it has the capacity to do work. When a body possessing energy does some work, part of its energy is used up. Conversely if some work is done upon an object, the object will be given some energy. Energy and work are mutually convertiable.

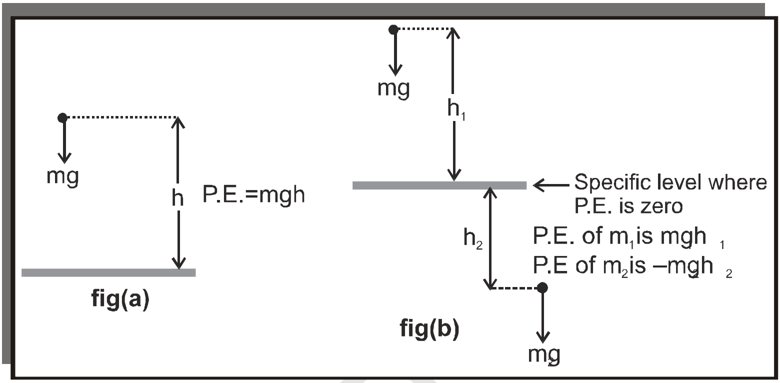
6.1 Kinetic Energy

$$K.E. = \frac{1}{2}mv^2$$

6.2 Potential Energy

W.D = $-\Delta U$

where ΔU is change in potential energy



6.2.1 Gravitational Potential Energy :

It is possessed by virtue of height.

$$GPE = \pm mgh$$

6.2.2 Elastic Potential Energy :
It is a property of stretched or compressed springs.

Elastic Potential Energy = $\frac{1}{2}kx^2$

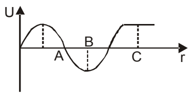
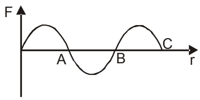
7. Relation Between Potential Energy (U) and Conservation Force $(\vec{F})$

(i) If U is a function of only one variable, then $F = -\frac{dU}{dr} = -$slope of U-r graph

(ii) If U is a function of three coordinate variables x, y and z, then

$$F = -\left[\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j} + \frac{\partial U}{\partial z}\hat{k}\right]$$

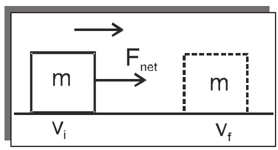
8. EQUILIBRIUM

Physical situation	Stable equilibrium	Unstable equilibrium	Neutral equilibrium
Net force	Zero	Zero	Zero
Potential energy	Minimun	Maximum	Constant
When displaced mean (equilibrium) position.	A restoring nature of force will act on the body, which brings the body back towards mean position.	A force will act which moves the body away from mean position.	Force is again zero
In U-r graph 	At point B	At point A	At point C
In F-r graph 	At point A	At point B	At point C

9. WORK ENERGY THEOREM :

$$W_{net} = \Delta K$$

Work done by net force F_{net} in displacing a particle equals to the change in kinetic energy of the particle i.e.



$(W.D)_c + (W.D)_{N.C} + (W.D)_{ext.} + (W.D)_{pseudo} = \Delta K$

where $(W.D)_c$ = work done by conservative force

10. Power of a Force

(i) Average power

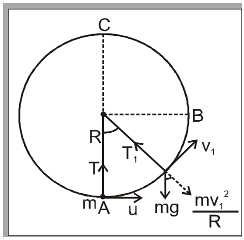
$$: P_{av} = \frac{\text{Total work done}}{\text{Total time taken}} = \frac{W_{Total}}{t}$$

(ii) Instantaneous power

$P_{ins.}$ = rate of doing work done $= \dfrac{dW}{dt}$ $= \vec{F}.\vec{v} = F\,v\cos\theta$

11. Vertical Circular Motion

A bob of mass *m* is suspended from a light string of length R as shown. if velocity at bottommost point of bob is u, then depending on the value of u following three cases are possible



(i) If $u \geq \sqrt{5gR}$, bob will complete the circle.

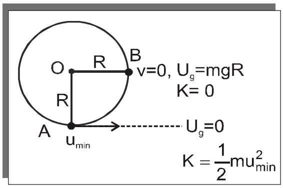
(ii) If $\sqrt{2gR} < u < \sqrt{5gR}$, string will slack between

(iii) If $u \leq \sqrt{2gR}$, bob will oscillate between CAB. In this case v = 0 but $T \neq 0$.

• If $u = \sqrt{5gR}$,bob will just complete the circle, In this case, velocity at topmost point is $v = \sqrt{gR}$, Tension in this critical case is zero at topmost point and 6 mg at bottommost point.

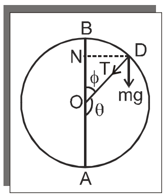
(iv) Condition for the body to reach B :

$\therefore$ if $u \leq \sqrt{2gR}$ then the body will oscillate about A.



(v) At height h from bottom velocity of bob will be,

$$v = \sqrt{u^2 - 2gh}$$



(vi) $\sqrt{2gR} < u < \sqrt{5gR}$ then $\cos\phi = \dfrac{u^2 - 2gR}{3gR}$ v

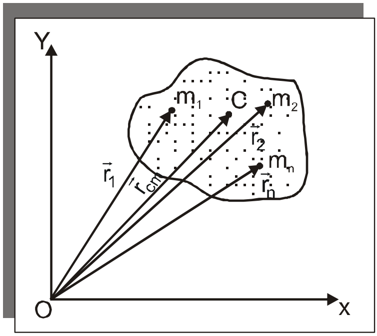
It is the angle from the vertical at which tension in the string vanishes to zero. And after that its motion is projectile.

(vii) **When $u \geq \sqrt{5gR}$ Tension at A** : $T_A = 6mg$ **Tension at B :** $T_B = 3mg$

CENTRE OF MASS

1. Centre of Mass of a System of 'N' Discrete Particles :

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + ... + m_n\vec{r}_n}{m_1 + m_2 + ... + m_n}$$

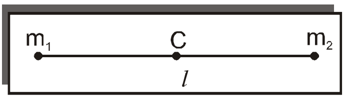


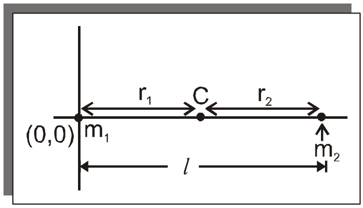
where, $m_i\vec{r}_i$ is called the **moment of mass** of particle with respect to origin.

$$x_{COM} = \frac{\sum_{i=1}^{n} m_i x_i}{M}$$

Similarly, $y_{COM} = \frac{\sum_{i=1}^{n} m_i y_i}{M}$ and $z_{COM} = \frac{\sum_{i=1}^{n} m_i z_i}{M}$

2. Position of COM of two particles : -





$$r_1 = \frac{0 + m_2 l}{m_1 + m_2} = \frac{m_2 l}{m_1 + m_2} \quad ...(1)$$

$$r_2 = l - \frac{m_2 l}{m_1 + m_2} = \frac{m_1 l}{m_1 + m_2} \quad ...(2)$$

$$m_1 r_1 = m_2 r_2 \quad ...(3)$$

Centre of mass of two particle system lie on the line joining the centre of mass of two particle system.

3. Centre of Mass of a Continuous Mass Distribution

For continuous mass distribution the centre of mass can be located by replacing summation sign with an integral sign. Proper limits for the integral are chosen according to the situation

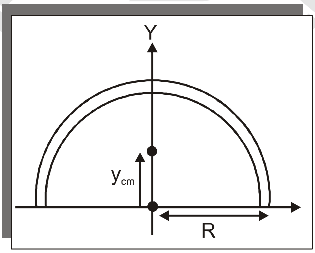
$$x_{cm} = \frac{\int x\,dm}{\int dm},\ y_{cm} = \frac{\int y\,dm}{\int dm},\ z_{cm} = \frac{\int z\,dm}{\int dm}$$

$$\int dm = M \text{ (mass of the body)}$$

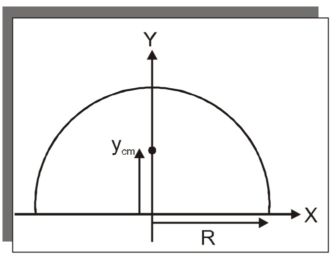
here x,y,z in the numerator is the coordinate of the centre of mass of the dm mass.

$$\vec{r}_{cm} = \frac{1}{M}\int \vec{r}\,dm$$

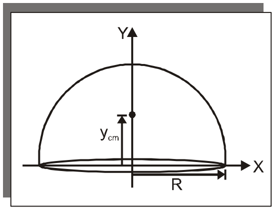
(a) C.O.M of a semicircular Ring $y_{cm} = \frac{2R}{\pi}$



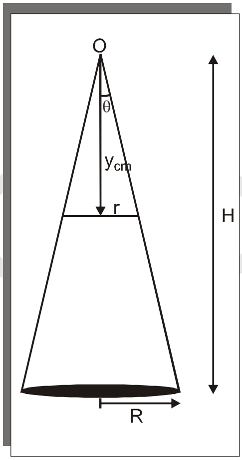
(b) C.O.M of a semicircular Disc $y_{cm} = \frac{4R}{3\pi}$



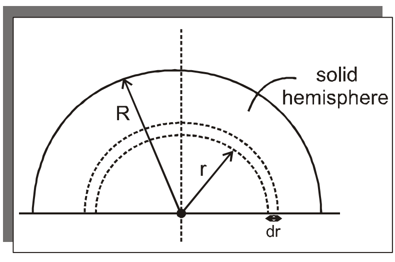
(c) C.O.M. of a Hollow Hemisphere $y_{cm} = \dfrac{R}{2}$



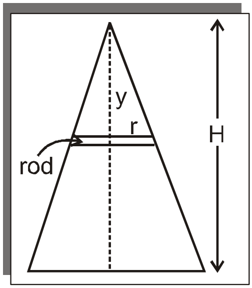
(d) C.O.M. of mass of a solid cone = $\dfrac{3H}{4}$



(e) C.O.M. of a solid Hemisphere $y_{cm} = \dfrac{3R}{8}$



(f) C.O.M. of mass of triangular plate $y_{cm} = \frac{2H}{3}$



4. CAVITY PROBLEMS :

If some mass or area is removed from a rigid body then the position of centre of mass of the remaining portion is obtained by assuming that in a remaining part +m & – m mass is there.

5. Velocity of C.O.M of system :

To find the velocity of centre of mass we differentiate equation (1) with respect to time

$$\frac{d\vec{r}_{com}}{dt} = \frac{m_1 \frac{d\vec{r}_1}{dt} + m_2 \frac{d\vec{r}_2}{dt} + m_3 \frac{d\vec{r}_3}{dt} + ...}{m_1 + m_2 + m_3 + ...}$$

$$\Rightarrow \vec{V}_{com} = \frac{m_1 \frac{d\vec{r}_1}{dt} + m_2 \frac{d\vec{r}_2}{dt} + m_3 \frac{d\vec{r}_3}{dt} + ...}{m_1 + m_2 + m_3 + ...}$$

$$\vec{V}_{com} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + ...}{m_1 + m_2 + m_3 + ...} \quad ...(2)$$

6. Acceleration of centre of mass of the system : -

To find the acceleration of C.O.M we differentiate equation (2)

$$\Rightarrow \frac{d\vec{V}_{com}}{dt} = \frac{m_1\frac{d\vec{v}_1}{dt} + m_2\frac{d\vec{v}_2}{dt} + m_3\frac{d\vec{v}_3}{dt} + ...}{m_1 + m_2 + m_3 + ...}$$

$$\vec{a}_{com} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 + ...}{m_1 + m_2 + m_3 + ...} \qquad ...(3)$$

Now $(m_1 + m_2 + m_3)$

$\vec{a}_{com}$ = $m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 + ...$

$$\vec{F}_{net(system)} = \vec{F}_{1net} + \vec{F}_{2net} + \vec{F}_{3net} + ...$$

The internal forces which the particles exert on one another play absolutely no role in the motion of the centre of mass.

Case I : If F_{net} = 0 then we conclude :

(a) The acceleration of centre of mass is zero ($\vec{a}_{com} = 0$)

If a_1, a_2, a_3.... is acceleration of m_1, m_2, m_3 mass in the system then a_1, a_2, a_3 may or may not be zero.

(b) K.E. of the system is not constant it may change due to internal force.

(c) Velocity of centre of mass is constant $(\vec{v}_{com} = \text{constant})$ but v_1, v_2, v_3 may or may not constant. It may be change due to internal force.

from eq (2)

$$m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + ... = \text{constant}$$

This is called momentum conservation.

"If resultant external force is zero on the system, then the net momentum of the system must remain constant".

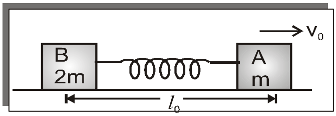
Case II : When centre of mass is at rest.

$\vec{V}_{com} = 0$ then

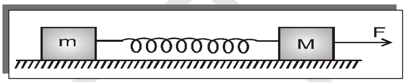
$$\frac{d\vec{r}_{com}}{dt} = 0 \quad \Rightarrow \vec{r}_{com} = \text{constant.}$$

i.e. $\vec{r}_1, \vec{r}_2, \vec{r}_3$ .... may or may not change

7. SPRING BLOCK SYSTEM :



Maximum extension $x_0 = v_0\sqrt{\frac{2}{3k}m}$



$$x_{max} = x_1 + x_2 = \frac{2mF}{k(m+M)}$$

8. IMPULSE :

Impulse of a force $\vec{F}$ acting on a body for the time interval $t = t_1$ to $t = t_2$ is defined as

$$\vec{I} = \int_{t_1}^{t_2} \vec{F}\, dt$$

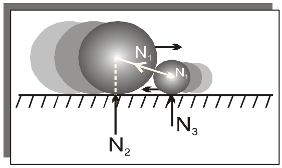
$$\vec{I} = \int \vec{F}\, dt = \int m\frac{d\vec{v}}{dt} dt = \int m\, d\vec{v}$$

$\vec{I} = m(\vec{v}_2 - \vec{v}_1) = \Delta \vec{P} =$ change in momentum due to force $\vec{F}$

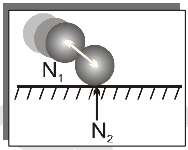
Also $\quad \vec{I}_{Re} = \int_{t_1}^{t_2} \vec{F}_{Res} dt = \Delta\vec{P}$ **(impulse - momentum theorem)**

9. Impulsive force :

A force, of relatively higher magnitude and acting for relatively shorter time, is called impulsive force. An impulsive force can change the momentum of a body in a finite magnitude in a very short time interval. Impulsive force is a relative term. There is no clear boundary between an impulsive and Non-Impulsive force.



N_1, N_3 = Impulsive; N_2 = non-impulsive



Both normals are Impulsive

10. COEFFICIENT OF RESTITUTION (e)

The coefficient of restitution is defined as the ratio of the impulses of reformation and deformation of either body.

$$e = \frac{\text{Impulse of reformation}}{\text{Impulse of deformation}} = \frac{\int F_r dt}{\int F_d dt}$$

$$e = \frac{\text{Velocity of separation of point of contact}}{\text{Velocity of approach of point of contact}}$$

11. Line of Motion

The line passing through the centre of the body along the direction of resultant velocity.

12. Line of Impact

The line passing through the common normal to the surfaces in contact during impact is called line of impact. The force during collision acts along this line on both the bodies.

Direction of Line of impact can be determined by :

(a) Geometry of colliding objects like spheres, discs, wedge etc.

(b) Direction of change of momentum.

If one particle is stationary before the collision then the line of impact will be along its motion after collision.

$$e = \frac{\text{velocity of seperation along line of impact}}{\text{velocity of approach along line of impact}}$$

13. Conservation of Linear Momentum

(i) For a single mass or single body

If net force acting on the body is zero. Then,

$\vec{p}$ = constant or $\vec{v}$ = constant

(if mass = constant)

(ii) **For a system of particles or system of rigid bodies**

If net external force acting on a system of particles or system of rigid bodies is zero, then,

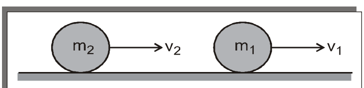
$\vec{P}_{CM} = \text{constant}$ or $\vec{v}_{CM}$ = constant

14. COLLISION

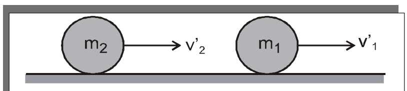
In every type of collision only linear momentum remains constant

(i) Head on elastic collision

In this case linear momentum and kinetic energy both are conserved. After solving two conservation equations. we get



Before collision



After collision

$$v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_1 + \left(\frac{2m_2}{m_1 + m_2}\right)v_2$$

and $$v'_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)v_2 + \left(\frac{2m_1}{m_1 + m_2}\right)v_1$$

(ii) **Head on inelastic collision**

(a) In an inelastic collision, the colliding particles do not regain their shape and size completely after collision.

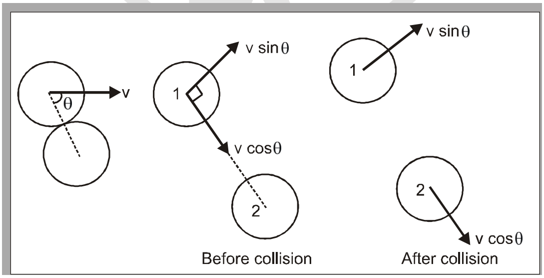
(b) Some fraction of mechanical energy is retained by the colliding particles in the form of deformation potential energy. Thus, the kinetic energy of the particles no longer remains conserved.

(c) However, in the absence of external forces, law of conservation of linear momentum still holds good.

(d) $(\text{Energy loss})_{\text{Perfectly Inelastic}} > (\text{Energy loss})_{\text{Partial Inelastic}}$

(e) $0 < e < 1$

(iii) **Oblique collision (both elastic and inelastic)**



Ball	Component along common tangent direction		Component along common normal direction	
	Before collision	After collision	Before collision	After collision
1	v sin θ	v sinθ	v cos θ	0
2	0	0	0	v cosθ

15. Variable Mass

(i) We now consider those mass is variable, i.e., those in which mass enters or leaves the system. A typical case is that of the rocket from which hot gases keep on escaping thereby continuously decreasing its mass.

(ii) Magnitude of thrust force is given by, $\vec{F}_r = \left|\vec{v}_r\left(\pm\frac{dm}{dt}\right)\right|$

Note : Direction of $\vec{F}_t$ is parallel to $\vec{v}_r$ if mass of system is increasing or $\frac{dm}{dt}$ is positive.

Direction of $\vec{F}_t$ is antiparallel to $\vec{v}_r$ if mass of system is decreasing or $\frac{dm}{dt}$ is negative.

(iii) Based on this fact velocity of rocket at time t is given by

$$v = u - gt + v_r \ln\left(\frac{m_0}{m}\right)$$

value of g has been assumed constant in above equation.

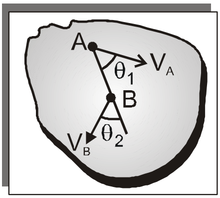
(iv) If mass is just dropped from a moving body then the mass which is dropped acquires the same velocity as that of the moving body.

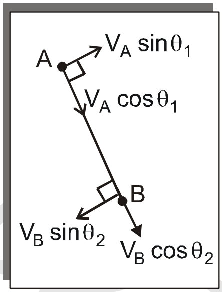
Hence, $\vec{v}_r = 0$ or no thrust force will act in this case.

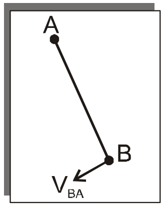
ROTATION

1. RIGID BODY :

Rigid body is defined as a system of particles in which distance between each pair of particles remains constant (with respect to time) that means the shape and size do not change, during the motion. Eg. Fan, Pen, Table, stone and so on.



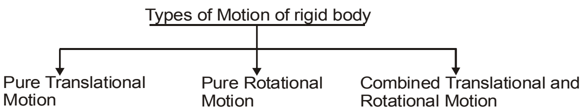




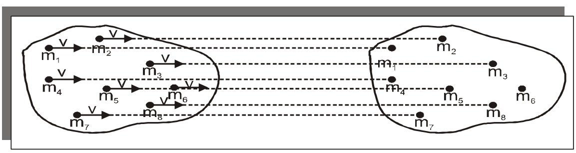
If the above body is rigid

$V_A \cos\theta_1 = V_B \cos\theta_2$

2. TYPES OF MOTION OF RIGID BODY



2.1 Pure Translational Motion :

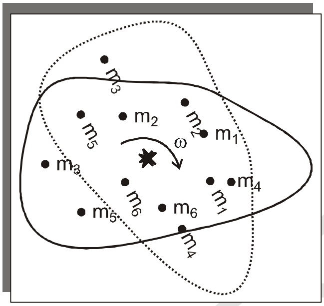


$$\vec{F}_{ext} = m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 + ...$$

$$\vec{F}_{ext} = M\vec{a}$$

$$\vec{P} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + ...$$

2.2 Pure Rotational Motion :



Total Kinetic Energy

$$= \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + ... \quad = \frac{1}{2}I\omega^2$$

Where I = Moment of Inertia = $m_1r_1^2 + m_2r_2^2 + ...$

ω = angular speed of body.

2.3 Combined translational and rotational Motion

A body is said to be in translational and rotational motion if all the particles rotates about an axis of rotation and the axis of rotation moves with respect to the ground.

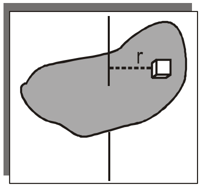
3. MOMENT OF INERTIA

Moment of inertia depends on :

(i) density of the material of body (ii) shape & size of body

(iii) axis of rotation

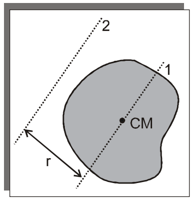
4. Moment of Inertia of Rigid Bodies



$$I = \int r^2\, dm$$

where the integral is taken over the system.

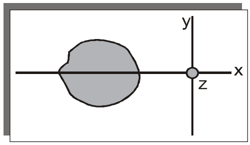
5. Two Theorems



(i) Theorem of parallel axes

$I = I_{CM} + mr^2$

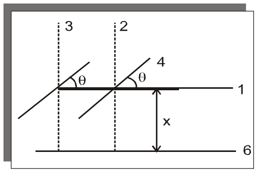
or $I_2 = I_1 + mr^2$



(ii) Theorem of perpendicular axes

$I_z = I_x + I_y$

6. Moment of Inertia of different body :

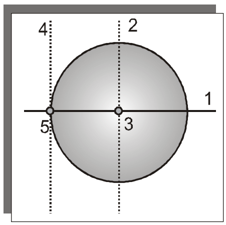


(i) Thin rod

$$I_1 = 0, \quad I_2 = \frac{ml^2}{12}, \quad I_3 = \frac{ml^2}{3}$$

$$I_4 = \frac{ml^2}{12}\sin^2\theta, \quad I_5 = \frac{ml^2}{3}\sin^2\theta$$

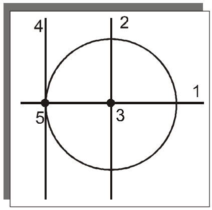
(ii) **Circular disc**



$$I_1 = I_2 = \frac{mR^2}{4}, \quad I_3 = I_1 + I_2 = \frac{mR^2}{2}$$

$$I_4 = I_2 + mR^2 = \frac{5}{4}mR^2, \quad I_5 = I_3 + mR^2 = \frac{3}{2}mR^2$$

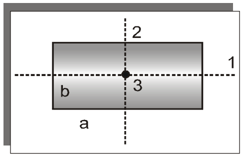
(iii) **Circular ring**



$$I_1 = I_2 = \frac{mR^2}{2}, \quad I_3 = I_1 + I_2 = mR^2$$

$$I_4 = I_2 + mR^2 = \frac{3}{2}mR^2, \; I_5 = I_3 + mR^2 = 2mR^2$$

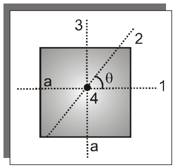
(iv) **Rectangular slab**



$$I_1 = \frac{mb^2}{12}, \quad I_2 = \frac{ma^2}{12}$$

$$I_3 = I_1 + I_2 = \frac{m}{12}(a^2 + b^2)$$

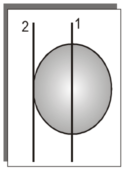
(v) **Square slab**



$$I_1 = I_2 = I_3 = \frac{ma^2}{12}$$

$$I_4 = I_1 + I_3 = \frac{ma^2}{6}$$

(vi) **Solid sphere**

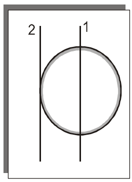


$$I_1 = \frac{2}{5}mR^2$$

$$I_2 = I_1 + mR^2 = \frac{7}{5}mR^2$$

m = mass of sphere

(vii) **Hollow sphere**



$$I_1 = \frac{2}{3}mR^2$$

7. Torque about point :

Torque of force $\vec{F}$ about a point $\vec{\tau} = \vec{r} \times \vec{F}$

8. Torque about axis :

$$\vec{\tau} = \vec{r} \times \vec{F}$$

where $\vec{\tau}$ = torque acting on the body about the axis of rotation

$\vec{r}$ = position vector of the point of application of force about the axis of rotation.

$\vec{F}$ = force applied on the body.

$$\vec{\tau}_{net} = \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_3 + ...$$

9. BODY IS IN EQUILIBRIUM : -

We can say rigid body is in equillibrium when it is in

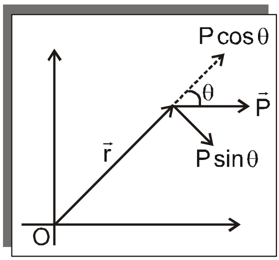
(a) Translational equilibrium

i.e. $\vec{F}_{net} = 0 \Rightarrow F_{net\,x} = 0$ and $F_{net\,y} = 0$ and

(b) Rotational equillibrium $\vec{\tau}_{net} = 0$, i.e., torque about any point is zero

10. ANGULAR MOMENTUM

10.1 Angular momentum of a particle about a point.



$\vec{L} = \vec{r} \times \vec{P} \quad \Rightarrow \quad L = r\,p\,\sin\theta$

$|\vec{L}| = r_{\perp} \times P$,

$|\vec{L}| = P_{\perp} \times r$

10.2 Angular Momentum of a rigid body rotating about a fixed axis

$$L = I\omega$$

Here, I is the moment of inertia of the rigid body about axis.

Note : Angular momentum about axis is the component of $I\vec{\omega}$ along the axis. In most of the cases angular momentum about axis is $I\omega$.

10.3 CONSERVATION OF ANGULAR MOMENTUM :

or $\vec{\tau}_{net} = \frac{d\vec{L}}{dt}$ Now, suppose that $\vec{\tau}_{net} = 0$,

then $\frac{d\vec{L}}{dt} = 0$, so that $\vec{L}$ = constant.

10.4 ANGULAR IMPULSE

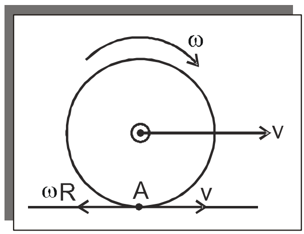
The angular impulse of a torque in a given time interval is defined as $\int_{t_1}^{t_2} \vec{\tau}\, dt$

$\vec{\tau} = \frac{d\vec{L}}{dt}$ $\therefore$ $\vec{\tau}\, dt = d\vec{L}$ or $\int_{t_1}^{t_2} \vec{\tau}\, dt$

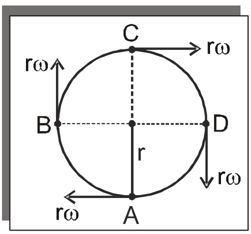
= angular impulse = $\vec{L}_2 - \vec{L}_1$

11. COMBINED TRANSLATIONAL AND ROTATIONAL MOTION OF A RIGID BODY:

To understand the concept of combined translational and rotational motion we consider a uniform disc rolling on a horizontal surface. Velocity of its centre of mass is V_{com} and its angular speed is ω as shown in figure. We divide our problem in two parts



(1) Pure Rotational + (2) Pure Translational about centre of mass.



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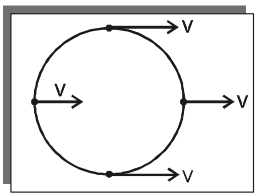
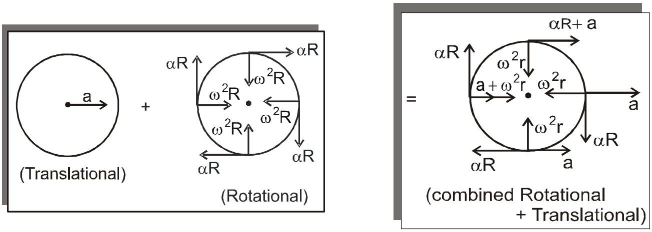


Acceleration of a point on the circumference of the body in R + T motion :

The net acceleration of different points on the rigid body.

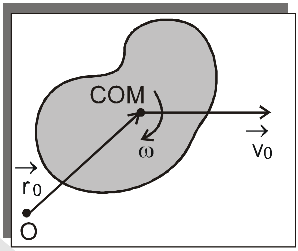


11.1 Angular momentum of a rigid body in combined rotation and translation

Let O be a fixed point in an inertial frame of reference.

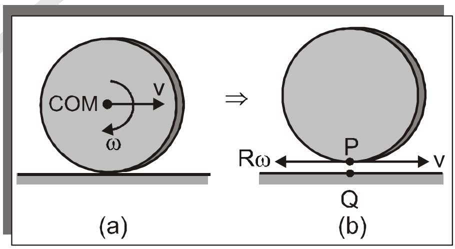
Angular momentum of the body about O is.

$$\vec{L} = \vec{L}_{cm} + M(\vec{r}_0 \times \vec{v}_0)$$



12. UNIFORM PURE ROLLING

Pure rolling means no relative motion (or no slipping at point of contact between two bodies.)



$v_p = v_Q$

or $v - R\omega = 0$

or $v = R\omega$

If $v_p > v_Q$ or $v > R\omega$,

the motion is said to be forward slipping and if $v_p < v_Q < R\omega$, the motion is said to be backward slipping.

Now, suppose an external force is applied to the rigid body, the motion will no longer remain uniform. The condition of pure rolling on a stationary ground is,

$a = R\alpha$

Thus, $v = R\omega$, $a = R\alpha$ is the condition of pure rolling on a stationary ground. Sometime it is simply said rolling.

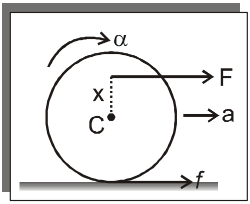
12.1 Pure rolling when force F act on a body :

a = linear acceleration, α = angular acceleration

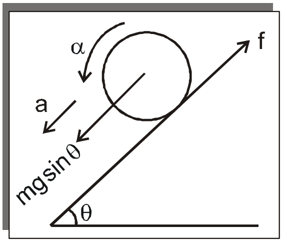
from linear motion : $F + f = Ma$

from rotational motion : $Fx - fR = I\alpha$

$$a = \frac{F(R+x)}{MR(C+1)}, \quad f = \frac{F(x-RC)}{R(C+1)}$$



12.2 Pure Rolling on an Inclined Plane:



$$a = \frac{g\sin\theta}{1+c}$$

So body which have low value of C have greater acceleration.

Note : We can represent the moment of inertia of a different rigid body in a following way.

$$I = CMR^2$$

value of C = 1 for circular ring (R),

$C = \frac{1}{2}$ for circular disc (D) and solid cylinder (S.C.)

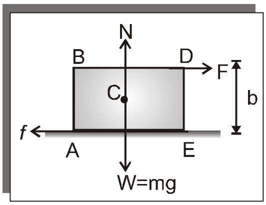
$C = \frac{2}{3}$ for Hollow sphere (H.S) , $C = \frac{2}{5}$ for solid sphere (S.S)

13. TOPPLING

Torque about E

$$Fb = (mg)\,a$$

or $$a = \frac{Fb}{mg}$$



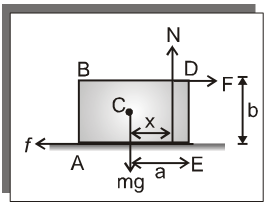
F increases

$$Fb + N(a - x) = mg\,a$$

if x = a

$$F_{max} b = mga$$

$$F_{max} = \frac{mga}{b}$$



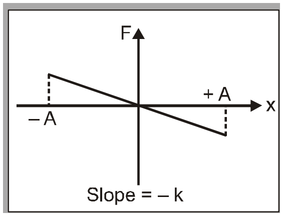
PART-II

SHM, WAVES

SIMPLE HARMONIC MOTION

1. Different Equations In SHM

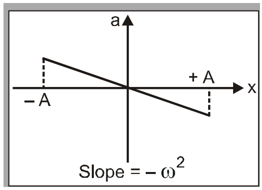
(i) $F = -kx$



(ii) $a = \frac{F}{m} = -\left(\frac{k}{m}\right)x = -\omega^2 x$

(iii) $\omega = \sqrt{\frac{k}{m}}$ = angular frequency of SHM

(iv) $\frac{d^2x}{dt^2} = -\omega^2 x$



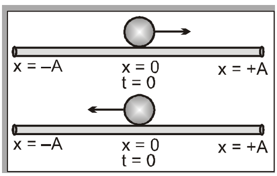
(v) $F \propto -x$ or $a \propto -x$

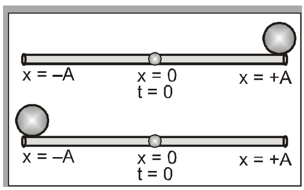
is the sufficient and necessary condition for a periodic motion to be simple harmonic.

(vi) If $x = A \sin \omega t$ then $v = \frac{dx}{dt} = \omega A \cos \omega t$ and $a = \frac{dv}{dt} = -\omega^2 A \sin \omega t$

From these three equations we can see that x-t, v-t and a-t all three functions oscillate simple harmonically with same angular frequency ω, Here x oscillates between +A and –A, v between $+\omega A$ and $-\omega A$ and a between $+\omega^2 A$ and $-\omega^2 A$.

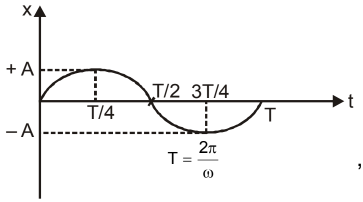
2.





$x = A \sin \omega t$, $x = -A \sin \omega t$ $\qquad$ $x = A \cos \omega t$, $x = -A \cos \omega t$

3. If $x = A\sin\omega t$. Then, $v = \omega A\cos\omega t$ and $a = -\omega^2 A\sin\omega t$. Corresponding x-t, v-t and a-t graphs are shown.



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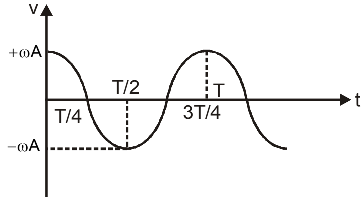
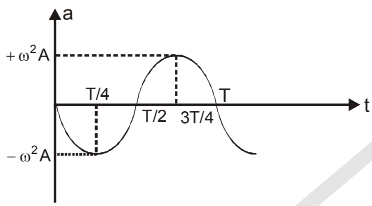


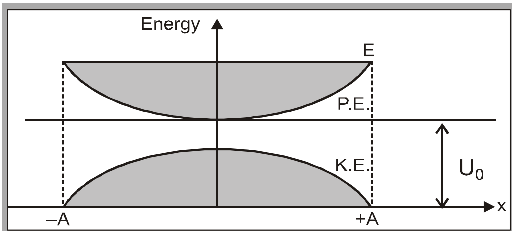


4.

Physical quantity	**At mean position**	**At extreme position**	**At general point**
Speed	ωA	zero	$\omega\sqrt{A^2 - x^2}$
Acceleration	zero	$\pm\omega^2 A$	$-\omega^2 x$
Force	zero	$\pm kA$	$-kx$
Kinetic energy	$\frac{1}{2}kA^2$ $= \frac{1}{2}m\omega^2 A^2$	zero	$\frac{1}{2}k(A^2 - x^2)$
Potential energy	U_0	$U_0 + \frac{1}{2}kA^2$	$U_0 + \frac{1}{2}kx^2$
Total mechanical energy	$U_0 + \frac{1}{2}kA^2$	$U_0 + \frac{1}{2}kA^2$	$U_0 + \frac{1}{2}kA^2$

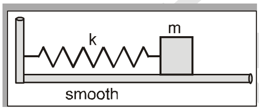
U_0 or potential energy at mean position can be zero.

5. ***Potential energy versus x or kinetic energy versus x graph is parabola. While total energy versus x graph is a straight line as it remains constant.***

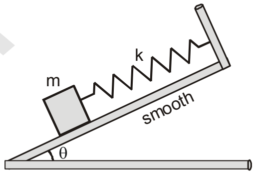


6. **Spring Block System**

(i) $\omega = \sqrt{\frac{k}{m}}, T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}, f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

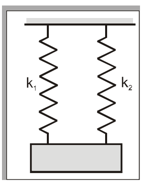
(ii) 

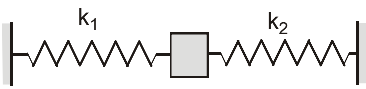
or 

or 

In all three cases, $T = 2\pi\sqrt{\frac{m}{k}}$

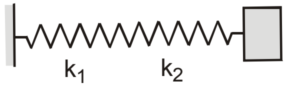
(iii) Parallel combination



or 

In both cases $k_e = k_1 + k_2$

(iv) Series combination

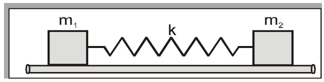


$$k_e = \frac{k_1 k_2}{k_1 + k_2} \quad \text{or} \quad \frac{1}{k_e} = \frac{1}{k_1} + \frac{1}{k_2}$$

(v) In case of two body oscillation,

$$T = 2\pi\sqrt{\frac{\mu}{k}}$$ where,

μ = Reduced mass of two blocks $= \dfrac{m_1 m_2}{m_1 + m_2}$

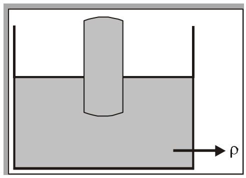


(vi) A plank of mass m and area of cross section A is floating in a liquid of density ρ. When depressed, it starts oscillating like a spring-block system.

Effective value of k in this case is

$$k = \rho A g$$

$$\therefore \quad T = 2\pi\sqrt{\frac{m}{\rho A g}}$$



(vii) If mass of spring m_s is also given, then $T = 2\pi\sqrt{\dfrac{m + \frac{m_s}{3}}{k}}$

(viii) Every wire is also like a spring of force constant given by $k = \dfrac{YA}{l}$.

From here itself we conclude that force constant of a spring is inversely proportional to its length. If length of spring is halved its force constant will become two times.

7. Pendulum

(i) Only small oscillations of a pendulum are simple harmonic in nature :

$$T = 2\pi\sqrt{\frac{l}{g}}.$$

(ii) Second's pendulum is one whose time period is 2 s and length is 1 m.

$x = A\sin\omega t$, $x = -A\sin\omega t$ $x = A\cos\omega t$, $x = -A\cos\omega t$

(iii) Time period of a pendulum of length of the order of radius of earth is

$$T = 2\pi\sqrt{\frac{1}{g\left(\frac{1}{l}+\frac{1}{R}\right)}}$$

From here we can see that

$T = 2\pi\sqrt{\frac{R}{g}}$ or 84.6 min if $l \to \infty$.

Hence time period of a pendulum of infinite length is $2\pi\sqrt{\frac{R}{g}}$

or 84.6 min, Further,

$T = 2\pi\sqrt{\frac{l}{g}}$ if l << R or $\frac{1}{l} >> \frac{1}{R}$.

(iv) If point of suspension has an acceleration $\vec{a}$, then $T = 2\pi\sqrt{\frac{l}{|g_e|}}$

Here $\vec{g}_e = \vec{g} - \vec{a} = \vec{g} + (-\vec{a})$

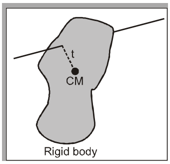
For example if point of suspension has an upward acceleration $\vec{a}$, then $(-\vec{a})$ is downwards or parallel to $\vec{g}$. Hence, $|\vec{g}_e| = g + a$

or $T = 2\pi\sqrt{\frac{l}{g+a}}$

(v) If a constant force $\vec{F}$ (in addition to weight and tension) acts on the bob then,

$T = 2\pi\sqrt{\frac{l}{|g_e|}}$ Here, $\vec{g}_e = \vec{g} + \frac{\vec{F}}{m}$

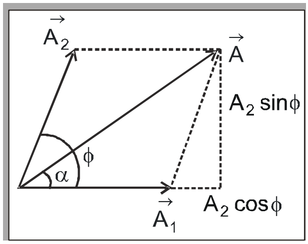
8. **Physical Pendulum**



$$T = 2\pi\sqrt{\frac{I}{mgl}}$$

9. **Combination of two S.H.M**

Suppose the two individual motions are represented by,



$x_1 = A_1 \sin \omega t$

and $x_2 = A_2 \sin (\omega t + \phi)$

Both the simple harmonic motions have same angular frequency ω.

Then the resultant motion is given by

$x = x_1 + x_2 = A_1 \sin \omega t + A_2 \sin (\omega t + \phi) = A \sin (\omega t + \alpha)$

Here, $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos\phi}$ and $\tan \alpha = \dfrac{A_2 \sin\phi}{A_1 + A_2 \cos\phi}$

Important point to remember before solving the questions.

1. Convert all the trignometric ratios into **sine** form and ensure that ωt term is with +ve sign.

2. Make the sign between two term +ve.

3. A_1 is the amplitude of that S.H.M whose phase is small.

4. Then resultant $x = A_{net} \sin (\text{phase of } A_1 + \alpha)$

Where A_{net} is the vector sum of A_1 & A_2 with angle between them is the phase difference between two S.H.M.

WAVES

1. In any type of wave, oscillations of a physical quantity y are produced at one place and these oscillations (along with energy and momentum) are transferred to other places also.

2. **Classification of Waves**

A wave may be classified in following three ways.

First a transverse wave is one in which oscillations of y are perpendicular to wave velocity. Electromagnetic waves are transverse in nature. Sound wave are longitudinal in nature.

Second Mechanical waves require medium for their propagation. Sound waves are mechanical in nature. Non mechanical waves do not require medium for their propagation.

Third Transverse string wave is one dimensional. Transverse waves on the surface of water are two dimensional. Sound wave due to a point source is three dimensional.

3. **Wave Equation**

In any wave equation $y = f(x, t)$

Only those functions of x and t represent a wave equation which satisfy

$$\frac{\partial^2 y}{\partial x^2} = (\text{constant}) \frac{\partial^2 y}{\partial t^2}$$

Here constant $= \frac{1}{v^2}$ where v is the wave speed.

All functions of x and t of type, $y = f(ax \pm bt)$ then

(i) Wave speed $v = \frac{\text{coefficient of t}}{\text{coefficient of x}} = \frac{b}{a}$

(ii) Wave travels along positive x-direction. If ax and bt have opposite signs and it travels along negative x-direction if they have same signs.

4. Plane Progressive Harmonic Wave

General equation of this wave is, $y = A\sin(\omega t \pm kx \pm \phi)$

or $y = A\cos(\omega t \pm kx \pm \phi)$

In these equations,

$T = \frac{2\pi}{\omega}, \omega = 2\pi f$ and $f = \frac{1}{T} = \frac{\omega}{2\pi}$

(i) k is wave number, $k = \frac{2\pi}{\lambda}$

(ii) Wave speed $v = \frac{\omega}{k} = f\lambda$

5. Particle Speed (v_p) And Wave Speed (v) In case Of Harmonic Wave

(i) $y = f(x,t)$ where x and t are two variables. So, $v_p = v\frac{\partial y}{\partial t}$

(ii) In harmonic wave, particles are in SHM.

(iii) **Relation between v_p and v** ; $v_p = -v\frac{\partial y}{\partial x}$

6. Phase Difference ($\Delta\phi$)

case I

$\Delta\phi = \omega(t_1 - t_2)$ or $\Delta\phi = \frac{2\pi}{T}.\Delta t$ = phase difference of one particle at a time interval of Δt.

Case II

$\Delta\phi = k(x_1 \sim x_2) = \frac{2\pi}{\lambda}.\Delta x$ =phase difference at one time between two particles at a path difference of Δx.

7. Energy Density (u), Power [P] And Intensity (I) In Harmonic Wave

(i) Energy density $u = \frac{1}{2}\rho\omega^2 A^2$ = energy of oscillation per unit volume.

(ii) Power $P = \frac{1}{2}\rho\omega^2 A^2 Sv$ = energy transferred per unit time (S = Area)

(iii) Intensity $I = \frac{1}{2}\rho\omega^2 A^2 v$ = energy transferred per unit time per unit area.

8. Longitudinal Wave

(i) There are three equations associated with any longitudinal wave y(x, t), ΔP (x,t) and $\Delta\rho$ (x,t)

(ii) y represents displacement of medium particles from their mean position parallel to direction of wave velocity.

(iii) From y(x,t) equation, we can make $\Delta P(x,t)$ or $\Delta\rho(x,t)$ equation by using the fundamental relation between them,

$\Delta P = -B.\frac{\partial y}{\partial x}$ and $\Delta\rho = -\rho.\frac{\partial y}{\partial x}$

B = Bulk Modulus.

(iv) ΔP_0 = BAk and $\Delta\rho_0$ = ρAk

(v) $\Delta P(x,t)$ and $\Delta\rho(x,t)$ are in same phase. But y(x,t) equation has a phase difference of $\frac{\pi}{2}$ with rest two equations.

9. Wave Speed

(i) Speed of transverse wave on a stretched wire : $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{\rho s}}$

(ii) Speed of longitudinal wave : $v = \sqrt{\frac{E}{\rho}}$

(a) In solids, E = Y = Young's modulus of elasticity : $v = \sqrt{\frac{Y}{\rho}}$

(b) In gases, According to Newton.

$E = B_r$ = Isothermal bulk modulus of elasticity = P $\qquad \therefore\ v = \sqrt{\dfrac{P}{\rho}}$

But results did not match with this formula.

Laplace made correction in it. According to him,

$\gamma = \dfrac{C_p}{C_v} = 1 + \dfrac{2}{f}$ (f = degree of freedom)

$E = B_S$

= Adiabatic bulk modulus of elasticity = γp

$$\therefore \qquad v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{\gamma kT}{m}}$$

10. Effect of Temperature, Pressure And Relative Humidity in Speed of Sound In Air (On In A Gas)

(i) With tempeature $v \propto \sqrt{T}$

(ii) **With pressure** Pressure has no effect on speed of sound as long as temperature remains constant.

(iii) **With relative humidity** : With increase in relative humidity in air, density decreases, Hence, speed of sound increase.

11. Sound Level (L)

$L = 10 \log_{10} \dfrac{I}{I_0}$ (in dB)

Here I_0 = intensity of minimum audible sound

= 10^{-12} watt/m^2

While comparing loudness of two sounds we may write,

$$L_2 - L_1 = 10 \log_{10} \frac{I_2}{I_1}$$

In case of point source, $I \propto \frac{1}{r^2}$ or, $\frac{I_2}{I_1} = \left(\frac{r_1}{r_2}\right)^2$,

In case of line source, $I \propto \frac{1}{r}$ or, $\frac{I_2}{I_1} = \left(\frac{r_1}{r_2}\right)$

12. Doppler Effect In sound : $f' = f\left(\frac{v \pm v_m \pm v_0}{v \pm v_m \pm v_s}\right)$

13. Beats : $f_b = f_1 - f_2$ $(f_1 > f_2)$

14. Stationary Waves

(i) Stationary waves are formed by the superposition of two identical waves travelling in opposite directions.

(ii) Formation of stationary waves is really the interference of two waves in which coherent (same frequency) source are required.

(iii) By the word 'identical waves' we mean that they must have same value of v, ω, and k. Amplitudes may be different, but same amplitudes are preferred.

(iv) In stationary waves all particles oscillate with same value of ω but amplitudes varying from $A_1 + A_2$ to $A_1 - A_2$. Points where amplitude is maximum (or $A_1 + A_2$) are called antinodes (or points of constructive interference) and points where amplitude is minimum (or $A_1 - A_2$) are called nodes (or points of destructive interference).

(v) If $A_1 = A_2 = A$, them amplitude at antinode is 2A and at node is zero. In this case point at node do not oscillate.

(vi) Points at antinodes have maximum energy of oscillation and points at nodes have minimum energy of oscillation (zero when $A_1 = A_2$).

(vii) Points lying between two successive nodes are in same phase. They are out of phase with the points lying between two neighbouring successive nodes.

(viii) Equation of stationary wave is of type,

$y = 2A \sin kx \cos \omega t$ or $y = A \cos kx \sin \omega t$ etc

This equation can also be written as,

$y = A_x \sin \omega t$ or $y = A_x \cos \omega t$

If x = 0 is a node then, $A_x = A_0 \sin kx$, If x = 0 is an antinode then, $A_x = A_0 \cos kx$

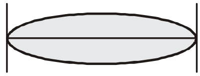
Here A_0 is maximum amplitude at antinode.

(ix) Energy of oscillation in a given volume can be obtained either by adding energies due to two individual waves travelling in opposite directions or by integration. Because in standing wave amplitude and therefore energy of oscillation varies point to point.

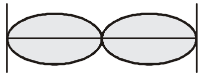
15. Oscillation of Stretched Wire or Organ Pipes

(i) Stretched wire

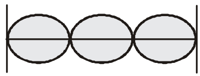
Fundamental tone or first harmonic (n = 1)



First overtone or second harmonic (n = 2)



Second overtone or third harmonic (n = 3)



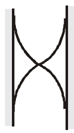
$f = n\left(\frac{v}{2l}\right)$. Here, n = 1, 2, 3, ...

Even and odd both harmonics are obtained.

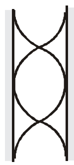
Here, $v = \sqrt{\frac{T}{\mu}}$ or $\sqrt{\frac{T}{\rho S}}$

(ii) **Open organ pipe**

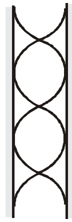
Fundamental tone or first harmonic (n = 1)



First overtone or second harmonic (n = 2)



Second overtone or third harmonic (n = 3)



$$f = n\left(\frac{v}{2l}\right), \text{ n = 1, 2, 3, ...}$$

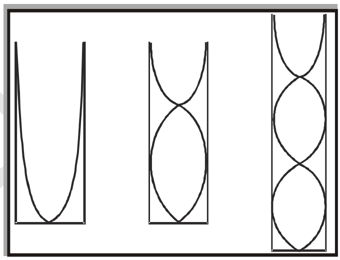
Even and odd both harmonics are obtained.

Here, v = speed of sound in air.

v will be either given in the question, otherwise calculate from $v = \sqrt{\frac{\gamma RT}{M}}$.

(iii) **Closed organ pipe**

Fundamental tone or first harmonic (n = 1)



First overtone or third harmonic (n = 3)

Second overtone or fifth harmonic (n = 5)

$$f = n\left(\frac{v}{4l}\right) \text{ n = 1, 3, 5, ...}$$

Note : (i) Stationary transverse wave are formed in stretched wire and longitudinal stationary waves are formed in organ pipes.

(ii) Open end of pipe is displacement antinode, but pressure and density nodes. Closed end of pipe is displacement node, but pressure and density antinodes.

(iii) Laplace correction e = 0.6r (in closed pipe and 2e = 1.2r (in open pipe)

Hence, $f = n\left[\frac{v}{2(l+1.2r)}\right]$ (in open pipe) and $f = n\left[\frac{v}{4(l+0.6r)}\right]$ (in closed pipe)

(iv) If an open pipe and a closed pipe are of same lengths then fundamental frequency of open pipe is two times the fundamental frequency of closed pipe.

PART-III

GRAVITATION, FLUID, HEAT

GRAVITATION

1. Gravitational force between two point Masses is

$$F = G\frac{m_1 m_2}{r^2}$$

2. Acceleration Due to Gravity

(i) On the surface of earth $g = \frac{GM}{R^2} = 9.81 ms^{-2}$

(ii) At height h from the surface of earth.

$$g' = \frac{g}{\left(1+\frac{h}{R}\right)^2} = g\left(1-\frac{2h}{R}\right) \text{ if } h << R$$

(iii) At depth d from the surface of earth.

$$g' = g\left(1-\frac{d}{R}\right)$$

$g' = 0$ if d = R ie, at centre of earth

(iv) Effect of rotation of earth at latitude ϕ,

$$g' = g - R\omega^2 \cos^2 \phi$$

At equator $\phi = 0$, $g' = g - R\omega^2$ = minimum value

At poles, $\phi = 90°$, $g' = g$ = maximum value.

At equator effect of rotation of earth is maximum and value of g is minimum.

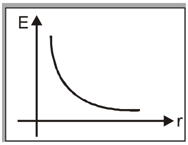
At pole effect of rotation of earth is zero and value of g is maximum.

3. Field Strength

(i) Gravitation field strength at a point in gravitational field is defined as,

$\vec{E} = \frac{\vec{F}}{m} =$ gravitational force per unit mass.

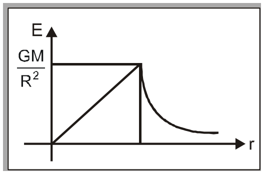
(ii) **Due to a point mass**



$E = \frac{GM}{r^2}$ (towards the mass) or $E \propto \frac{1}{r^2}$

(iii) **Due to solid sphere**

Inside points



$$E_i = \frac{GM}{R^3}r$$

At r = 0, E = 0 i.e., centre

At r = R, $E = \frac{GM}{R^2}$ ie, on surface

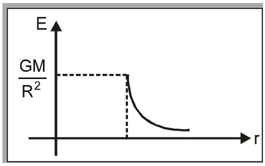
Outside points, $E_O = \frac{GM}{r^2}$ or $E_O \propto \frac{1}{r^2}$

At r = R, $E = \frac{GM}{R^2}$ ie, on surface

As $r \to \infty, E \to 0$

On the surface E-r graph is continuous.

(iv) **Due to a spherical shell**



Inside points, $E_i = 0$

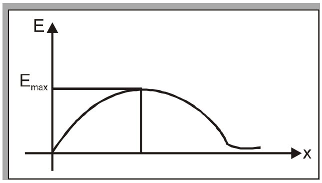
Outside point, $E_O = \frac{GM}{r^2}$

Just outside the surface, $E = \frac{GM}{R^2}$

On the surface E-r graph is discontinuous.

(v) **On the axis of a ring**

$$E_x = \frac{GMx}{(R^2 + x^2)^{3/2}}$$



At x = 0, E = 0 ie, at centre

If x >> R, $E = \frac{GM}{x^2}$

ie, ring behaves as a point mass

As $x \to \infty$, $E \to 0$

$$E_{max} = \frac{2GM}{3\sqrt{3}R^2} \text{ at } x = \frac{R}{\sqrt{2}}$$

4. Gravitational Potential

(i) Gravitational potential at a point in a gravitational field is defined as the negative of work done by gravitational force in moving a unit mass from infinity to that point. Thus.

$$V_P = \frac{W_{\infty} \to P}{m}$$

(ii) **Due to a point mass**

$$V = -\frac{Gm}{r}$$

$V \to -\infty$ as $r \to 0$

and $V \to 0$ as $r \to \infty$

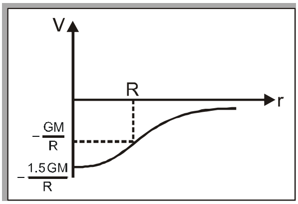
(iii) **Due to solid sphere Inside points**

$$V_i = -\frac{GM}{R^3}\left(1.5R^2 - 0.5r^2\right)$$

At r = R, $V = -\frac{GM}{R}$ ie, on surface

At r = 0, $V = 1.5\frac{GM}{R}$ ie, at centre.

V-r graph is parabolic for inside points and potential at centre is 1.5 times the potential at surface.

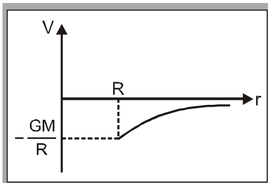


Outside points $V_o = -\frac{GM}{r}$

At r = R, $V = -\frac{GM}{R}$ ie, on surface

As $r \to \infty$, $V \to 0$

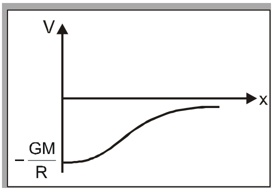
(iv) **Due to a spherical shell**



Inside points $V_i = -\frac{GM}{R}$ = constant

Outside points $V_o = -\frac{GM}{r}$

(v) **On the axis of a ring**



$$V_x = -\frac{GM}{\sqrt{R^2 + x^2}}$$

At x = 0, $V = -\frac{GM}{R}$ ie, at centre,

This is the minimum value.

As $x \to \infty$, $V \to 0$

5. Gravitation Potential Energy

(i) This is negative of work done by gravitational forces in making the system from infinite separation to the present position.

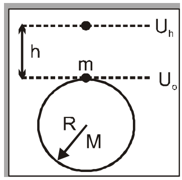
(ii) Gravitational potential energy of two point masses, is $U = -\frac{Gm_1 m_2}{r}$

(iii) To find gravitational potential energy of more than two point masses we have to make pairs of masses, Neither of the pair should be repeated. For example, in case of four point masses.

$$U = -G\left[\frac{m_4m_3}{r_{43}} + \frac{m_4m_2}{r_{42}} + \frac{m_4m_1}{r_{41}} + \frac{m_3m_2}{r_{32}} + \frac{m_3m_1}{r_{31}} + \frac{m_2m_1}{r_{21}}\right]$$

For n point masses, total number of pairs will be $\frac{n(n-1)}{2}$.

(iv) If a point mass m is placed on the surface of earth,



the potential energy here is $U_0 = -\frac{GMm}{R}$ and potential

energy at height h is $U_h = -\frac{GMm}{(R+h)}$

The difference in potential energy would be

$$\Delta U = U_h - U_o \text{ or } \Delta U = \frac{mgh}{1+\frac{h}{R}}$$

If h << R, $\Delta U = mgh$

6. Relation Between Field Strength $\vec{E}$ and Potential V

(i) If V is a function of only one variable (say r) then,

$$E = -\frac{dV}{dr} = \text{-slope of V-r graph}$$

(ii) If V is a function of three coordinate variables x, y and z then,

$$\vec{E} = -\left[\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right]$$

7. Escape Velocity

(i) From the surface of earth,

$$v_e = \sqrt{2gR} = \sqrt{\frac{2GM}{R}} \quad \left(\text{as } g = \frac{GM}{R^2}\right) = 11.2 \text{ kms}^{-2}$$

(ii) Escape velocity does not depend upon the angle at which particle is projected from the surface.

8. Motion of Satellites

(i) Orbital speed $v_o = \sqrt{\frac{GM}{r}}$

(ii) Time period $T = \frac{2\pi}{\sqrt{GM}} r^{3/2}$

(iii) Kinetic energy $K = \frac{GMm}{2r}$

(iv) Potential energy $U = -\frac{GMm}{r}$

(v) Total mechanical energy $E = -\frac{GMm}{2r}$

* Near the surface of earth, r = R and $v_0 = \sqrt{\frac{GM}{R}} = \sqrt{gR} = 7.9\,kms^{-1}$

This is the maximum speed of earth's satellite.

* Time period of such a satellite would be,

$$T = \frac{2\pi}{\sqrt{GM}} R^{3/2} = 2\pi\sqrt{\frac{R}{g}} = 84.6\,min.$$

This is the minimum time period of any earth's satellite.

9. Kepler's laws

Keplar's three empirical laws describe the motion of planets.

First law Each planet moves in an elliptical orbit, with the sun at one focus of the ellipse.

Second law The radius vector, drawn from the sun to a planet, sweeps out equal areas in equal time interval ie, areal velocity is constant. This law is derived from law of conservation of angular momentum.

$$\frac{dA}{dt} = \frac{L}{2m} = \text{constant}$$

Here L is angular momentum and m is mass of planet.

Third law $T^2 \propto r^3$ where r is semi-major axis of elliptical path.

* Circle is a special case of an ellipse. Therefore second and third laws can also be applied for circular path. In third law, r is radius of circular path.

HEAT

1. Calorimetry

(i) $Q = ms\Delta\theta = c\Delta\theta$, when temperature changes without change in state.

(ii) Q = mL, when state changes without change in temperature.

(iii) s = specific heat of any substance = heat required to increase the temperature of unit mass by 1ºC or 1 K.

(iv) c = heat capacity of a body = ms = heat required to increase the temperature of whole body by 1ºC or 1 K.

(v) Specific heat of water is 1 $calg^{-1}$.ºC between 14.5ºC and 15.5ºC.

(vi) ***L = latent heat*** = heat required to convert unit mass of that substance from one to another state.

(vii) ***Water equivalent of a vessel*** It is mass of equivalent water which takes same amount of heat as take by the vessel for same rise of temperature.

2. Thermal Expansion

(i) $\Delta l = l\alpha\Delta\theta, \Delta s = s\beta\Delta\theta$ and $\Delta V = V\gamma\Delta\theta$

(ii) $\beta = 2\alpha$ and $\gamma = 3\alpha$ for isotropic medium.

(iii) **Time period of pendulum**:

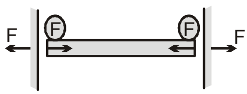
$$T = 2\pi\sqrt{\frac{l}{g}} \quad \text{or} \quad T \propto \sqrt{l}$$

increasing Temperature $\Delta\theta$:

$$\Delta T = (T' - T) = \frac{1}{2}T\alpha\Delta\theta$$

Time lost/ gained $\qquad \Delta t = \frac{\Delta T}{T'} \times t$

(iv) **Thermal Stress** : strain $\frac{\Delta l}{l} = \alpha\Delta\theta$



$\therefore \qquad F = A \times \text{stress} = Y A\alpha\Delta\theta$

Rod applies this much force on wall to expand.

3. **Effect of temperature on different physical quantities**

(i) **On Density** : $\rho' = \rho(1 - \gamma.\Delta\theta)$ if $\gamma.\Delta\theta << 1$

(ii) **In Fluid Mechanics**

Case 1 : $\rho_s < \rho_l$

fraction $f = \frac{\rho_s}{\rho_l}$

When temperature is increased, ρ_s and ρ_l both will decreases, Hence, fraction may increase, decrease or remains same. At higher temperature, $f' = f\left(\frac{1+\gamma_l\Delta\theta}{1+\gamma_s\Delta\theta}\right)$

If $\gamma_l > \gamma_s$, $f' > f$ or immersed fraction will increase.

Case 2 : $\rho_s > \rho_l$ $W_{app} = W - F$

Here $F_s = \rho_l g$

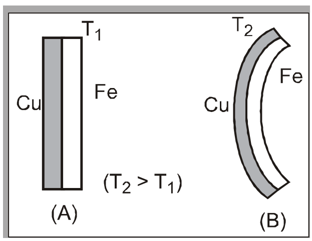
With increases in temperature V_s will increase and ρ_l will decrease, while g will remain unchanged. Therefore upthrust may increase, decrease or remain same.

At some higher temperature. $F' = F\left(\frac{1+\gamma_s\Delta\theta}{1+\gamma_l\Delta\theta}\right)$

If $\gamma_s > \gamma_l$, upthrust will increase. Therefore apparent weight will decrease.

(iii) Bimatellic Strip

Note If two strips of equal length but of different metals are placed on each other and riveted, the single strip so formed is called '**bimetallic strip'** [see given fig.].



4. **Heat Transfer**

Heat conduction through a rod

(i) Heat flow in steady state $Q = \frac{kA(\theta_1 - \theta_2)}{l}t$

(ii) Rate of flow of heat = heat current or $H = \frac{dQ}{dt} = \frac{TD}{R}$

Here TD = temperature difference = $\theta_1 - \theta_2$ and R = thermal resistance = $\frac{l}{KA}$

5. **SLABS IN PARALLEL AND SERIES**

(a) ***Slabs in series (in steady state) :***

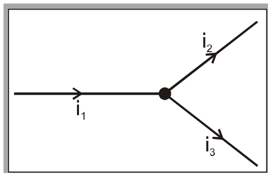
$R = R_1 + R_2 + R_3 + ...$

(b) ***Slabs in parallel :***

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ...$$

6. **Junction Law**

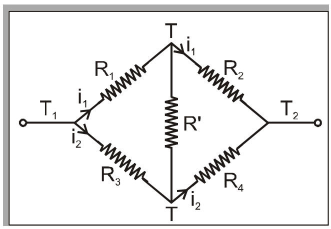
According to the Junction law the sum of all the heat current directed towards a point is equal to the sum of all the heat currents directed away from the points.



$i_1 = i_2 + i_3$

WHEAT STONE BRIDGE

$R_1 R_4 = R_2 R_3$



7. Radiation

(i) Absorptive power $a = \frac{\text{energy absorbed}}{\text{energy incident}}$ $a \le 1$, a = 1 for perfectly black body

(ii) Spectral absorptive power a_λ = absorptive power of wavelength λ.

$a_\lambda \le 1$, a_λ = 1 for perfectly black body

(iii) **Emissive power e** Energy radiated per unit area per unit time is called emissive power of a body.

(iv) **Spectral emissive power e_λ** Emissive power of wavelength λ is known as

spectral emissive power $e = \int_0^\infty e_\lambda \, d_\lambda$.

(v) **Kirchhoff's law** If different bodies (including a perfectly block body) are kept at same temperature, than

$e_\lambda \propto a_\lambda$ or $\frac{e_\lambda}{a_\lambda} = \text{constant}$

or $= (e_\lambda)_{\text{Perfectly black body}}$

* Good absorbers of a particular wavelength λ are also good emitters of same wavelength λ.

* At a given temperature, ratio of e_λ and a_λ for any body is constant. This ratio is equal to e_λ of perfectly black body at that temperature.

(vi) **Stefan's law** Emissive power of a body is given by, $e = e_r \sigma T^4$

Here e_r = emissivity, emittance, relative emissivity or relative emittance.

$e_r \le 1$, e_r = 1 for a perfectly black body.

(vii) Total energy radiated by a body, $E = e_r \sigma T^4 At$

(viii) $a = e_r$ or absorptivity of a body = its emissivity.

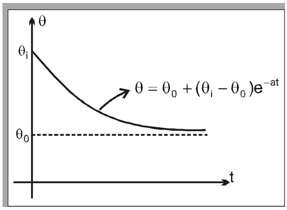
(ix) **Cooling of a body by radiation**

(a) Rate of cooling = $-\frac{dT}{dt} = \frac{e_r A \sigma}{ms}\left(T^4 - T_0^4\right)$ or $-\frac{dT}{dt} \propto T^4 - T_0^4$

(b) **Newton's law of cooling** when ΔT is small then

rate of cooling $\propto$ temperature difference.

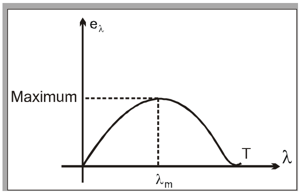
(c) If body cools by radiation according to Newton, then temperature of body decreases exponentially.



(d) Shortcut approximation of e^{-at} then

$$\left(\frac{\theta_1 - \theta_2}{t}\right) = \alpha\left[\frac{\theta_1 + \theta_2}{2} - \theta_0\right]$$

(x) **Wien's displacement law**



$\lambda_m \propto \frac{1}{T}$ or $\lambda_m T =$ constant

= Wien's constant 'b' = 2.89×10^{-3} m-K

Further, area of this graph will give total emissive power which is proportional to T^4.

8. Kinetic theory of Gases

(i) $pV = nRT = \frac{m}{M}RT$ (m = mass of gas in gms)

(ii) Density $\rho = \frac{m}{V}$ (General) $= \frac{pM}{RT}$ (for ideal gas)

(iii) Four speeds, $v = \sqrt{\frac{ART}{M}} = \sqrt{\frac{AkT}{m}} = \sqrt{\frac{Ap}{\rho}}$

Here, m = mass of one gas molecule.

A = 3 for rms speed of gas molecules, $= \frac{8}{\pi} \approx 2.5$ for average speed of gas molecules

= 2 for most probable speed of gas molecules $= \gamma = \frac{C_p}{C_v}$ for speed of sound in a gas

(iv) $p = \frac{1}{3}\frac{mn}{V}v_{rms}^2$

(v) $p = \frac{2}{3}E$. Here, E = total translational kinetic energy per unit volume

(vi) f = degree of freedom = 3 for monoatomic gas

= 5 for diatomic and linear polyatomic gas = 6 for nonlinear polyatomic gas

(a) Vibrational degree of freedom is not taken into consideration.

(b) Translational degree of freedom for any type of gas is three.

(vii) Total internal energy of a gas is, $U = \frac{nf}{2}RT$

Here, n = total number of gram moles

(viii) $C_V = \frac{dU}{dT}$ (where U = internal energy of one mole of a gas $= \frac{f}{2}RT$)

$\therefore \qquad C_V = \frac{f}{2}R = \frac{R}{\gamma - 1}$

(ix) $C_p = C_v + R = \left(1 + \frac{f}{2}\right)R = \left(\frac{\gamma}{\gamma - 1}\right)R$ (x) $\gamma = \frac{C_p}{C_v} = 1 + \frac{2}{f}$

(xi) Internal energy of 1 mole in one degree of freedom of any gas is $\frac{1}{2}RT$

(xii) Translational kinetic energy of one mole of any type of gas is $\frac{3}{2}RT$

(xiii) Rotational kinetic energy of 1 mole of monoatomic gas is zero of dia or linear polyatomic gas is $\frac{2}{2}RT$ or RT, of nonlinear polyatomic gas is $\frac{3}{2}RT$.

9. THERMODYNAMICS

9.1 Gas laws

(a) **Boyle's law** Is applied when T = constant, $p \propto \frac{1}{V}$

(b) **Charle's law** p = constant or, process is isobaric, $V \propto T$

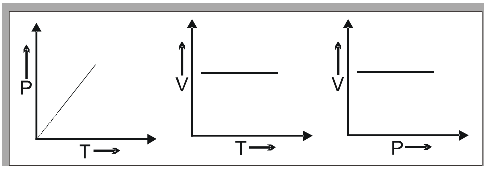
(c) **Pressure law or Gay Lussac's law** Is applied when V = constant or process is isochoric, $p \propto T$

9.2 **First law of thermodynamics** It is a law of conservation of energy given by, $Q = \Delta U + W$

PROCESS :

(i) **Isochoric process :**

Since change in volume is zero therefore, $dW = 0$, $dV = 0$

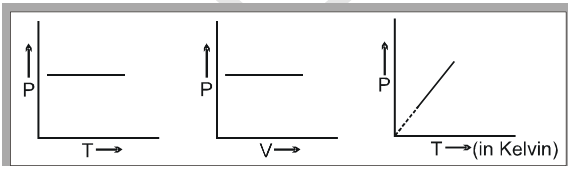


$\Delta U = n\frac{f}{2}R\ \Delta T$, $\Delta Q = \Delta U = n\frac{f}{2}R\ \Delta T$

(ii) **Isobaric Process :** Pressure remains constant in isobaric process

$\therefore$ P = constant $\Rightarrow$ $\frac{V}{T} = \text{constant}$

Indicator diagram of isobaric process :



$\Delta W = P\ \Delta V = P\ (V_{final} - V_{initial}) = nR(T_{final} - T_{initial})$

$\Delta U = n\ \frac{f}{2}R\Delta T$, $\Delta Q = \Delta U + \Delta W$

$\Delta Q = n\ \frac{f}{2}R\ \ \Delta T + P[V_f - V_i] = n\frac{f}{2}R\ \Delta T + nR\ \Delta T$

Above expression gives an idea that to increase temperature by ΔT in isobaric process heat required is more than in isochoric process.

(iii) Cyclic Process :

In the cyclic process initial and final states are same therefore initial state = final state

Work done = Area enclosed under P-V diagram.

If the process on P-V curve is clockwise, then net work done is (+ve) and vice-versa.

(iv) Free Expansion

$\Delta Q = 0$, $\Delta U = 0$ and $\Delta W = 0$. Temperature in the free expansion remains constant.

10. MOLAR HEAT CAPACITY

C = heat required to raise the temperature of 1 mole of any substance by 1°C or 1 K. $= \dfrac{Q}{n\Delta T}$

Molar heat capacity of solids and liquids is almost constant.

In case of gases C is process dependent. It varies from O to ∞.

In isothermal process $C = \infty$ as $\Delta T = 0$

In adiabatic process C = 0 as Q = 0

C_P (molar heat capacity of isobaric process and C_V (molar heat capacity of isochoric process) are commonly used. In a general process pV^x = constant, molar heat capacity is given by,

$$C = \frac{R}{\gamma - 1} + \frac{R}{1 - x}$$

11. MIXTURE OF NON REACTIVE GASES

(a) $n = n_1 + n_2$

(b) $p = p_1 + p_2$

(c) $U = U_1 + U_2$

(d) $\Delta U = \Delta U_1 + \Delta U_2$

(e) $C_v = \dfrac{n_1 C_{V_1} + n_2 C_{v_2}}{n_1 + n_2}$

(f) $C_p = \dfrac{n_1 C_{p_1} + n_2 C_{p_2}}{n_1 + n_2} = C_v + R$

(g) $\gamma = \dfrac{C_p}{C_v}$ or $\dfrac{n}{\gamma - 1} = \dfrac{n_1}{\gamma_1 - 1} + \dfrac{n_2}{\gamma_2 - 1}$

(h) $M = \dfrac{n_1 M_1 + n_2 M_2}{n_1 + n_2}$

ELASTICITY

1. **Stress** $= \frac{F}{A}$ = restoring force per unit area.

2. **Strain** $= \frac{\Delta x}{x}$ = change per unit original.

3. **Modulus of Elasticity** $E = \frac{\text{Stress}}{\text{Strain}}$

4. Materials which offer more resistance to external deforming forces have higher value of modulus of elesticity.

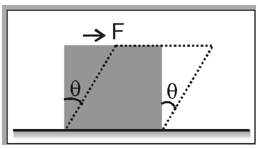
5. **Young's Modulus of Elasticity**

$$Y = \frac{F/A}{\Delta l/l} = \frac{Fl}{A\Delta l} \qquad F = Mg \text{ and } A = \pi r^2$$

6. **Bulk Modulus of Elasticity**

$$B = \frac{F/A}{\Delta V/V} = -\frac{\Delta p}{\Delta V/V} \text{ or } -\frac{dp}{dV/V}$$

7. **Shear Modulus of Elasticity or Modulus of Rigidity**



$$\gamma = \frac{F/A}{\theta}$$

8. Solids have all three modulii of elasticities, Young's modulus, bulk modulus and shear modulus. Whereas liquids and gases have only bulk modulus.

9. Every wire is like a spring whose force constant is equal to, $k = \frac{YA}{l}$ or $k \propto \frac{1}{l}$

10. Potential energy stored in a stretched wire

$$U = \frac{1}{2}k(\Delta l)^2 = \frac{1}{2}\left(\frac{YA}{l}\right)(\Delta l)^2$$

11. Potential energy stored per unit volume (also called energy density) in a stretched wire,

$$U = \frac{1}{2} \times \text{Stress} \times \text{Strain}$$

12. **Change in Length of a Wire** $\Delta l = \frac{Fl}{AY}$

Here F is tension in the wire. If wire is having negligible mass, tension is uniform throughout the wire and change in length is obtained directly. Otherwise by integration.

13. In case of solids and liquids bulk modulus is almost constant. In case of a gas, it is process dependent.

In isothermal process $B = B_T = p$

In adiabatic process $B = B_s = \gamma p$

14. **Compressibility** = 1/B

15. When pressure is applied on a substace, its volume decreases, while mass remains constant. Hence its density will increaes,

$$\rho' = \frac{\rho}{1 - \Delta p/B} \text{ or } \rho' = \rho\left(1 + \frac{\Delta p}{B}\right) \text{ if } \frac{\Delta p}{B} << 1$$

FLUID MECHANICS

1. The pressure P is defined at that point as the normal force per unit area, i.e.,

$$P = \frac{dF_{\perp}}{dA}$$

2. Relative Density (or Specific Gravity) of any Substance

$$RD = \frac{\text{Density of that substance}}{\text{Density of water}}$$

$$= \frac{\text{Weight in air}}{\text{Change in weight in water when completely immersed in it}}$$

3. Effect of Temperature on Density :

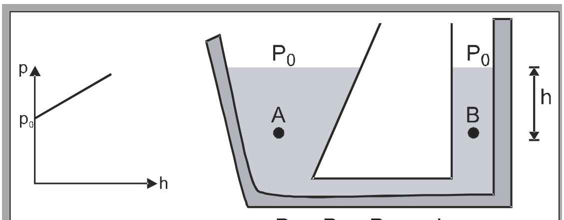
$$\rho' = \frac{\rho}{1+\gamma\Delta\theta}$$

4. Effect of Pressure on Density :

$$\rho' = \frac{\rho}{1-\dfrac{\Delta p}{B}}$$

5. 1 Pa = 1 Nm^{-2}, 1 Bar = 10^5 Pa,
1 atm = 1.013 × 10^5 Pa
Gauge pressure = absolute pressure – atmospheric pressure

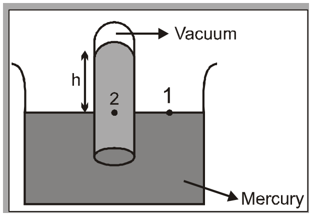
6. Pressure at depth h below the surface of water, $p = p_0 + \rho gh$



$P_A = P_B = P_0 + \rho gh$

Change in pressure per unit depth $\frac{dp}{dh} = \rho g$

7. Barometer



$$p_1 = p_2$$

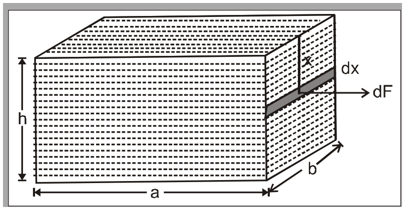
$$\therefore \quad p_0 = h\rho g$$

or $$h = \frac{p_0}{\rho g}$$

h is approximately 76 cm of mercury.

8. Force on Side Wall of Vessel

Net force can be evaluated by integrating equation



$$F = \int dF = \int_0^h x\rho g b dx$$

$$F = \frac{\rho g b h^2}{2}$$

8.1 Average Pressure on Side Wall

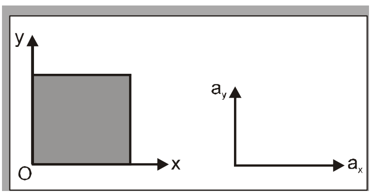
$$< p >_{av} = \frac{F}{bh} = \frac{1}{2}\frac{\rho g b h^2}{bh} = \frac{1}{2}\rho g h$$

8.2 Torque on the Side Wall due to Fluid Pressure

This net torque is

$$\tau = \int d\tau = \int_0^h \rho g b(hx - x^2)dx = \frac{1}{6}\rho g b h^3$$

8.3 Pressure Distribution in an Accelerated Frame

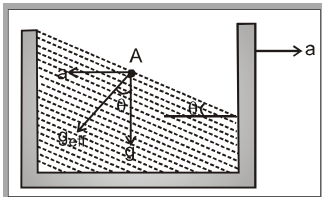


If container has an acceleration component a_x in x-direction and a_y in y-direction. Then,

$$\frac{dp}{dx} = -\rho a_x \text{ and } \frac{dp}{dy} = -\rho(a_y + g)$$

Due to acceleration of container, liquid filled in it experiences a pseudo force relative to container and due to this the free surface of liquid which normal to the gravity

$$\theta = \tan^{-1}\left(\frac{a}{g}\right)$$

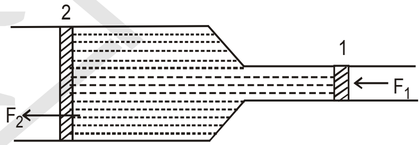


pressure at a point A in the acceleratd container as shown in figure is given as

$$P_A = P_0 + h\rho\sqrt{a^2 + g^2}$$

9. PASCAL'S PRINCIPLE

"*The pressure applied at one point in an enclosed fluid is transmitted uniformly to every part of the fluid and to the walls of the container.*"



$$F_2 = p_2 \times S_2 \Rightarrow F_2 = \frac{F_1}{S_1} \times S_2$$

10. ARCHIMEDE'S PRINCIPLE

If a heavy object is immersed in water, it seems to weight less than when it is in air. This is because the water exerts an upward force called **buoyant force.** It is equal to the weight of the fluid displaced by the body.

A body wholly or partially submerged in a fluid is buoyed up by a force equal to the weight of the displaced fluid.

This result is known as **Archimedes' principle.**

(i) **Upthrust** $F = V_i \rho_l g_e$

(ii) When a solid whose density is less than the density of liquid floats in it then, some fraction of solid remains immeresed in the liquid. In this case,

(i) Weight = Upthrust (ii) Fraction of volume immersed $f = \frac{\rho_s}{\rho_l}$

(iii) When a solid whose density is more than the density of liquid is completely immersed in it then upthrust acts on the 100% volume and apparent weight is less than its actual weight. $W_{app} = W - F$

11. *ASSUMPTIONS OF IDEAL FLUID*

(1) Fluid is incompressible : density of fluid remain constant through out the fluid.

(2) Fluid is non-viscous : fluid friction is absent

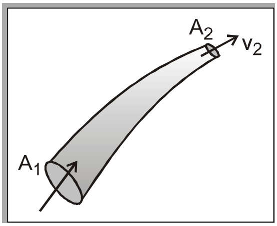
(3) Doesn't show rotational effect : If we release any body in the flowing section there it will not rotate about its C.O.M.

(4) Stream line flow : velocity of fluid at any particular point remains constant with time It may vary with position.

12. EQUATION OF CONTINUITY

According to the equation of continuity, if flow is steady *mass of fluid entering at end A_1 per second*

= mass of fluid leaving the end A_2 per second.

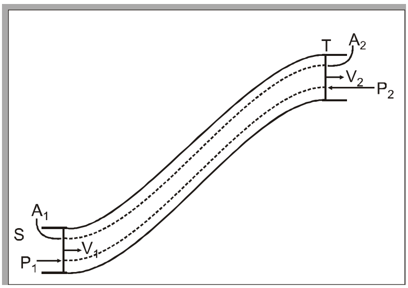


$$\frac{dV}{dt} = A_1v_1$$

According to the definition of steady flow

$$A_1 v_1 \rho = A_2 v_2 \rho \quad \text{or } A_1v_1 = A_2v_2 \quad \text{or} \quad v \propto \frac{1}{A}$$

13. BERNOULLIS EQUATION



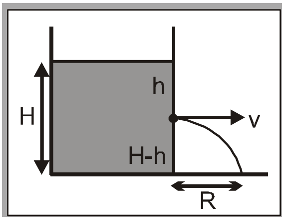
It relates the variables describing the steady laminer of liquid. It is based on energy conservation.

Assumptions

The fluid is incompressible, non-viscous, non rotational and streamline flow.

$$p + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$$

or $p_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$



(i) $v = \sqrt{2gh} = \sqrt{2gh_{top}}$

(ii) $t = \sqrt{\frac{2(H-h)}{g}} = \sqrt{\frac{2h_{bottom}}{g}}$

(iii) $R = vt = 2\sqrt{h(H-h)} = 2\sqrt{h_{top} \times h_{bottom}}$

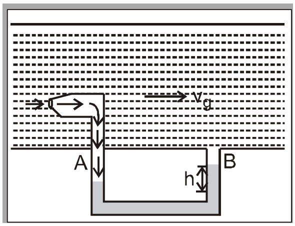
(iv) $R_{max} = H$ at $h = \frac{H}{2}$

(v) Time taken to empty a tank if hole is made at bottom.

$$t = \frac{A}{a}\sqrt{\frac{2H}{g}}$$

14. Pitot Tube

It is a device used to measure flow velocity of fluid.



It if the a liquid of density ρ, then

$$P_A - P_B = h(\rho - \rho_g)g$$

Now if we apply Bernoulli's equation at ends A and B we'l have

$$\frac{1}{2}\rho v_g^2 = P_A - P_B = h\rho g$$

Note : *Pitot tube is also used to measure velocity of aeroplanes with respect to wind. It can be mounted at the top surface of the plain and hence the velocity of wind can be measured with respect to plane.*

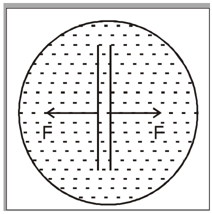
15.(a) COHESIVE FORCE

The force of attraction between the molecules of the same substance is called cohesive force

15. (b) ADHESIVE FORCE

The force at attraction between molecules of different substances is called adhesive force

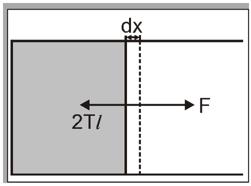
16. SURFACE TENSION



The property of a liquid at rest due to which its free surface tries to have minimum surface area and behaves as if it were under tension somewhat like a stretched elastic membrane is called surface tension.

$$T = (F/L)$$

16.1 SURFACE ENERGY



When the surface area of a liquid is increased, the molecules from the interior rise to the surface. This requires work against force of attraction of the molecules just below the surface. This work is stored in the form of potential energy.

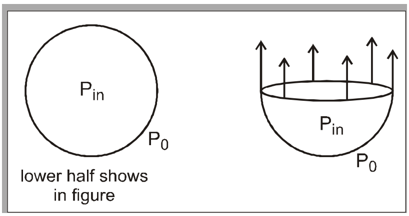
One has to apply an external force F equal and opposite to it to keep the wire in equilibrium. Thus,

$$F = 2Tl$$

But (2l) (dx) is the total increase in area of both the surface of the film. Let is be dA. Then,

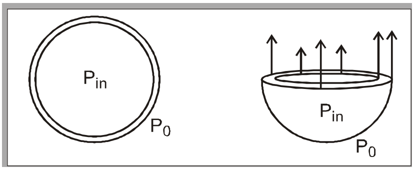
$$dW = T\,dA \qquad \text{or} \qquad T = \frac{dW}{dA}$$

16.2 Excess Pressure Insider a liquid drop :



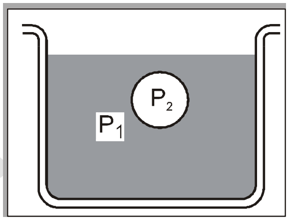
$$P_{in} - P_0 = \frac{2T}{R} = \text{Excess Pressure}$$

16.3 Excess pressure inside a bubble



$$P_{in} - P_0 = \frac{4T}{R} = \text{Excess pressure}$$

Note : (1) If we have an air bubble inside a liquid, a single surface is formed. on the convex side. The pressure in the concave side (that is in the air) is greater than the pressure in the convex side



(that is in the liquid) by an amount $\frac{2T}{R}$.

$$\therefore \qquad P_2 - P_1 = \frac{2T}{R}$$

16.4. For any curved surface excess pressure on the concave side = $T\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

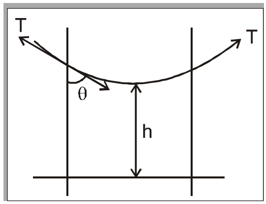
where R_1 & R_2 are radius of curvature of the surface in two perpendicular direction of instead of liquid surface, liquid film is given then above exression will be

$P = 2T\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$ For spherical curved surface R_1, R_2

16.5 Pressure inside a charged bubble :

$$P_{in} = P_0 + \frac{4T}{R} - \frac{\sigma^2}{2\varepsilon_0}$$

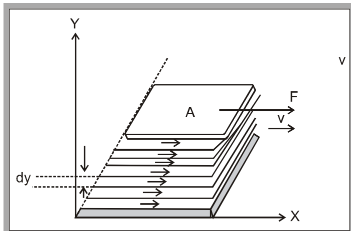
17. **CAPILLARY ACTION :** Capillary rise or fall.



$$h = \frac{2T}{R\rho g} = \frac{2T\cos\theta}{r\rho g}$$

$$\left(\text{as } R = \frac{r}{\cos\theta}\right)$$

18. VISCOSITY AND NEWTON'S LAW OF VISCOUS FORCE

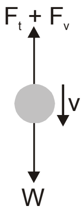


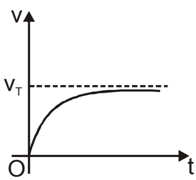
$$F \propto A\frac{dv}{dy}$$

or $$F = -\eta A\frac{dv}{dy}$$

The negative sign shows that viscous force on a liquid layer acts in a direction opposite to the relative velocity of flow of fluid. Above Eq. is known as Newton's law of viscous force.

19. TERMINAL VELCOITY (V_T)





$$v_T = \frac{2}{9}\frac{r^2(\rho-\sigma)g}{\eta}$$

PART-IV
OPTICS

1. PROPERTIES OF LIGHT

(i) Speed of light in vaccum, denoted by c, is equal to 3×10^8 m/s approximately

(ii) Light is electromagnetic wave (proposed by Maxwell). It consists of varying electric field and magnetic field.

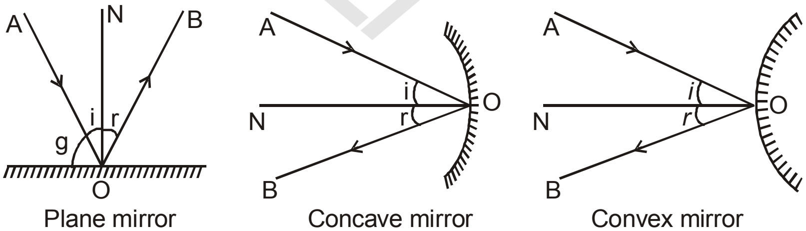
(iii) Light carries energy and momentum.

(iv) The formula $v = f\lambda$ is applicable to light.

2. REFLECTION

When a ray of light is incident at a point on the surface, the surface throws partly or wholly the incident energy back into the medium of incidence. This phenomenon is called reflection.

Surfaces that cause reflection are known as mirrors or reflectors. Mirrors can be plane or curved.



2.1 Law of reflection

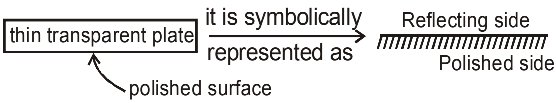
(i) The incident ray, the reflected ray and the normal to the reflecting surface at the point of incidence, all lie in the same plane.

(ii) The angle of incidence is equal to the angle of reflection, i.e., $\angle i = \angle r$

These laws hold good for all reflecting surfaces either plane or curved.

3. PLANE MIRROR

Plane mirror is formed by polishing one surface of a plane thin glass plate. It is also said to be silvered on one side.



PLANE MIRROR

A beam of parallel rays of light, incident on a plane mirror will get reflected as a beam of parallel reflected rays.

Formation of image by a plane mirror.

from the argument of similar triangles

OM = *I*M

i.e., perpendicular distance of the object from the mirror = perpendicular distance of the image from the mirror

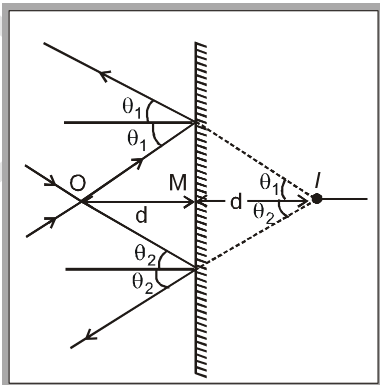
Steps to draw the image :

(1) Drop a perpendicular on the mirror and extend it on the back side of the mirror.

(2) Image always lie on this extended line

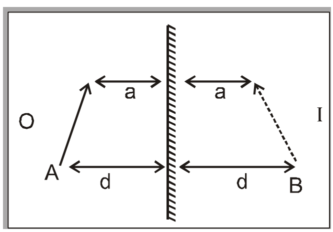
(3) To exactly locate the image, use the concept :

Perpendicular distance of the object from the mirror is equals to the perpendicular distance from the mirror of the image.



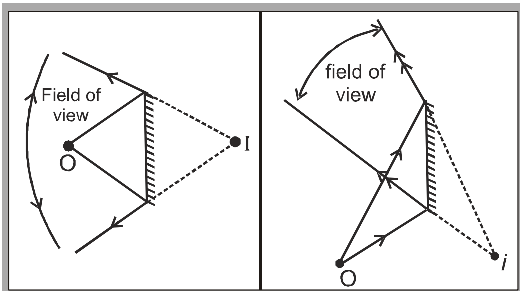
3.1 Image of an extended linear object :

Draw the images of the extreme points and join them with a straight line



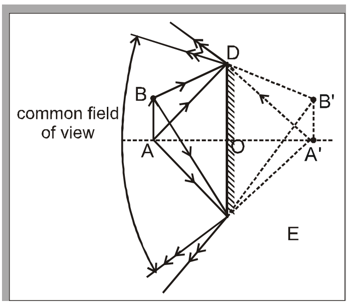
3.2 Field of view :

Area in which reflected rays exists is called field of view. It is the area from which an observer can see the image of an object. If the observer is outside this area he will not be able to see the image although the image will be there.



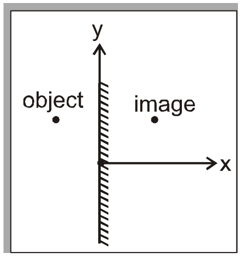
Most of the problems in optics involving geometry can be solved by using similar triangles.

3.3 Field of view of extended linear object



Common field of view of extreme points of the object will be the field of view of extended linear object

3.4 Relation between velocity of object and image :



$$\Rightarrow v_{mG} = \frac{v_{iG} + v_{oG}}{2}$$

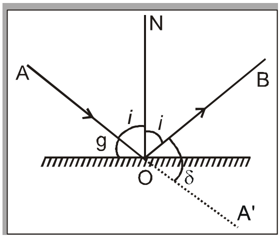
here : v_{iG} = velocity of image with respect to ground

v_{OG} = velocity of object with respect to ground.

v_{mG} = velocity of mirror with respect to ground.

☛ Valid only for perpendicular component of velocity to the mirror.

3.5 Deviation produced by a Plane mirror



Deviation is defined as the angle between directions of the incident ray d the reflected ray (or, the emergent ray). It is generally denoted by δ.

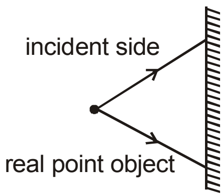
Here, $\angle A'OB = \delta = \angle AOA' - \angle AOB = 180° - 2i$

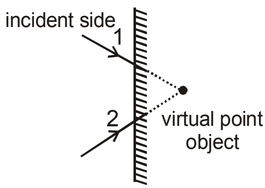
or, $\delta = 180° - 2i$

3.6 Real or virtual image/Object

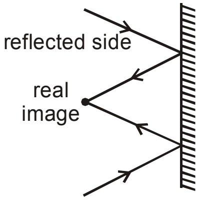
Object and Image

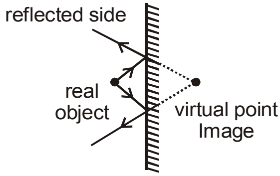
Object is defined as point of intersection of incident rays. Image is defined as point of intersection of reflected rays (in case of reflection) or refracted rays (in case of refraction).





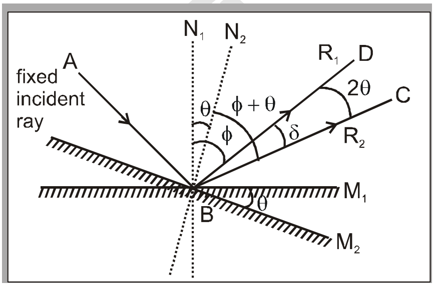
Rays 1 and 2 have originated from a point source





4. ROTATION OF MIRROR

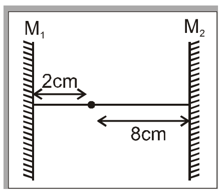
For a fixed incident light ray, if the mirror be rotated through an angle θ (about an axis which lies in the plane of mirror and perpendicular to the plane of incidence), the reflected ray turns through an angle 2θ in same sense.



5. IMAGES FORMED BY TWO PLANE MIRRORS

If rays after getting reflected from one mirror strike second mirror, the image formed by first mirror will function as an object for second mirror, and this process will continue for every successive reflection.

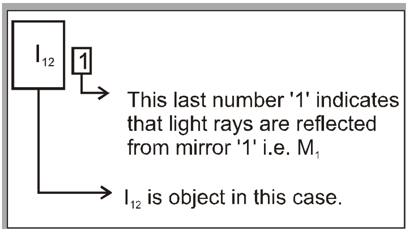
5.1 Images due to parallel plane mirrors :

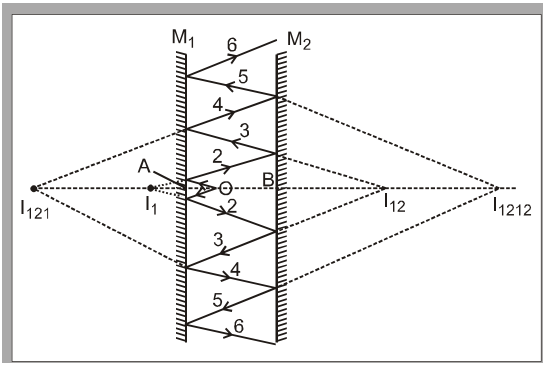


Ex. ***Figure shows a point object placed between two parallel mirrors. Its distance from M_1 is 2 cm and that from M_2 is 8 cm. Find the distance of images from the two mirrors considering reflection on mirror M_1 first.***

Sol. To understand how images are formed see the following figure and table.

You will require to know what symbols like I_{121} stands for. See the following diagram.

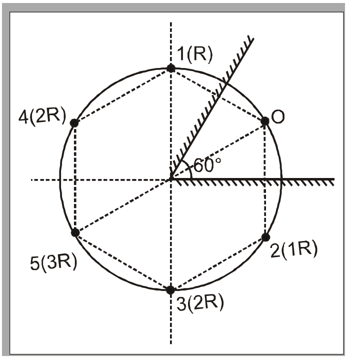




Incident rays	Ref.by	Ref. rays	Object	Image	Object distance	Image distance
Rays 1	M_1	Rays 2	O	I1	AO=2cm	AI_1=2cm
Rays 2	M_2	Rays 3	I1	I_{12}	BI_1=12cm	BI_{12}=12cm
Rays 3	M_1	Rays 4	I_{12}	I_{121}	AI_{12}=22cm	AI_{121}=22cm
Rays 4	M_2	Rays 5	I_{121}	I_{1212}	BI_{121}=32cm	BI_{1212}=32cm

Similarly images will be formed by the rays striking mirror M_2 first. Total number of images = ∞.

5.2 Circle concept



All the images formed will lie on a circle whose centre is the intersection point of the mirror and radius equal to distance of object from the intersection point

6. NUMBER OF IMAGES FORMED BY TWO INCLINED MIRRORS.

(i) if $\frac{360°}{\theta}$ = even number.; number of image = $\frac{360°}{\theta} - 1$

(ii) If $\frac{360°}{\theta}$ = odd number ;number of image = $\frac{360°}{\theta} - 1$,

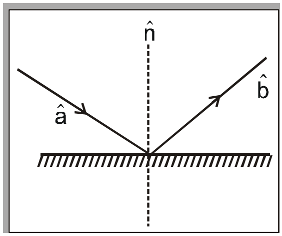
If the object is placed on the angle bisector.

(iii) If $\frac{360°}{\theta}$ = odd number ; number of image = $\frac{360°}{\theta}$,

If the object is not placed on the angle bisector.

(iv) If $\frac{360°}{\theta} \neq \text{integer}$, then the number of images = nearest even integer.

VECTOR - FORM



$\hat{a}$ = Unit vector along the incident ray

$\hat{n}$ = Unit normal vector

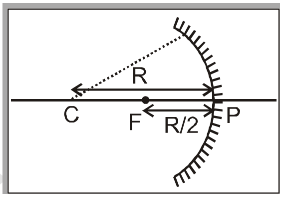
$\hat{b}$ = Unit vector along the reflected Ray

$$\hat{\mathbf{b}} = \hat{\mathbf{a}} - 2(\hat{\mathbf{a}}.\hat{\mathbf{n}})\hat{\mathbf{n}}$$

7. SPHERICAL MIRROR

7.1 RELATION BETWEEN u, v AND R FOR SPHERICAL MIRRORS

where R is the radius of the curvature of the mirror



$$\frac{1}{u} + \frac{1}{v} = \frac{2}{R}$$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

IMPORTANT POINT

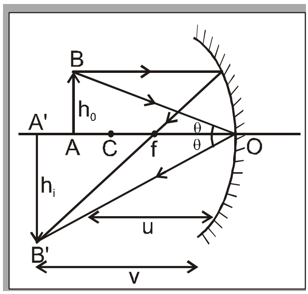
1. Above formula is only valid for paraxial rays

2. u, v and f should be put along with sign

7.2 MAGNIFICATION :

7.2.1 Transverse Magnification

Δ ABO ~ Δ A'B'O



$$x = \frac{h_i}{v} = \frac{h_0}{u} \quad \Rightarrow \quad m = \frac{h_i}{h_0} = -\frac{v}{u}$$

* The above formula is valid for both concave and convex mirror.

* Above the optical axis is considered positive and below to be negative

* h_i, h_0, v and u should be put with sign.

7.2.2 Longitudinal Magnification

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

By differentiating

$$\Rightarrow \qquad -\frac{dv}{v^2}-\frac{du}{u^2}=0 \Rightarrow \frac{dv}{du}=-\frac{v^2}{u^2}$$

Longitudinal magnification when the size of object is quite less with respect to its distance from the pole.

Above formula is valid only when the length of object is very small as compared to the distance of object from the pole.

dv $\rightarrow$ length of image

du $\rightarrow$ length of object

u $\rightarrow$ object distance from the pole.

v $\rightarrow$ Image distance from the pole.

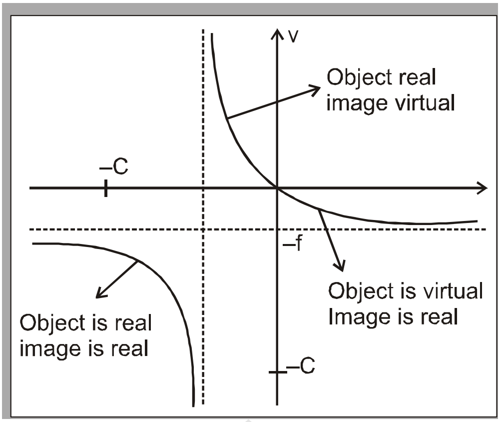
FOR CONCAVE MIRROR

S.No	Position of object	Details of images
1.	At ∞	At F, real, inverted $\|m\| << 1$
2.	Between C and ∞	Beetween F and C, real inverted, $\|m\| < 1$
3.	At C	At C, real, inverted $\|m\| = 1$
4.	Between F and C	Between C and ∞, real, inverted, $\|m\| > 1$
5.	At F	At infinity, real, inverted, $\|m\| >> 1$
6.	Between F and P	Behind the mirror, virtual, erect $\|m\| > 1$

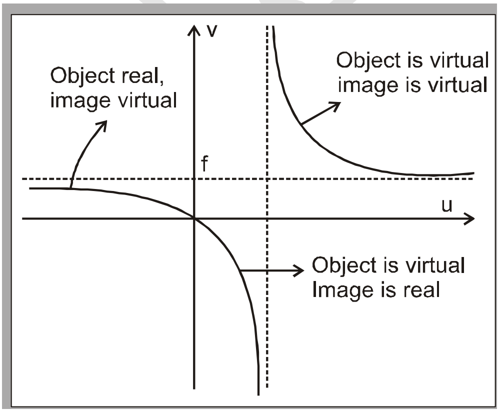
FOR CONCAVE MIRROR

S.No	Position of object	Details of images
1.	At infinity	At F, virtual, erect, $\|m\| << 1$
2.	In front of mirror	Beetween P and F, virtual, erect, $\|m\| < 1$

u - v curve for spherical mirror



FOR CONCAVE MIRROR



FOR CONVEX MIRROR

7.3 Velocity in Spherical Mirror :

Velocity of image

(a) Object moving along principal axis :

$$V_{IM} = -\frac{v^2}{u^2}(V_{OM})$$

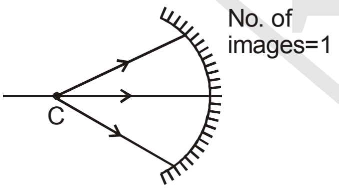
(b) Object moving perpendicular to principal axis :

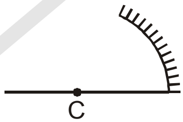
$$\frac{dh_i}{dt} = -\frac{v}{u}\frac{dh_o}{dt}$$

(c) Object moving parallel to Principal axis :

$$v_y = \frac{dh_i}{dt} = -h_0\left[\frac{dv}{dt}.\frac{1}{u} - \frac{v}{u^2}\frac{du}{dt}\right]$$

8. CUTTING OF MIRRORS





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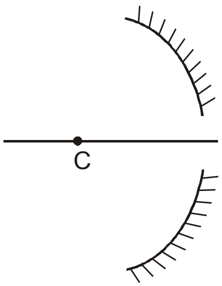

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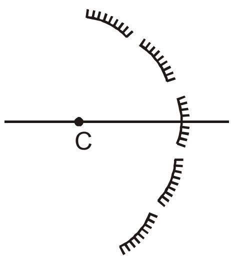

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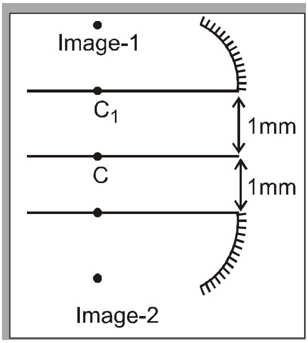


9. REFRACTION OF LIGHT

$$\mu = \frac{c}{v} = \frac{\text{speed of light in vacuum}}{\text{speed of light in medium}}$$

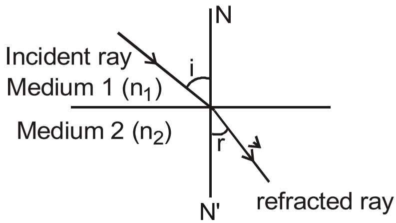
9.1 Laws of Refraction :

(a) The incident ray, the normal to any refracting surface at the point of incidence and the refracted ray all lie in the same plane called the plane of incidence or plane of refraction.

(b) $\frac{\sin i}{\sin r}$ = Constant for any pair of media

and for light of a given wavelength.

This is known as ***Snell's*** *Law.*



Also, $\frac{\sin i}{\sin r} = \frac{n_2}{n_1} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$

For applying in problems remember

$$\mathbf{n_1 \sin i = n_2 \sin r}$$

$\frac{n_2}{n_1} = {}_1n_2$ = Refractive Index of the second medium with respect to the first medium.

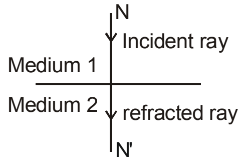
C = speed of light in air (or vacuum) = 3×10^8 m/s.

i & r should be taken from normal.

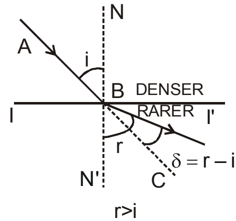
Special cases :

- Normal incidence : i = 0

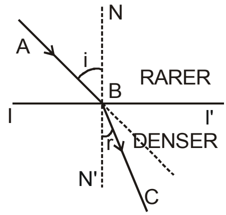
  from snell's law : r = 0



- When light moves from denser to rarer medium it bends away from normal.



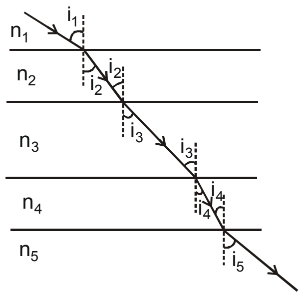
- When light moves from rarer to denser medium it bends towards the normal.



9.2 Plane Refraction :

Prove that $n_1 \sin i_1 = n_2 \sin i_2 = n_3 \sin i_3 = n_4 \sin i_4$

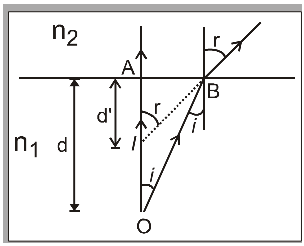
= **(Remember this).** Also Prove that if $n_1 = n_4$ then light rays in medium n_1 and in medium n_4 are parallel.



10. APPARENT DEPTH AND NORMAL SHIFT

Case I : When the object is in denser medium and the observer is in rarer medium (near normal incidence)

$$\frac{n_2}{n_1} = \frac{d'}{d} = \frac{\text{apparent depth}}{\text{Real depth}}$$



: 1. The above formula is valid only for paraxial rays.

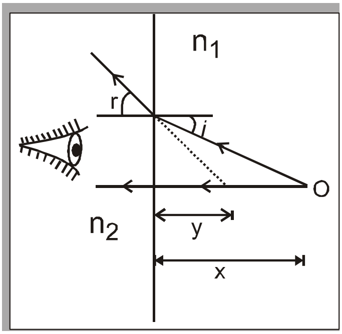
2. distances should be taken from surface

3. n_2 is the reflective index of the medium, where ray is going and n_1 from where ray is coming

10.1 Velocity of the image in case of plane refraction :

$$\frac{n_2}{n_1} = \frac{y}{x} \qquad \Rightarrow \quad y = \frac{n_2}{n_1}.x$$

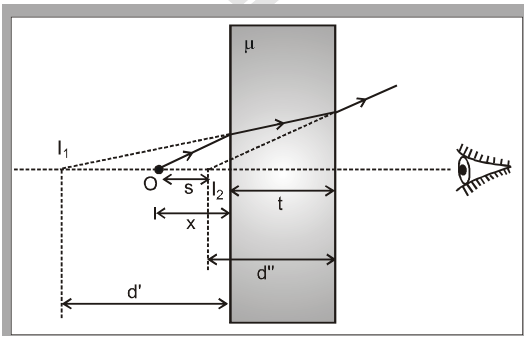
$$\frac{dy}{dt} = \frac{n_2}{n_2}\frac{dx}{dt} \qquad \Rightarrow \quad V_{IS} = \frac{n_2}{n_1}V_{OS}$$



11. REFRACTION THROUGH A GLASS SLAB

When a light ray passes through a glass slab having parallel faces, it gets refracted twice before finally emerging out of it.

11.1 Apparent shift due to slab when object is seen normally through the slab

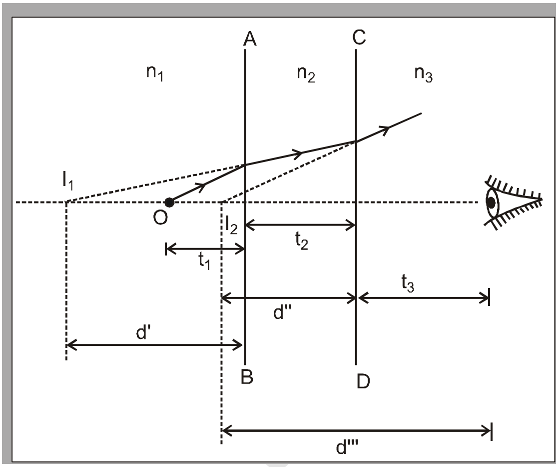


$$S = t\left[1 - \frac{\mu_{surroundig}}{\mu_{slab}}\right]$$

Important points

1. Rays should be paraxial
2. Medium on both side of the slab should be same.
3. Shift comes out from the object
4. Shift is independent of the distance of the object from the slab.
5. If shift comes out +ve then shift is towards the direction of incident rays and vice versa.

11.2 Apparent distance between object and observer when both are in different medium.

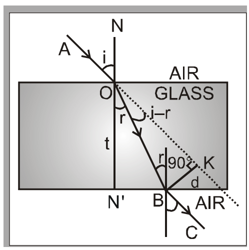


$$d''' = n_3\left[\frac{t_1}{n_1} + \frac{t_2}{n_2} + \frac{t_3}{n_3}\right]$$

If object and observer are in same medium then shift formula should be used and if both are in different medium then the above formula of apparent distance should be used.

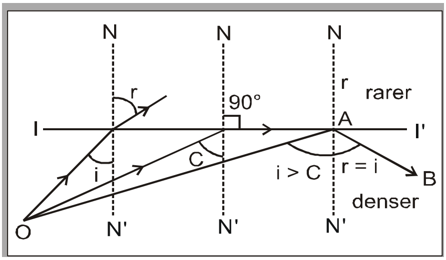
11.3 LATERAL SHIFT

$$d = \frac{t}{\cos r}\sin(i - r)$$



12. CRITICAL ANGLE AND TOTAL INTERNAL REFLECTION

Critical angle is the angle made in denser medium for which the angle of refraction in rarer medium is 90°. When angle in denser medium is more then critical angle the light ray re.flects back in denser medium following the laws of reflection and the interface behaves like a perfectly reflecting mirror. In the figure.



$$\therefore \qquad C = \sin^{-1}\frac{n_r}{n_d}$$

12.1 Conditions of T.I.R. :

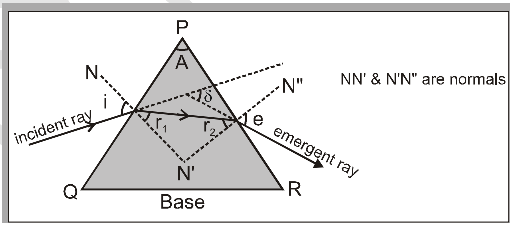
(a) light is incident on the interface from denser medium.

(b) Angle of incidence should be greater than the critical angle (i > c).

13. PRISM

(a) PQ and PR are refracting surfaces.

(b) $\angle QPR = A$ is called refracting angle or the angle of prism (also called Apex angle.)



(c) δ = angle of deviation

(d) For refraction of a monochromatic (single wave length) ray of light through a prism;

$$\delta = (i + e) - (r_1 + r_2)$$

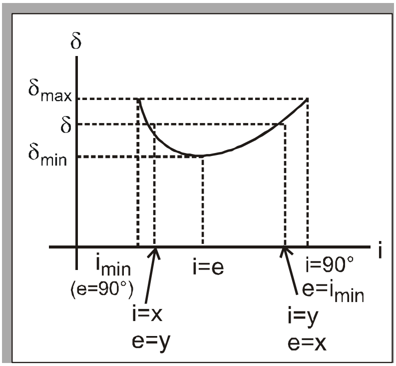
and $\qquad r_1 + r_2 = A$

$$\therefore \qquad \delta = i + e - A.$$

Note **I.** If ray crosses two surface which are inclined to each other then we use the concept of prism

II. If ray crosses two plain parallel surfaces then we use concept of slab.

13.1 Graph between ∠δ and ∠*i*



(1) Variation of δ versus i (shown in diagram).

For one δ (except δ min) there are two values of angle of incidence. If i and e are interchanged then we get the same value of δ because of reversibility principle of light

(2) There is one and only one angle of incidence for which the angle of deviation is minimum.

(3) Right hand side part of the graph is more tilted then the left hand side.

13.2 Minimum Deviation and Condition for Minimum Deviation :

The condition for minimum deviation is

$i = e$ and $r_1 = r_2$ ....(19)

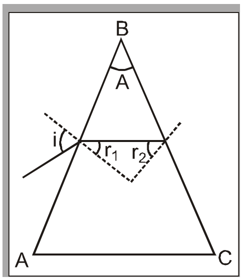
13.3 Relation Between Refractive index and the angle of Minimum Deviation

If surrounding medium has refractive index = n_s

then $$\frac{n_p}{n_s} = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\frac{A}{2}}$$

13.4 CONDITION FOR PRISM :

(a) Relation between prism angle A & critical angle C such that ray will always show T I R at BC :



For this $(r_2)_{min} > C$ ...(i)

For $(r_2)_{min}$, r_1 should be maximum and

for $(r_1)_{max} \Rightarrow i_{max} = 90°$

$(r_1)_{max} = C$

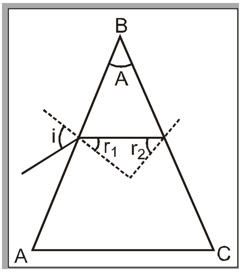
$(r_2)_{max} = A - C$

Now from eq. (i) $A - C > C$

$A > 2C$

i.e. $A > 2C$, all rays are reflected back from the second surface.

(b) The relation between A & C such that ray will always cross surface BC.



For this $(r_2)_{max} < C$

$(A - r_1)_{max} < C$

$A - (r_1)_{min} < C$ ..(2)

$(r_1)_{min} = 0$ when $i_{min} = 0$

from eq. (2) $A - 0 < C$

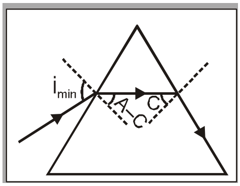
$A < C$

i.e. If $A \leq C$, no rays are reflected back from the second surface i.e. all rays are refracted from second surface.

(c) If $2C \geq A > C$, some rays are reflected back from the second surface and some rays are refracted from second surface, depending on the angle of incidence.

δ is maximum for two values of i

$\Rightarrow$ i_{min} (corresponding to e = 90º) and i = 90º (corresponding to e_{min}).



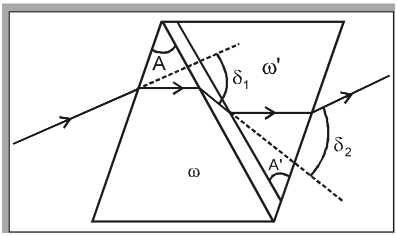
For i_{min} : $n_s \sin i_{min} = n_p \sin (A - C)$

If $i < i_{min}$ then T.I.R. takes pla.ce at second refracting surface PR.

13.5 Dispersive power

Dispersive power (ω) of the medium of the material of prism is given by :

$$\omega = \frac{n_v - n_r}{n_y - 1}$$



Dispersion without average deviation and average deviation without dispersion

Dispersion without Average Deviation

$$(\mu_y - 1)A = (\mu'_y - 1)A'$$

$$\delta_v - \delta_r = (\mu_y - 1)A(\omega - \omega')$$

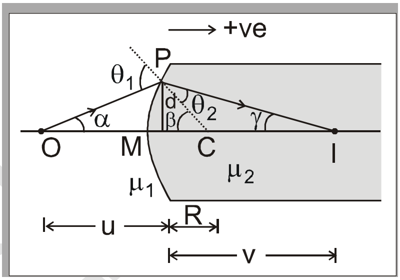
Average Deviation without Dispersion.

$$\frac{(\mu'_y - 1)}{(\mu_y - 1)}\frac{A'}{A} = \frac{\omega}{\omega'}$$

$$\delta = (\mu_y - 1)A\left(1 - \frac{\omega}{\omega'}\right)$$

14. REFRACTION FROM A SPHERICAL SURFACE

$$\boxed{\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}}$$

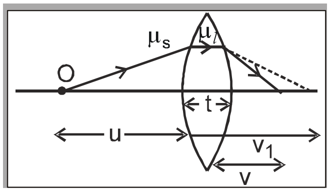


Important point for above formula

- Above formula is valid only for paraxial ray.
- u,v,R should be put along with sign
- μ_2 is r.i. of medium in which rays is going and μ_1 is the r.i. of medium from which rays are coming.

15. REFRACTION THROUGH THIN LENSES

15.1 Lens Maker formula



$$\frac{\mathbf{1}}{\mathbf{v}} - \frac{\mathbf{1}}{\mathbf{u}} = \left(\frac{\boldsymbol{\mu}_\ell}{\boldsymbol{\mu}_s} - \mathbf{1}\right)\left[\frac{\mathbf{1}}{\mathbf{R_1}} - \frac{\mathbf{1}}{\mathbf{R_2}}\right]$$

Impoartnt points for the above formula :

(1) Rays should be paraxial

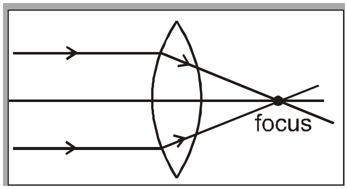
(2) v, u, R_1 and R_2 should be put with the sign.

(3) R_1 is the radius of curvature of that surface on which the ray strikes first.

(4) Lens should be thin.

(5) Medium on both sides of the lens should be same.

15.2 Focus :

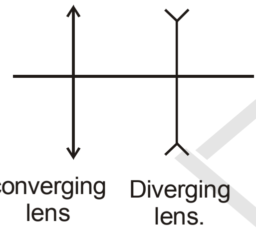


$$\frac{1}{f} = \left(\frac{\mu_\ell}{\mu_s} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Substituting in the lens maker formula :

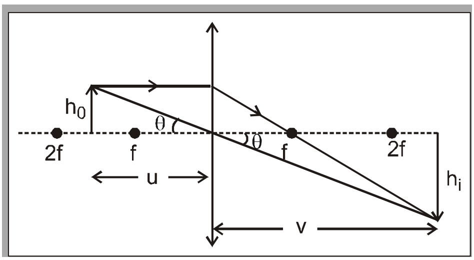
$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$ (lens formula)

➤ Focal length of lens depends on surrounding medium.



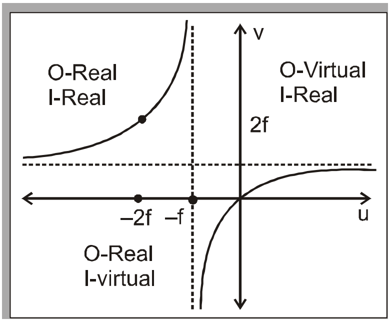
➤ If f = + ve implies converging and if f = – ve implies diverging lens.

15.3 Transverse Magnification

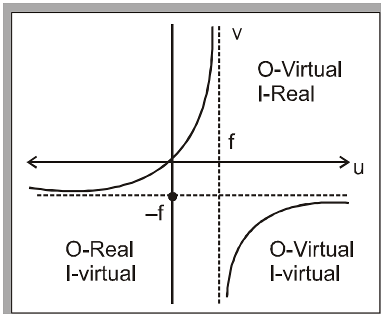


$$m = \frac{h_i}{h_0} = \frac{v}{u}$$

Graphs for converging lens



Graphs for diverging lenses.

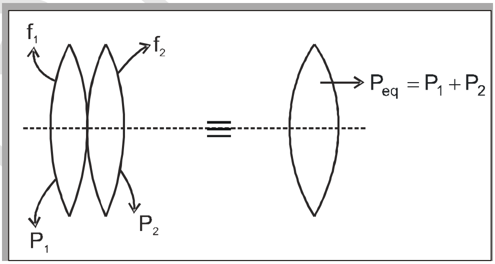


15.4 Velocity of the image formed by a lens

$$v_{IL} = \frac{v^2}{u^2} V_{OL}$$

15.5 COMBINATION OF LENS.

$$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2}$$



Important points :

(1) Rays should be paraxial

(2) Lens should be thin

(3) Lenses should be kept in contact

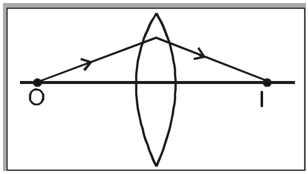
(4) f_1, f_2, f_3, ... should be put with sign.

(5) f_1, f_2, f_3 .... are the focal length of lenses in the surrounding medium.

(6) If f_{eq} = + ve then system will behave as converging system.

If f_{eq} = – ve then system will behave as diverging system.

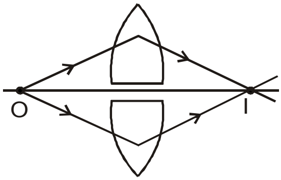
16. CUTTING OF LENS.

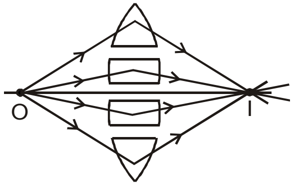


16.1 Parallel Cutting

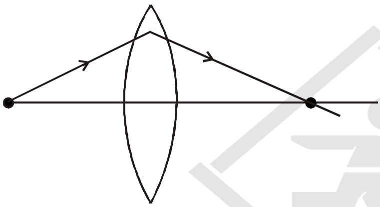
No. of images in all the cases = 1

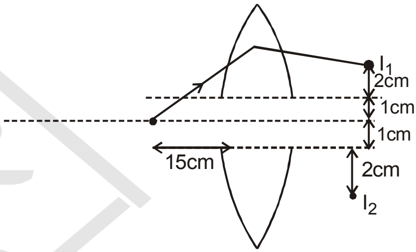
$\because$ Principle axis does not shift





(ii)





f = 10 cm

16.2 Power of a lens.

$$\textbf{Power} = \frac{1}{f}\ \textbf{(diopter)}$$

where f $\rightarrow$ meter and f should be put with sign

power of converging lens. = + ve Power of diverging lens. = – ve

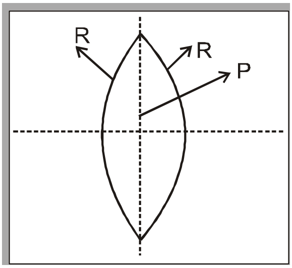
16.3 Power of Mirror

$$\textbf{Power} = -\frac{1}{f}\ \textbf{(diopter)}$$

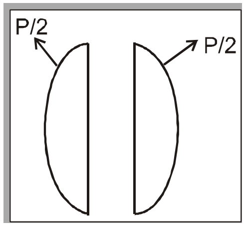
where $f \rightarrow$ meter and f should be put with sign.

Power of converging mirror = + ve Power of diverging mirror = – ve

16.4 Perpendicular cutting

(1) 

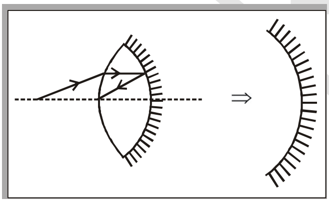
$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R} - \left(-\frac{1}{R}\right)\right) \Rightarrow \frac{1}{f} = (\mu - 1)\frac{2}{R}$$



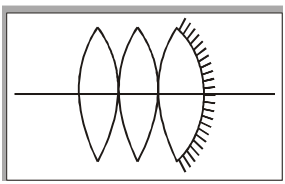
$$\frac{1}{f_1} = (\mu - 1)\left(\frac{1}{R} - 0\right) = \frac{(\mu - 1)}{R}$$

$f' = 2f$

17. SILVERGING OF LENS.



$$\mathbf{P_{eq} = 2P_\ell + P_m} \Rightarrow -\frac{1}{f_{eq}} = \frac{2}{f_\ell} - \frac{1}{f_m}$$



$$\Rightarrow \quad \frac{1}{f_{eq}} = \frac{1}{f_m} - \frac{2}{f_\ell} \qquad \frac{1}{f_{eq}} = \frac{1}{f_m} - 2\left[\frac{1}{f_{\ell_1}} + \frac{1}{f_{\ell_2}} + ...\right]$$

Important points :

(1) Rays should be paraxial

(2) Lenses should be thin

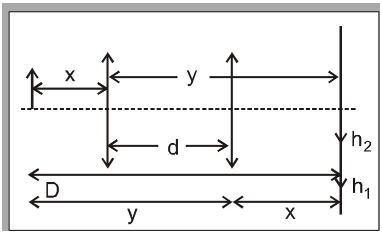
(3) All the lenses should be in contact.

(4) f_ℓ, f_m should be put along with the sign.

(5) If f_{eq} = – ve $\Rightarrow$ concave, f_{eq} = + ve $\Rightarrow$ convex,

If $f_{eq} = \infty$ $\Rightarrow$ plane mirror

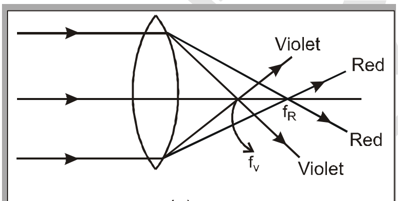
18. DISPLACEMENT METHOD



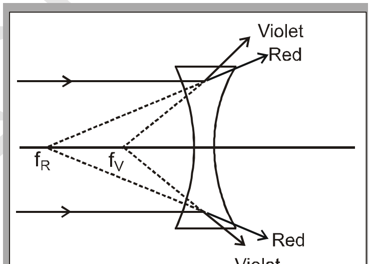
$$h_0 = \sqrt{h_1 h_2}$$

$$f = \frac{D^2 - d^2}{4D}$$

19. CHROMATIC ABERRATION AND ACHROMATISM



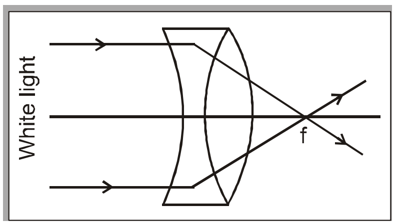
(a)



(b)

CONDITION FOR ACHROMATISM

$$\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0$$



WAVE OPTICS

1. WAVE FRONT

- Wave front is a locus of particles having same phase.
- Direction of propagation of wave is perpendicular to wave front.
- Every particle of a wave front act as a new source & is known as secondary wavelet.

Coherent source

If the phase difference due to two source at a particular point remains constant with time, then the two sources are considered as coherent source.

2. INTERFERENCE :

$\Rightarrow$ $$A_{net}{}^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\phi$$

$\therefore$ $$I_{net} = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\phi \quad (\text{as } I \propto A^2)$$

For I_{net} to be maximum

$\cos\phi = 1 \Rightarrow \phi = 2n\pi$

where $n = \{0,1,2,3,4,5..........\}$

$$\frac{2\pi}{\lambda}\Delta x = 2n\pi \Rightarrow \Delta x = n\lambda$$

For constructive interference

$$I_{net} = (\sqrt{I_1} + \sqrt{I_2})^2$$

When $I_1 = I_2 = I$

$I_{net} = 4I$

$A_{net} = A_1 + A_2$

For I_{net} to be minimum,

$\cos\Delta\phi = -1$

$\Delta\phi = (2n + 1)\pi$ where $n = \{0,1,2,3,4,5..........\}$

$$\frac{2\pi}{\lambda}\Delta x = (2n + 1)\pi \Rightarrow \Delta x = (2n+1)\frac{\lambda}{2}$$

For destructive interfence

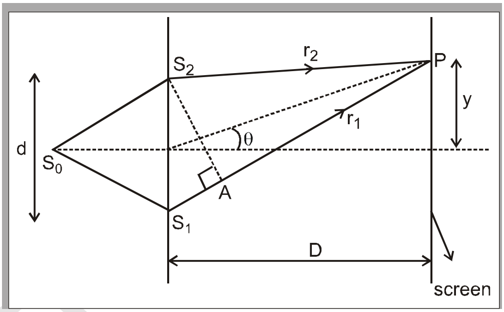
$$I_{net} = (\sqrt{I_1} - \sqrt{I_2})^2$$

If $I_1 = I_2$ $\quad I_{net} = 0 \quad A_{net} = A_1 - A_2$

3. YOUNG'S DOUBLE SLIT EXPERIMENT (Y.D.S.E.) :

Path difference

$\Delta p = S_1P - S_2P$



$$= \sqrt{\left(y + \frac{d}{2}\right)^2 + D^2}$$

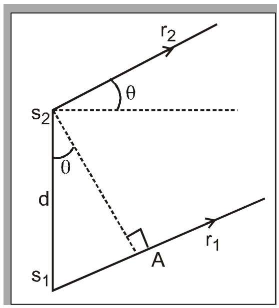
$$- \sqrt{\left(y - \frac{d}{2}\right)^2 + D^2} \qquad ...(1)$$

Approximation I :

For D >> d, we can approximate rays $\vec{r_1}$ and $\vec{r_2}$ as being approximately parallel, at angle θ to the principle axis.

Now, $S_1P - S_2P = S_1A = S_1 S_2 \sin\theta$

$\Rightarrow$ path difference = $d \sin\theta$ ...(2)



Approximation II :

further if θ is small, i.e.,

$y << D, \sin\theta \simeq \tan\theta = \dfrac{y}{D}$

and hence, path difference = $\dfrac{dy}{D}$ ...(3)

for maxima

$$\Delta p = \frac{d.y}{D} = n\lambda$$

for minima

$$\Delta p = \pm\frac{\lambda}{2}, \pm\frac{3\lambda}{2} \pm\frac{5\lambda}{2}$$

4. FRINGE WIDTH :

$$\beta = \frac{\lambda D}{d}$$

Note : As vertical distance y is related to θ by $\theta = \frac{y}{D}$ so $\Delta\theta = \frac{\Delta y}{D}$ which is referred as angular fringe width

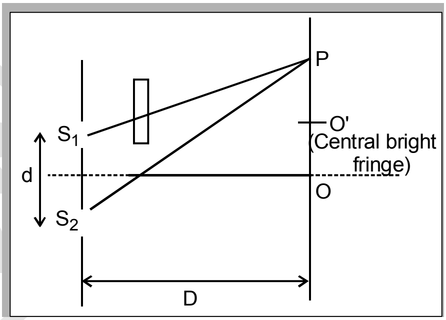
$$B_\theta = \frac{\beta}{D} = \frac{\lambda}{d}$$

5. ON INTRODUCTION OF A GLASS SLAB IN THE PATH OF THE LIGHT COMING OUT OF THE SLITS –

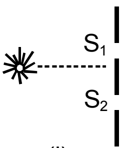
for central bright fringe;

$$\Delta p = 0$$

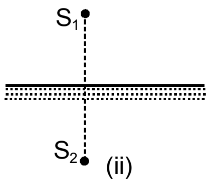
$$\Rightarrow \quad \frac{yd}{D} = t(\mu - 1)$$



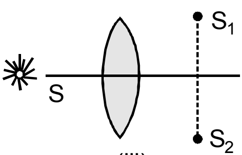
A single source is split in two coherent sources to obtain sustained interference in light. This can be done in following different ways.



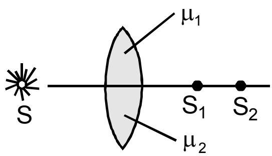
(i)



(ii)



(iii)



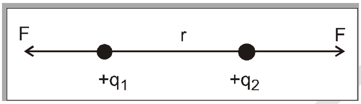
(iV)

PART-V
ELECTROMAGNETISM

ELECTROSTATICS

1. COULOMB'S LAW :

$$F = \frac{Kq_1q_2}{r^2} \Rightarrow K = \frac{1}{4\pi\varepsilon_0} = 9\times10^9\ Nm^2/c^2$$



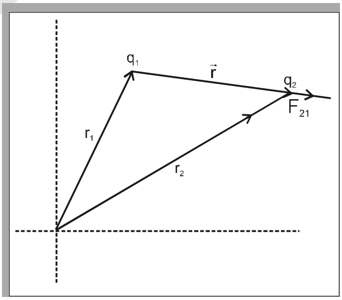
This force F acts along the line joining the two charges and is repulsive if q_1 and q_2 are of same sign and is attractive if they are of opposite sign.

1.1 Vector forms of Coloumb's law

(F_{21} : force on q_2 due to q_1)

$$\vec{F}_{21} = \frac{k\,q_1q_2}{r^2}\hat{r} = \frac{k\,q_1q_2}{r^3}\vec{r}$$

$$\vec{F}_{21} = \frac{k\,q_1q_2}{\left[\vec{r}_2 - \vec{r}_1\right]^3}\left(\vec{r}_2 - \vec{r}_1\right)$$



- Head of $\vec{r}$ points at that position where force has to be calculated.
- $\vec{r}_2$ & $\vec{r}_1$ depend on origin but $\vec{r}$ does not.
- q_1 and q_2 should be put along with sign.

1.2 Coloumb's law in a medium :

$$F_{med} = \frac{1}{4\pi \in_0 \in_r}\frac{q_1q_2}{r^2}$$

or $\quad F_{med} = \dfrac{F_{air}}{\in_r} \quad$ as $\quad \in_r > 1 \quad \Rightarrow \quad F_{med} < F_{air}$

2. ELECTRIC FIELD :

If a charge q_0 placed at a point in electric field, experiences a net force $\vec{F}$ on it, then electric field strength at that point can be

or $$\vec{E} = \frac{\vec{F}}{q_0} \qquad ....(1)$$

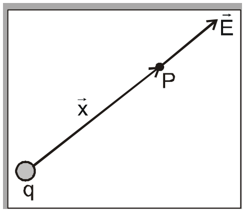
[$q_0 \rightarrow$ test charge]

(a) Electric Field Strength due to Point Charge :

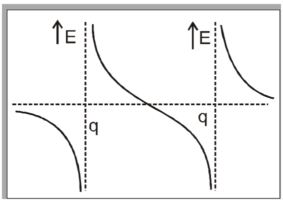
$$E = \frac{Kq}{x^2}$$

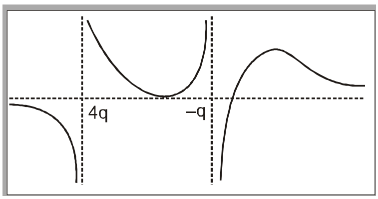
(b) Vector Form of Electric field due to a Point Charge :

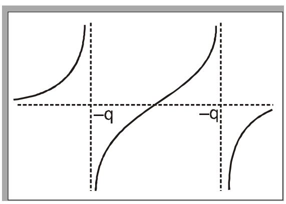
$$\vec{E} = \frac{Kq}{x^3}.\vec{x}$$



2.1 Graph of electric field due to binary charge configuration

1.


2.


3.


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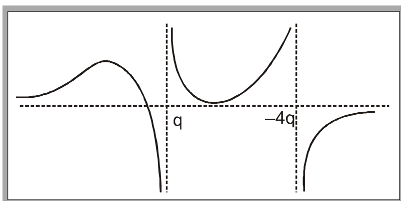
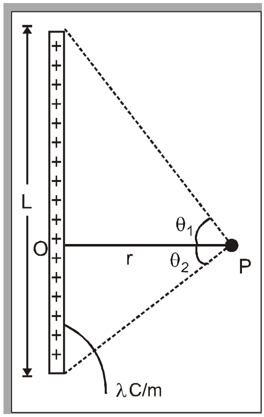


2.2 Electric field Strength at a General Point due to a Uniformly Charged Rod :



in ||-direction $E_x = \frac{KQ}{Lr}(\cos\theta_2 - \cos\theta_1)$

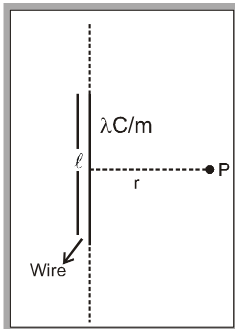
$$= \frac{k\lambda}{r}(\cos\theta_2 - \cos\theta_1)$$

in $\perp$-direction $E_y = \frac{KQ}{Lr}(\sin\theta_1 + \sin\theta_2)$

$$= \frac{k\lambda}{r}(\sin\theta_1 + \sin\theta_2)$$

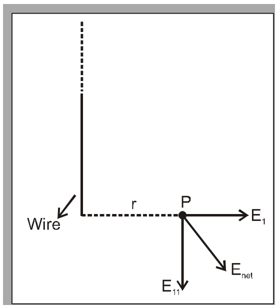
- r is the perpendicular distance of the point from the wire
- θ_1 and θ_2 should be taken in opposite sense

2.3 Electric field due to infinite wire ($\ell >> r$)



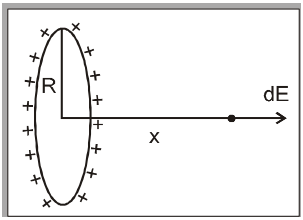
$$E_{net} \text{ at } P = \frac{k\lambda}{r}(1+1) = \frac{2k\lambda}{r}$$

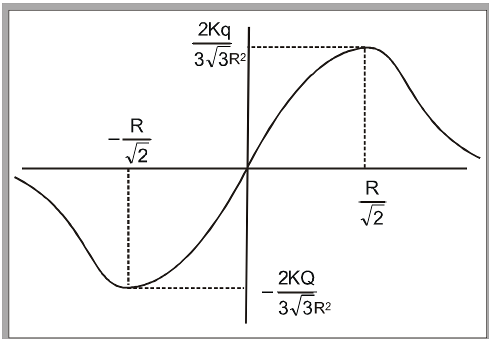
2.4 Electric field due to semi infinite wire



$$E_{net} \text{ at } P = \frac{\sqrt{2}k\lambda}{r}$$

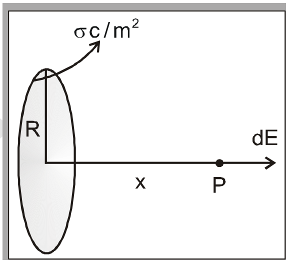
2.5 Electric field due to Uniformly Charged Ring :





$$E_P = \frac{KQx}{(R^2 + x^2)^{3/2}}$$ (along the axis)

2.6 Electric field Strength due to a Uniformly Surface Charged Disc :



$$E = \frac{\sigma}{2\in_0}\left[1 - \frac{x}{\sqrt{x^2 + R^2}}\right]$$ [along the axis]

Case (i) : If x >> R

$$E = \frac{KQ}{x^2}$$

Case (ii) : If x << R

$$E = \frac{\sigma}{2\varepsilon_0}[1 - 0] = \frac{\sigma}{2\varepsilon_0}$$

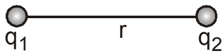
i.e. behaviour of the disc is like ***infinite sheet.***

2.7 Electric Field Strength due to a Uniformly charged Hollow Hemispherical Cup :

$$E_0 = \frac{\sigma}{4\in_0}$$ (along the axis)

3. ELECTROSTATIC POTENTIAL ENERGY :

3.1 Interaction Energy of a System of Two Charged Particles :

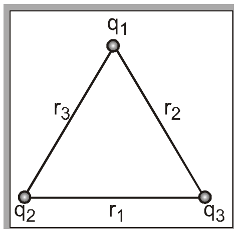


$$W = \frac{Kq_1q_2}{r} = U \qquad \text{[Interaction energy]}$$

If the two charges here are of opposite sign, the potential energy will be negative as

$$U = -\frac{Kq_1q_2}{r}$$

3.2 Potential Energy for a System of charged Particles :



$$U = \frac{Kq_1q_2}{r_3} + \frac{Kq_1q_3}{r_2} + \frac{Kq_2q_3}{r_1}$$

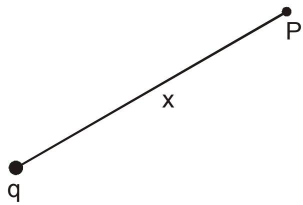
4. ELECTRIC POTENTIAL :

If at a point in electric field a charge q_0 has potential energy U, then electric potential at that point can be given as

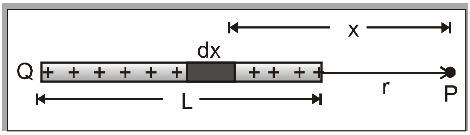
$$V = \frac{U}{q_0} \text{ joule/coulomb}$$

(a) Electric Potential due to a Point Charge in its Surrounding :

$$V_P = \frac{Kq}{x}$$

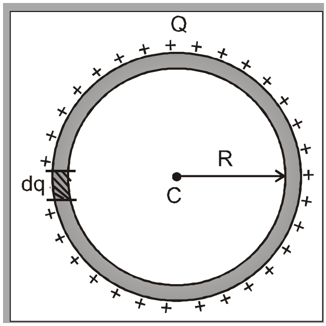


(b) Electric Potential due to a Charge Rod :



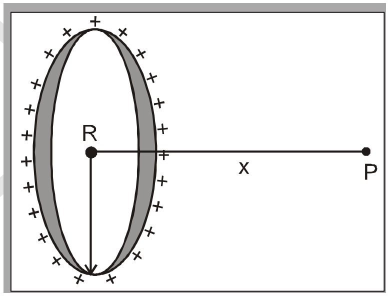
$$V = \frac{KQ}{L}\ell n\left(\frac{r+L}{r}\right)$$

(c) Electric Potential due to a Charged Ring :



Case I : At its centre

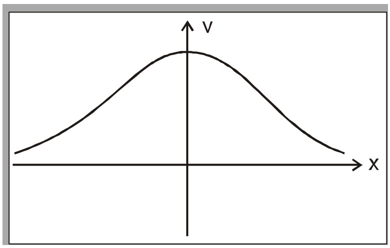
$$V = \frac{KQ}{R}$$



Case II : At a Point on Axis of Ring

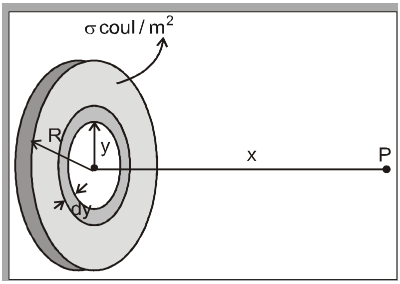
$$V_P = \frac{KQ}{\sqrt{R^2 + x^2}}$$

GRAPH



(d) Electric Potential due to a Uniformly Charged Disc :

$$V_P = \frac{\sigma}{2\epsilon_0}[\sqrt{x^2+R^2} - x]$$



5. RELATION BETWEEN ELECTRIC FIELD INTENSITY AND ELECTRIC POTENTIAL :

(a) For uniform electric field :

$$V_B - V_A = -\vec{E}.\vec{AB}$$

(b) Non uniform electric field

$$E = -\left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right]V = -\nabla V = -\text{grad } V$$

☛ Area under E - x curve gives negative of change in potential.

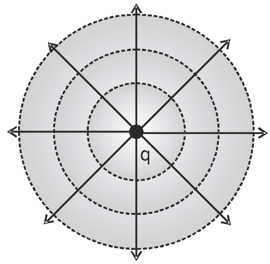
☛ Negative of slope of V - x curve gives the electric field at that point.

6. ELECTRIC LINES OF FORCE

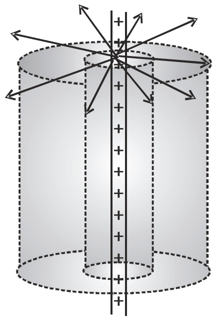
- Electric lines of forces originate from +ve charge or infinite and terminate on –ve charge or infinity
- If density of lines is high then electric field intensity will also high and vice versa.
- Tangent at any point on the electric field line will give u direction of force or electric field that's why these lines are known as electric lines of forces.
- Two electric field lines never cut each other because at the point at intersection there is two positon direction of electric field which is not possible.
- Electric field lines does not exist in closed loop because electrostatic force is a conservation force
- No. of lines originating or terminating is directly proportional to the magnitude of charge.

7. EQUIPOTENTIAL SURFACES :

1. Equipotential surface is the locus of points where potential of all the points is same.
2. Potential difference between any two points of the same equipotential surface is zero that's why work done by electric field on a charge moving from one point to another point on same equipotential surface is zero.
3. E.f lines is always perpendicualr to the equipotential surface.
4. If we take a particle from one equipotential surface to another equipotential surface then there is work done due to electric forces.
5. Electric field lines move from higher equipotential surface to lower equipotential surface.



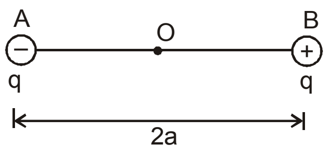
Point charge
Spherical equipotential surfaces



Line charge
Cylindrical equipotential surfaces

8. ELECTRIC DIPOLE

Electric dipole $\vec{P} = q.2\vec{a}$, Vector quantity SI unit Cm



Direction of dipole moment($\vec{p}$) is form negative charge to positive charge

9.(a) Electric Field on Axial Line of an Electric Dipole

$$E = \frac{1}{4\pi\varepsilon_0} \frac{2qr}{(r^2 - a^2)^2}$$

When a << r, $\boxed{E_{axial} = \frac{1}{4\pi\varepsilon_0} \frac{2p}{r^3}}$

$\boxed{E_{axial} \text{ is along the direction of dipole moment}}$

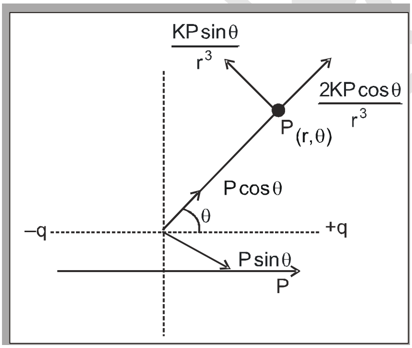
9.(b) Electric Field on Equatorial Line of an Electric Dipole

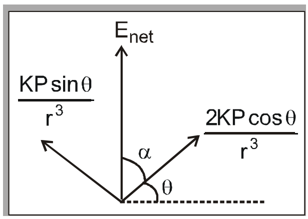
$$E = \frac{1}{4\pi\varepsilon_0} \frac{q \times 2a}{(r^2 + a^2)^{3/2}}$$ When a << r. $\boxed{E_{equiatoral} = \frac{1}{4\pi\varepsilon_0} \frac{P}{r^3}}$

$E_{equiatoral}$ is along the negative direction of dipole moment.

- Remembers, when a is very small, dipole is known as 'point dipole'.

9.(c) Electric field at a general Point due to a dipole





$$E_{net} = \sqrt{\left(\frac{2KP\cos\theta}{r^3}\right)^2 + \left(\frac{KP\sin\theta}{r^3}\right)^2} = \frac{kP}{r^3}\sqrt{1 + 3\cos^2\theta}$$

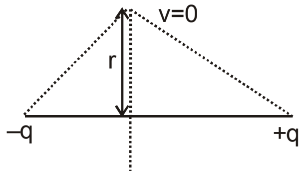
$$\tan\alpha = \frac{\tan\theta}{2}$$

9.(d) Electric potential due to a dipole.

1. At an axial point

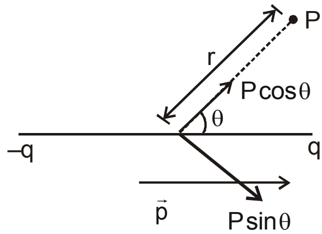
$$V_{net} = \frac{kp}{r^2} \text{ (As P = 2aq)}$$

2. At a point on perpendicular bisector



(3) Potential due to dipole at a general Point

Potential at P due to dipole = $\frac{kp\cos\theta}{r^2}$



10. Dipole in a Uniform External Electric Field

A system of equal and opposite charges separated by a small distance is called a dipole.

(a) In vector form, $\vec{\tau} = \vec{p} \times \vec{E}$

(b) $U = -pE\cos\theta$

At $\theta = 0°$, $U = U_{min}$ At $\theta = 180°$, $U = U_{Max}$

$U = -\vec{p}.\vec{E}$

(c) Dipole Moment, p = q × 2a cm

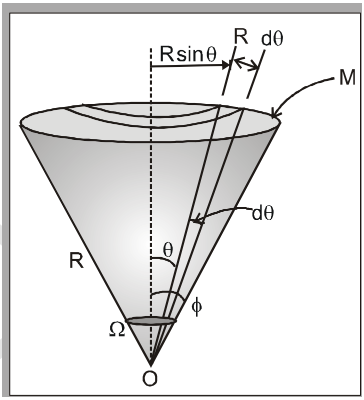
$\vec{p} = q \times 2a\ \hat{p}$

ELECTROSTATICS-2

1. ELECTRIC FLUX

$$\text{Flux} = \phi = \int \vec{E}.d\vec{A}, \qquad \text{If } \vec{E} \text{ is constant, } \phi = \vec{E}.\vec{A}$$

- Basically flux is the count of number of lines of electric field crossing an area.
- For open surface we choose one direction as a area vector & stick to it for the whole problem

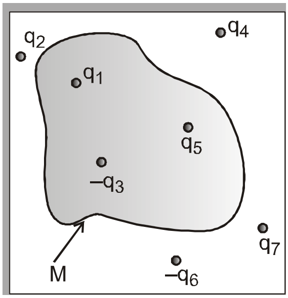


(2) Relation in Half Angle of cone and Solid Angle at Vertex :

$\Omega = 2\pi\,(1 - \cos\phi)$

Ω = solid angle ϕ = half angle of cone

3. GAUSS' LAW



$$\phi_E = \oint \vec{E}.d\vec{s} = \frac{q_{enclosed}}{\varepsilon_0}$$

$$\oint_M \vec{E}.d\vec{S} = \frac{q_1 + q_5 - q_3}{\epsilon_0}$$

[Gauss's Law]

(a) Electric field near a thin flat sheet of charge

$E = \dfrac{\sigma}{2\varepsilon_0}$, where σ = surface charge density

(b) Electric field near a conductor of any shape

$$E = \frac{\sigma}{\varepsilon_0}$$

4. CONDUCTOR

TYPE OF MATERIALS

- conductors (All electrons are free)
- Semi-conductors (Some electrons are free)
- Insulators (all electrons are bounded)

(i) Conductors are materials which contains large number of free electrons which can move freely inside the conductor.

(ii) In electrostatics conductors are always equipotential surfaces.

(iii) Charge always resides on outer surface of conductor.

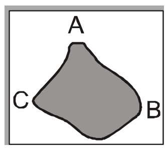
(iv) If there is a cavity inside the conductor having no charge then charge will always reside only on outer surface of conductor.

(v) Electric field is always perpendicular to conducting surface.

(vi) Electric lines of force never enter into conductors.

(vii) Electric field intensity near the conducting surface is given by formula

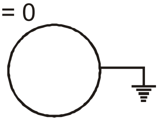
$$\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$$



σ is the local charge density.

$$\vec{E}_A = \frac{\sigma_A}{\varepsilon_0}\hat{n} \; ; \; \vec{E}_B = \frac{\sigma_B}{\varepsilon_0}\hat{n} \text{ and } \vec{E}_C = \frac{\sigma_C}{\varepsilon_0}\hat{n}$$

(viii) When a conductor is grounded its potential becomes zero.



(ix) When an isolated conductor is grounded then its charge becomes zero.

(x) When two conductors are connected there will be charge flow till their potential becomes equal.

(xi) Electric pressure : Electric pressure at the surface of a conductor is given by formula $P = \dfrac{\sigma^2}{2\varepsilon_0}$ where σ is the local surface charge density.

5. ELECTRIC FIELD DUE TO SPHERICAL BODIES

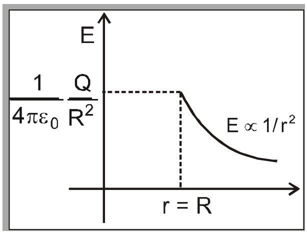
- Conducting hollow sphere
- Conducting Solid sphere
- Non Conducting Hollow sphere
- Non Conducting Solid sphere

(Conducting hollow sphere, Conducting Solid sphere, Non Conducting Hollow sphere:) Will behave in the same fashion because $\vec{E}$ & potential depends on charge distribution & all the above three spheres have same

(a) Electric field due to a charged spherical shell of radius R

Inside = 0,

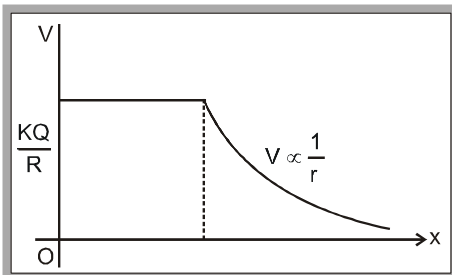
outside = $\dfrac{q}{4\pi\varepsilon_0 r}$



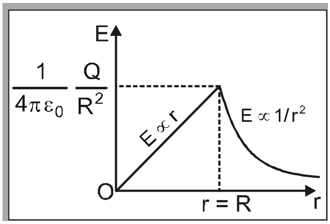
(b) Potential due to a charged spherical shell of radius R

inside = $\dfrac{q}{4\pi\varepsilon_0 R}$,

outside = $\dfrac{q}{4\pi\varepsilon_0 r}$



(c) Electric field due to a solid sphere of charge density ρ



inside = $\frac{KQr}{R^3}$, outside = $\frac{Q}{4\pi\varepsilon_0 r^2}$

r is distance of point from origin.

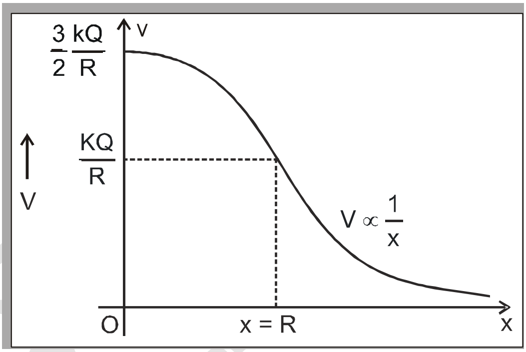
(d) Potential due to a solid sphere of charged density ρ, and charge Q : –

inside = $\frac{KQ}{2R^3}[3R^2 - r^2]$

outside = $\frac{KQ}{r}$

at the centre = $\frac{3}{2}\frac{KQ}{R}$

r = distance of point from the centre of sphere.



(e) Self energy of a Hollow, conducting, solid conducting & hollow non conducting sphere.

$$U = \frac{KQ^2}{2R}$$

(f) Self Energy of a Uniformly Charged Non-conducting Sphere :

$$U_{self} = \frac{3}{5}\frac{KQ^2}{R}$$

6. CHARGE INDUCTION IN METAL CAVITIES :

(1) Whenever a charge is placed inside a metal cavity, an equal and opposite charge is induced on the inner surface of cavity.

(2) A similar charge is induced on the outer surface of body with surface charge density inversely proportional to radius of curvature of body

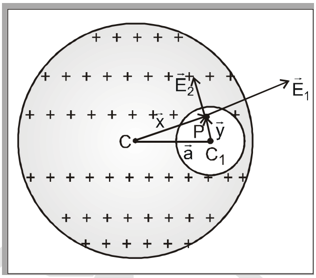
(3) When the charge inside is displaced, the induced charge distribution on inner surface of body changes in such a way that its centre of charge can be assumed to be at the point charge so as to nullify the electric field in outer region.

(4) Due to movement in the point charge inside the body. The charge distribution on outer surface of body does not change.

(5) If another charge is brought to the body from outside, it will only affect the outer distribution of charges not on the charge distribution inside the cavity.

(a) Electric Field Inside a Cavity of Non-conducting Charged Body :

$$\vec{E}_{net} = \frac{\rho\vec{a}}{3 \in_0}$$

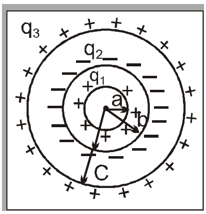


7. Potential due to concentric spheres

(i) At a point r > c

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q_1 - q_2 + q_3}{r}$$

(ii) At a point b < r < c



$$V = \frac{1}{4\pi\varepsilon_0}\frac{q_1 - q_2}{r} + \frac{1}{4\pi\varepsilon_0}\frac{q_3}{c}$$

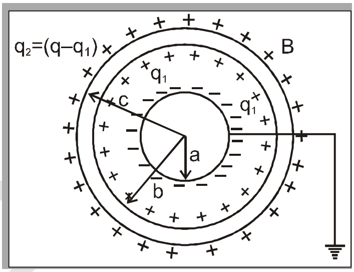
(iii) At a point a < r < b

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r} - \frac{1}{4\pi\varepsilon_0}\frac{q_2}{b} + \frac{1}{4\pi\varepsilon_0}\frac{q_3}{c}$$

(iv) At a point r < a

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{a} - \frac{q_2}{b} + \frac{q_3}{c}\right]$$

8. Potential difference between the two concentric spheres when one of them is earthed



(i) $V = \frac{1}{4\pi\varepsilon_0} q\left[\frac{1}{a} - \frac{1}{b}\right]$

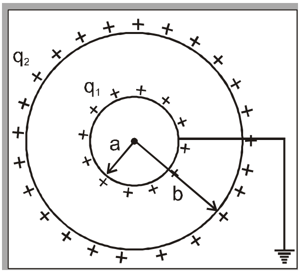
(ii) $V = \frac{1}{4\pi\varepsilon_0} q_1\left[\frac{1}{a} - \frac{1}{b}\right]$

$$\frac{q_2}{c} = q_1\left(\frac{1}{a} - \frac{1}{b}\right) \quad .....(i)$$

$$q_1 + q_2 = q \quad ....(ii)$$

Solving (i) and (ii) we can get q_1 and q_2

9. POTENTIAL DIFFERENCE BETWEEN TWO CONCENTIC UNIFORMLY CHARGED METALLIC SPHERES.

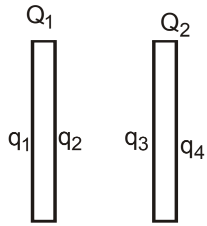


$$V_{in} = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{a} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{b}$$

$$V_{out} = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{b} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{b}$$

$$\Delta V = V_{in} - V_{out} \Rightarrow \Delta V = \frac{q_1}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

10. COMBINATION OF CONDUCTING PLATES :



where $q_1 + q_2 = Q_1$

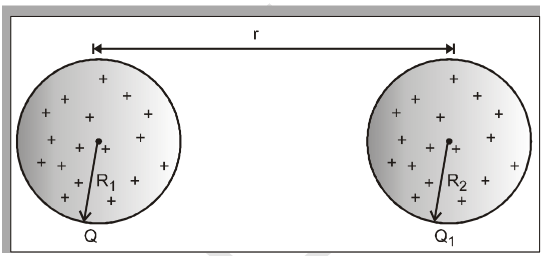
$q_3 + q_4 = Q_2$

$q_2 = -q_3$

$$q_1 = q_4 = \frac{Q_1 + Q_2}{2}$$

11. TOTAL ELECTROSTATIC ENERGY OF A SYSTEM OF CHARGES :

Total electrostatic potential energy of system of charges can be given as U = $\sum$ self energy of all charged bodies + $\sum$ Interaction energy of all pairs of charged bodies



$$U = U_{self} + U_{interaction}$$

$$U = \frac{3KQ_1^2}{5R_1} + \frac{3KQ_2^2}{5R_2} + \frac{KQ_1Q_2}{r}$$

CURRENT ELECTRICITY

1. ELECTRIC CURRENT

$$I = \lim_{\Delta t \to 0} \frac{\Delta q}{\Delta t} = \frac{dq}{dt}$$

2. ELECTRIC CURRENT IN A CONDUCTOR

$$I = nAev_d$$

$$v_d = \frac{\lambda}{\tau}$$

$$v_d = \frac{\frac{1}{2}\left(\frac{eE}{m}\right)\tau^2}{\tau} = \frac{1}{2}\frac{eE}{m}\tau$$

v_d = drift velocity

E = electric field

e = electronic charge

m = mass of electron

τ = relaxation time

λ = mean free path.

3. CURRENT DENSITY

$$\vec{J} = \frac{di}{ds}\vec{n} \qquad \int di = \int \vec{J}.d\vec{s}$$

So, $di = \vec{J}.d\vec{s}$

4. ELECTRICAL RESISTANCE

$$i = neAv_d = neA\left(\frac{eE}{2m}\right)\tau = \left(\frac{ne^2\tau}{2m}\right)AE$$

$$i = \left(\frac{ne^2\tau}{2m}\right)\left(\frac{A}{\ell}\right)V = \left(\frac{A}{\rho\ell}\right)V = \frac{V}{R} \quad \Rightarrow V = IR$$

$$V = I \times \frac{\rho \ell}{A}$$

$$E = \frac{V}{\ell} = \frac{I\rho}{A} \Rightarrow E = J\rho \quad J = \text{current density}$$

$$\rho = \frac{2m}{ne^2\tau} = \frac{1}{\sigma}, \quad \sigma \text{ is called conductivity}$$

5. DEPENDENCE OF RESISTANCE ON VARIOUS FACTORS

$$R = \frac{\rho\ell}{A} = \frac{2m}{ne^2\tau}.\frac{\ell}{A}$$

Various cases

(a) On stretching a wire (volume constant)

If length of wire is changed then $\frac{R_1}{R_2} = \frac{\ell_1^2}{\ell_2^2}$

(b) If radius of cross section is changed then $\frac{R_1}{R_2} = \frac{\ell_2^4}{\ell_1^4}$, where R_1 and R_2 are initial and final resistance and ℓ_1, ℓ_2 are initial and final lengths.

and r_1 and r_2 are the initial and final radii respectively.

(c) **Effect of percentage change in length of wire**

$$\frac{R_2}{R_1} = \frac{\ell^2\left[1 + \frac{x}{100}\right]^2}{\ell^2}$$

where ℓ is original length and x is % increment

If x is quite small (say < 5%) then % change in R is

$$\frac{R_2 - R_1}{R_1} \times 100 = \left\{\left(1 + \frac{x}{100}\right)^2 - 1\right\} \times 100 = 2x\%$$

6. TEMPERATURE DEPENDENCE OF RESISTIVITY AND RESISTANCE

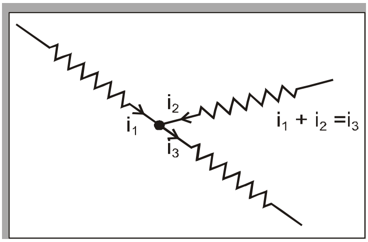
$\rho(T) = \rho_0 [1 + \alpha(T - T_0)]$

$R(T) = R_0 [1 + \alpha(T - T_0)]$

The $\rho - T$ equation written above can be derived from the relation

$$\rho = \rho_0 e^{\alpha(T - T_0)}$$

7 KIRCHHOFF'S CURRENT LAWS



(a) The sum of the currents that flow into a junction is equal to the sum of the currents that flow out of the junction.

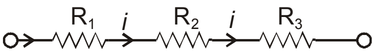
(b) The sum of the emf's around a loop is equal to the sum of the IR potential drops around the loops.

In writing $\sum IR = 0$, then current into a node (or junction) is taken to be positive.

In applying Kirchhoff's voltage law ($\sum E = \sum IR$), if there are more than one source when the directions do not agree, the voltage of source is taken as positive, if it is in the direction of assumed current.

8. RESISTANCE IN SERIES

When the resistance (or any type of elements) are connected end to end then they are said to be in series. The current through each element is same.



$R_{eq} = R_1 + R_2 + R_3 + ...+ R_n$

9. RESISTANCE IN PARALLEL

A parallel circuit of resistor is one in which the same voltage is applied across all the components, in a parallel grouping of resistors R_1, R_2, R_3

$$\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} \qquad \boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

Note :

In parallel combination

(a) Potential difference across each resistor is same

(b) $I = I_1 + I_2 + I_3 + ...+ I_n$

(c) Effective resistance (R) then

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ...\frac{1}{R_n}.$$

(R_{eq} is less than each resistor)

(d) Current in different resistors is inversely proportional to the resistance.

$$I_1 : I_2 : ....: I_n = \frac{1}{R_1} : \frac{1}{R_2} : \frac{1}{R_3} : .... : \frac{1}{R_n}$$

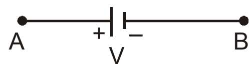
$$I_1 = \frac{G_1}{G_1 + G_2 + \ldots + G_n} I,\ I_2 = \frac{G_2}{G_1 + G_2 + \ldots + G_n} I, \text{ etc}$$

where $G = \frac{1}{R}$ = conductance of a resistor, [Its unit is Ω^{-1}]

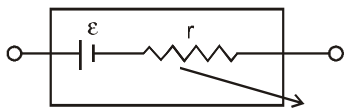
10. ELECTROMOTIVE FORCE (E.M.F.) AND POTENTIAL DIFFERENCE ACROSS A BATTERY.

(a) **Representation for battery**

Ideal cell : Cell in which there is no heating effect.

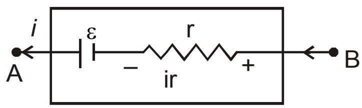


Non ideal cell : Cell in which there is heating effect inside due to opposition to the current flow internally.



(b) **Battery acting as a source (or battery is discharging)**

$V_A - V_B = \varepsilon - ir$



$V_A - V_B \Rightarrow$ it is also called terminal voltage

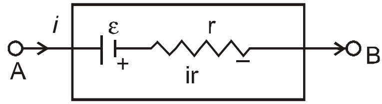
The rate at which the chemical energy of the cell is consumed = ε i

The rate of which heat is generated inside the battery or cell = i^2 r

electric power output = $\varepsilon i - i^2 r = (\varepsilon - ir) i$

(c) Battery acting as a load (or battery charging)

$V_A - V_B = \varepsilon + ir$



the rate at which chemical energy stored in the cell = ε/thermal power generated inside the cell = i^2 r electric power input = $\varepsilon i + i^2 r = (\varepsilon + ir) i$

$= (V_A - V_B) i$

(d) Electromotive force of a cell is equal to potential difference between its terminals when no current is passing throgh the circuit.

When cell in open circuit

i = 0 as resistance of open circuit is infintie (∞)

So,

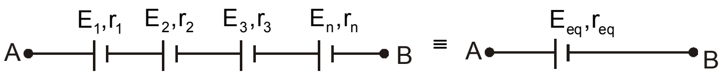
V = ε, so open circuit terminal potential difference is equal to emf of the cell.

(e) Short circuiting : Two points, in an electric circuit directly connected by a conducting wire are called short circuited, under such condition both points are at same potential.

$i = \frac{\varepsilon}{r}$ and V = 0, short circit current of a cell is maximum.

11. GROUPING OF CELLS

(a) Cells in series



Equivalent EMF

$E_{eq} = E_1 + E_2 + ...+ E_n$

[write EMF's with polarity]

Equivalent internal resistance

$r_{eq} = r_1 + r_2 + r_3 + r_4 + ...+ r_n$

In n cells each of emf E are arranged in series and if r is internal resistance of each cell, then total emf = nE

so current in the circuit I = $\dfrac{nE}{R + nr}$

If nr << R then I = $\dfrac{nE}{R}$ $\rightarrow$ Series combination is advantageous

If nr >> R then I = $\dfrac{E}{r}$ $\rightarrow$ Series combination is not advantageous

(b) Cells in parallel

$$E_{eq} = \frac{\varepsilon_1/r_1 + \varepsilon_2/r_2 + ... + \varepsilon_n/r_n}{1/r_1 + 1/r_2 + ... + 1/r_n}$$

$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + ... + \frac{1}{r_n}$$

If m cells each of emf E and internal resistance r be connected in parallel and if this combination is connected to an external resistance then the emf of the circuit = E.

internal resistance of the circuit = $\dfrac{r}{m}$ and $I = \dfrac{E}{R + \dfrac{r}{m}} = \dfrac{mE}{mR + r}$

If mR << r ; I = $\dfrac{mE}{r}$ $\rightarrow$ Parallel combination is advantageous

If mR > > r ; I = $\dfrac{E}{R}$ $\rightarrow$ Parallel combination is not advantageous

(c) Cells is multiple Arc

n = number of rows

m = number of cells in each row

Current $I = \dfrac{mE}{R + \dfrac{mr}{n}}$

for maximum current nR = mr

12. ELECTRICAL POWER

$$\text{Power} = \frac{V.dq}{dt} = VI \qquad P = I^2R = VI = \frac{V^2}{R}$$

$$W = VIt = I^2\,Rt = \frac{V^2}{R}t \qquad H = I^2\,Rt \text{ Joule} = \frac{I^2Rt}{4.2}\text{calorie}$$

13. GALVANOMETER

BINA sin θ = C ϕ

Here I $\propto$ ϕ

B = magnetic field

A = area of the coil

I = current

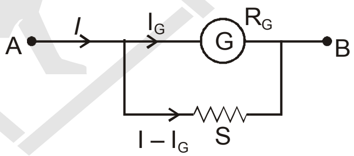
C = torsoinal constant

N = number of turns

ϕ = angle rotated by coil

14. AMMETER

A shunt (small resistance) is connected is parallel with galvanometer to convert it into ammeter.



$$S = \frac{I_G R_G}{I - I_G}$$

15. VOLTMETER

A high resistance is put in series with galvanometer. It is used to measure potential difference across a resistor in a circuit.

$$I_G = \frac{V}{R_G + R}$$

16. POTENTIOMETER

Potential gradient (x) $\rightarrow$ potential difference per unit length of wire

$$x = \frac{V_A - V_B}{L} = \frac{\varepsilon}{R + r} \cdot \frac{R}{L}$$

Application of potentiometer

(a) To find emf of unknown cell and compare emf of two cells

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{\ell_1}{\ell_2}$$

(b) To find current if resistance is known

$$I = \frac{x\ell_1}{R_1}$$

(c) To find the internal resistance of a cell

$$r' = \left[\frac{\ell_1 - \ell_2}{\ell_2}\right] R$$

17. METER BRIDGE (USED TO MEASURE UNKNOWN REISTANCE)

$$x = \frac{100 - \ell}{\ell} R$$

CAPACITANCE

1. CAPACITOR

$q \propto V$ For $q = CV$

$V = 1, q = C$

The SI unit is Farad. The practical unit is μF (Micro Farad)

$$1\ \text{Farad} = \frac{1\text{Coulomb}}{1\text{volt}} = 1CV^{-1}$$

2. Capacitance of an Isolated Sphere

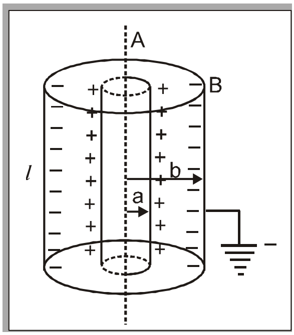
$$C = \frac{q}{V} = 4\pi\varepsilon_0 r$$

3. Calculation of Capacitance

$$\therefore \quad C = \frac{q}{V_{ab}} = \frac{q}{-\int_b^a \vec{E}.\vec{dr}}$$

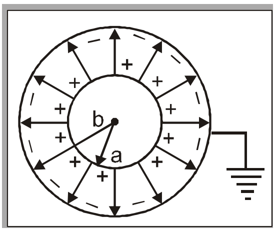
4. Cylindrical Capacitor

$$C = \frac{q}{V} = \frac{2\pi\varepsilon_0 l}{\left(\ln\frac{b}{a}\right)}$$



5. Spherical capacitor

$$C = \frac{Q}{V} = \frac{4\pi\varepsilon_0 ab}{b-a}$$



6. Parallel Plate Capacitor : $C = \frac{\varepsilon_0 A}{d}$

7. Energy Stored in a Charged Capacitor

$$U = \frac{q^2}{2C}$$

Putting, q = CV, $U = \frac{1}{2}CV^2$

Putting, $C = \frac{q}{V}$, $U = \frac{1}{2}qV$

8. Heat Generated :

(1) Work done by battery

W = QV Q = charge flow the battery V = EMF of battery

(2) W = +Ve (When Battery discharging)

W = –Ve (When Battery charging)

(3) $\because$ Q = CV (C = equivalent capacitance)

so W = CV × V = CV^2

Now energy on the capacitor $= \frac{1}{2}CV^2$

$\therefore$ Energy dissipated in form of heat (due to resistance)

Heat = Work done by battery — {final energy of capacitor - initial energy of capacitor}

9. DISTRIBUTION OF CHARGES ON CONNECTING TWO CHARGED CAPACITORS :

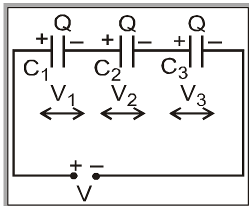
(a) Common potential :

$$V = \frac{Q_1 + Q_2}{C_1 + C_2} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} = \frac{\text{Total charge}}{\text{Total capacitance}}$$

(b) Heat loss during redistribution :

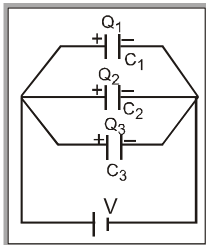
$$\Delta H = U_i - U_f = \frac{1}{2}\frac{C_1C_2}{C_1+C_2}(V_1 - V_2)^2$$

10. Capacitors in Series



$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

11. Capacitors in Parallel



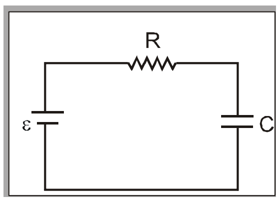
$$C = C_1 + C_2 + C_3$$

12. R - C CIRCUIT

Note : At time t = 0 (initial state) capacitor will behave as a short circuit wire.

At time t = ∞ (Final state or Steady state) capacitor will behave as a open circuit wire.

12.1 Charging of a condenser :



$$q = q_0[1 - e^{-(t/RC)}]$$

Where q_0 = maximum final value of charge at t = ∞.

- If t = RC = τ then

$$q = q_0\,[1 - e^{-(RC/RC)}] = q_0\left[1 - \frac{1}{e}\right]$$

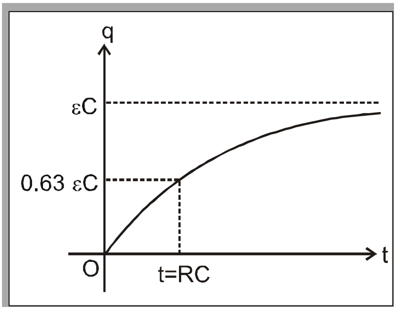
or $q = q_0\,(1 - 0.37) = 0.63\,q_0$

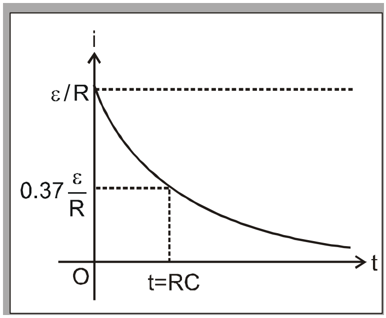
$= 63\%$ of q_0

- Time t = RC is known as time constant.

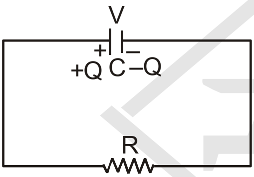
- The potential difference across the condenser plates at any instant of time is given by

$V = V_0[1 - e^{-(t/RC)}]$ volt $\qquad I = I_0[e^{-(t/RC)}]$ ampere

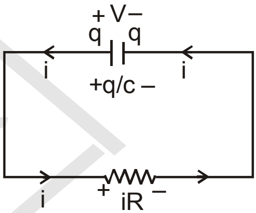




12.2 Discharging of a condenser :



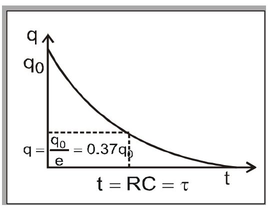
Initially

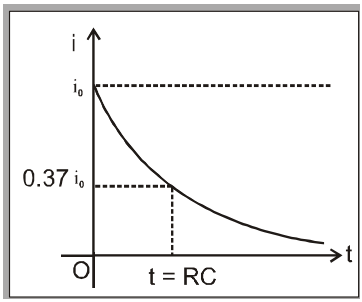


At any time t

The quantity of charge on the condenser at any instant of time t is given by $q = q_0 e^{-(t/RC)}$

i.e. the charge falls exponentially. $\qquad i = -\frac{dq}{dt} = \frac{Q}{RC}e^{-t/RC}$



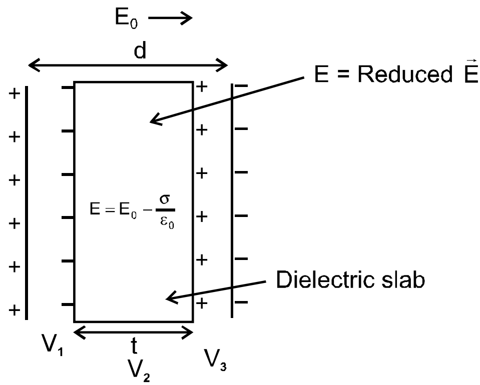


13. CAPACITOR WITH DIELECTRIC SLAB

$$C = \frac{q}{V}$$

$$= \frac{q}{\frac{q}{\varepsilon_0 A}\left(d - t + \frac{t}{K_D}\right)}$$

$$= \frac{\varepsilon_0 A}{d - t + \frac{t}{K_D}}$$

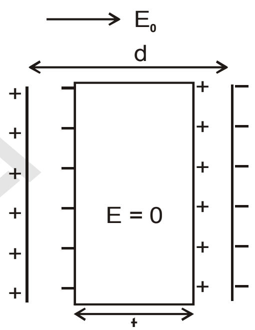


Capacitance of parallel plate capacitor with dielectric slab between plates

14. Capacitance of a Parallel Plate Capacitor with Conducting Slab between Plates

$$V = E_0\,(d - t)$$

$$\therefore \quad C = \frac{\varepsilon_0 A}{d\left(1 - \frac{t}{d}\right)} = \frac{C_0}{\left(1 - \frac{t}{d}\right)}$$



15. Introduction of a Dielectric slab of dielectric constant K between the plates

(a) When the battery is disconnected

(i) Charge remain constant, i.e., $q = q_0$, as in an isolated system charge is conserved.

(ii) Capacity increases, i.e., $C = KC_0$, as by the presence of a dielectric capacity becomes K times.

(iii) Potential difference between the plates decreases, i.e., $V = \left(\frac{V_0}{K}\right)$, as

$$V = \frac{q}{C} = \frac{q_0}{KC_0} = \frac{V_0}{K} \quad [\because\ q = q_0 \text{ and } C = KC_0]$$

(iv) Field between the plates decreases,

i.e., $E = \frac{E_0}{K}$, as

$$E = \frac{V}{d} = \frac{V_0}{Kd} = \frac{E_0}{K} \quad [\text{as } V = \frac{V_0}{K}] \quad \text{and} \quad E_0 = \frac{V_0}{d}$$

(v) Energy stored in the capacitor decreases i.e.

$U = \left(\frac{U_0}{K}\right)$, as

$$U = \frac{q^2}{2C} = \frac{q_0^2}{2KC_0} = \frac{U_0}{K} \quad (\text{as } q = q_0 \text{ and } C = KC_0)$$

b) When the battery remains connected (potential is held constant)

(i) Potential difference remains constant, i.e., $V = V_0$, as battery is a source of constant potential difference.

(ii) Capacity increases, i.e., $C = KC_0$, as by presence of a dielectric capacity becomes K times.

(iii) Charge on capacitor increases, i.e., $q = Kq_0$, as

$q = CV = (KC_0)V = Kq_0$ $[\because q_0 = C_0V]$

(iv) Electric field remains unchanged, i.e., $E = E_0$, as $E = \frac{V}{d} = \frac{V_0}{d} = E_0$ [as $V = V_0$

and $\frac{V_0}{d} = E_0$]

(v) Energy stored in the capacitor increases, i.e.,

$U = KU_0$, as

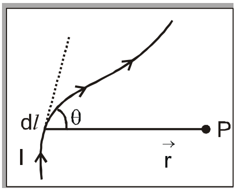
$$U = \frac{1}{2}CV^2 = \frac{1}{2}(KC_0)(V_0)^2 = KU_0$$

[as $C = KC_0$ and $U_0 = \frac{1}{2}C_0V_0^2$]

MAGNETISM

1. BIOT SAVART LAW

It states that the magnetic field strength (dB) produced due to a current element (of current I and length dl) at a point having position vector $\vec{r}$ relative to current element is



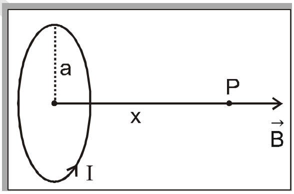
$$\vec{dB} = \frac{\mu_0}{4\pi}\frac{I\vec{dl}\times\vec{r}}{r^3}$$

μ_0 = permeability of free space = $4\pi \times 10^{-7}$

where θ is the angle between current element $I\vec{dl}$ and position vector $\vec{r}$. The direction of magnetic field $\vec{dB}$ is perpendicular to plane containing $I\vec{dl}$ and $\vec{r}$.

2. MAGNETIC FIELD DUE TO A CIRCUIT COIL

The magnetic field due to current carrying circular coil of N-turns, radius a, carrying current I at a distance x from the centre of coil is

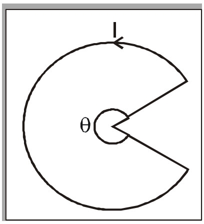


$$B = \frac{\mu_0 N I a^2}{2(a^2 + x^2)^{3/2}}$$ along the axis.

At centre x = 0

$$\therefore \quad B_C = \frac{\mu_0 NI}{2a}$$

In general the field produced by a circular arc subtending an angle θ at centre



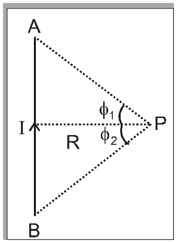
$$B_C = \frac{\mu_0 I}{2a}\cdot\frac{\theta}{2\pi}$$ (θ in radian)

3. AMPERE'S CIRCUITAL LAW

It states that the line integral of magnetic field induction along a closed path is equal to μ_0 - times the current enclosed by the path i.e.,

$$\oint \vec{B}.d\vec{l} = \mu_0 I$$

4. MAGNETIC FIELD DUE TO A STRAIGHT CONDUCTOR CARRYING A CURRENT



The magnetic field due to a straight current carrying wire of finite length is

$$B = \frac{\mu_0 I}{4\pi R}(\sin\phi_1 + \sin\phi_2)$$

The direction of magnetic field is given by right hand grip rule.

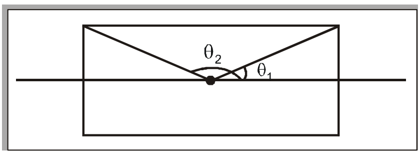
Special cases : If the wire is infinitely long. then $\phi_1 = \pi/2$, $\phi_2 = \pi/2$

$$B = \frac{\mu_0 I}{2\pi R}$$

If point is near one end of a long wire,

($\phi_1 = \frac{\pi}{2}$, $\phi_2 = 0$), then $B = \frac{\mu_0 I}{4\pi R}$

5. Magnetic field due to a Solenoid



$$B = \frac{\mu_0 n i}{2}[\cos\theta_2 - \cos\theta_1]$$

(i) At the axis of a long solenoid

$\theta_1 = 0$, $\theta_2 = \pi$

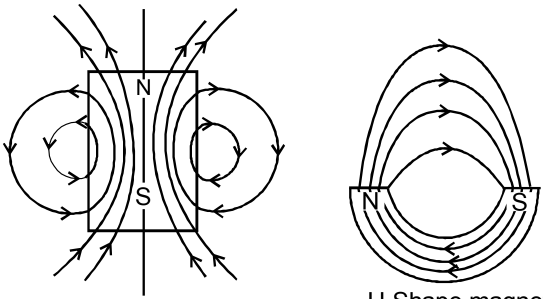
$B = \mu_0 n I$

where n = number of turns per metre length.

Magnetic field at one end of solenoid

$$B_{end} = \frac{\mu_0 n I}{2}$$

6. **MAGNETIC LINES AND THEIR CHARACTERISTICS**



Bar-magnet
(A)

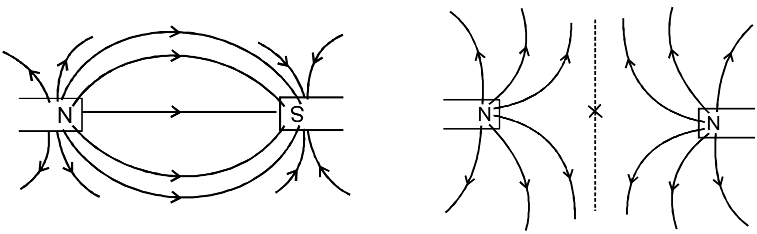
U-Shape magnet
(B)

(i) Outside a magnet, field are from north to south pole while inside from south to north, i.e., magnetic lines are closed curves i.e., they appear to converge or diverge at poles.

(ii) The number of magnetic lines of field originating or terminating on a pole is proportional to its strength.

(iii) Magnetic lines of field can never intersect each other because if they intersect at a point, intensity at that point will have two directions which is absurd.

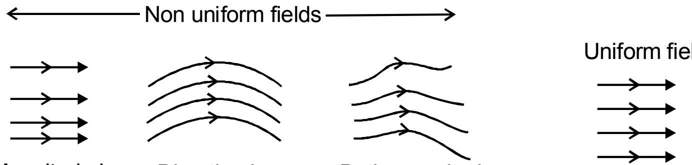
(iv) Magnetic lines of field have a tendency to contract longitudinally like a stretched elastic string (producing attraction between opposite poles) and repel each other laterally (resulting in repulsion between similar poles)



Attraction
(A)

Repulsion
(B)

(v) Number of lines of field per unit area, normal to the area at a point, represents the magnitude of field at that point. so crowded lines represent a strong field while distant lines represent weak field. Further, if the lines of force are equidistant and straight the field is uniform otherwise not



Magnitude is not constant (A)

Direction is not constant (B)

Both magnitude and direction are not constant (C)

Both magnitude and direction are constant (D)

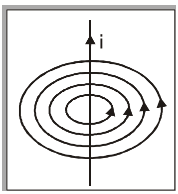
7. As monopoles do not exist, the total magnetic flux linked with a closed surface is always zero, i.e.,

$$\oint \vec{B}.\vec{ds} = \mu_0(0) = 0$$

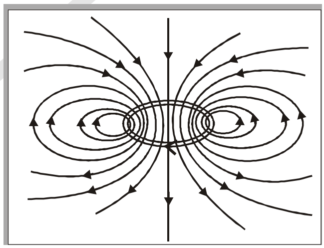
This law is called Gauss's law for magnetism.

8. ***MAGNETIC FIELD LINE DUE TO SOME IMPORTANT STRUCTURE***

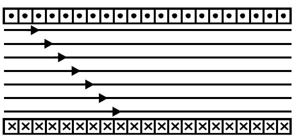
(a) Straight current carrying wire



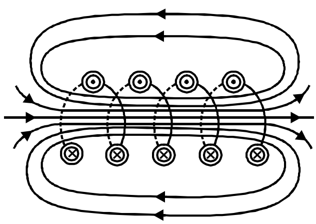
(b) Circular coil



(c) Solenoid



(ideal)



(Real)

9. MAGNETIC FIELD DUE TO A TOROID (ENDLESS SOLENOID)

Magntic field within the turns of toroid $B = \dfrac{\mu_0 N I}{2\pi r}$

where r is average radius.
Magnetic field outside the toroid is zero.

10. FORCE ON A CHARGED PARTICLE IN MAGNETIC FIELD

The force on a charged particle moving with velocity $\vec{v}$ in a uniform magnetic field $\vec{B}$ is given by $\vec{F}_m = q(\vec{v} \times \vec{B})$

The direction of this force is perpendicualr to both $\vec{v}$ and $\vec{B}$

When $\vec{v}$ is parallel to $\vec{B}$, then $\vec{F}_m = 0$

When $\vec{v}$ is perpendicular to $\vec{B}$,

then $\vec{F}_m$ is maximum, $(F_m) = qvB$.

11. FORCE ON A CHARGED PARTICLE IN SIMULTANEOUS ELECTRIC AND MAGNETIC FIELDS

The total force on a charged particle moving in simultaneous electric field $\vec{E}$ and magnetic field $\vec{B}$ is given by

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

This is called Lorentz force.

12. PATH OF CHARGED PARTICLE IN MAGNETIC FILED.

(i) If $\vec{v}$ is parallel to the direction of $\vec{B}$, then magnetic force = zero. So the path of particle is an undelfected straight line.

(ii) If $\vec{v}$ is perpendicular to $\vec{B}$, then magnetic field provides necessary centripetal force for **Circular path,** the radius r of path is given by

$$\frac{mv^2}{r} = qvB \qquad \Rightarrow \qquad r = \frac{mv}{qB}$$

If E is kinetic energy of particle $r = \dfrac{\sqrt{2mE}}{qB}$

If V is accelerating potential in volt, E = qV

$$\therefore \qquad r = \frac{\sqrt{2mE}}{qB} = \frac{l}{B}\sqrt{\frac{2mV}{q}}$$

Time period of path $$T = \frac{2\pi m}{qB}$$

(iii) If a particle's velocity $\vec{v}$ is oblique to magnetic field $\vec{B}$, then the particle follows a helical path of radius

$$r = \frac{mv\sin\theta}{qB} = \frac{mv_{\perp}}{qB}$$

Time period $T = \frac{2\pi m}{qB}$ and pitch $p = v'T = v\cos\theta\frac{2\pi m}{qB}$

where v' is a component of velocity parallel to the direction of magnetic field.

13. Simultaneous Electric and Magnetic Fields

If a particle of charge q enters in a region of simultaneous electric and magnetic field of strength $\vec{E}$ and $\vec{B}$ respectively. then the net force on charged particle is

$\vec{F} = \vec{F}_e + \vec{F}_m = q\vec{E} + q\vec{v} \times \vec{B}$ i.e., $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

14. Velocity Filter

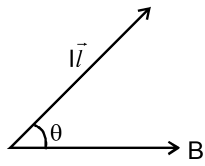
If electric and magnetic fields are mutually perpendicular and a charged particle enters this region with velocity $\vec{v}$ which is perpendicular to both electric and magnetic fields, then it may happen that the electric and magnetic forces are equal and opposite and charged particle with given velocity v remain undeflected in both fields. In such a codition.

$$qE = qvB \Rightarrow v = \frac{E}{B}$$

This arrangemnt is called velocity filter or velocity selector.

15. Magnetic Force on a Current Carrying Conductor or Length $\vec{l}$ is given by

$$\vec{F}_m = I\vec{l} \times \vec{B}$$



where the direction of $\vec{l}$ is along the direction of current.

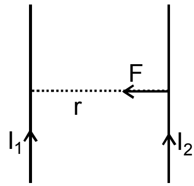
Magnitude of force

$F_m = IlB \sin\theta$

Direction of force $\vec{F}$ is normal to $\vec{l}$ and $\vec{B}$ given by rule of vector product. If $\theta = 0$ (i.e., $\vec{l}$ is parallel to $\vec{B}$), then magnetic force is zero.

16. FORCE BETWEEN PARALLEL CURRENT CARRYING CONDUCTORS

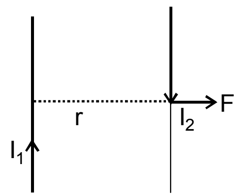
Parallel currents attract while antiparallel curents repel. The magnetic force per unit length on either current carrying conductor at separation 'r' given by



$$f = \frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r} \text{ newton/metre}$$

17. DEFINITION OF AMPERE IN S.I. SYSTEM

one ampere is the current which when flowing in each of the two parallel wires in vaccum at separation 1 m exert a force of



$\frac{\mu_0}{2\pi} = 2 \times 10^{-7}$ N/m on either wire.

18. TORQUE EXPERIENCED BY A CURRENT LOOP (OF AREA $\vec{A}$)

carrying current I in a uniform magnetic field $\vec{B}$ is given by

$$\vec{\tau} = NI\vec{A} \times \vec{B} = \vec{M} \times \vec{B}$$

where $\vec{M} = NI\vec{A}$ is magnetic moment of loop. The unit of magnetic moment in S.I. system is ampere × metre^2 (Am^2)

19. POTENTIAL ENERGY OF A CURRENT LOOP IN A MAGNETIC FIELD

When a current loop of magnetic moment M is placed in a magnetic field, then potenial energy of dipole is :

$$U = -\vec{M}.\vec{B} = -MB\cos\theta$$

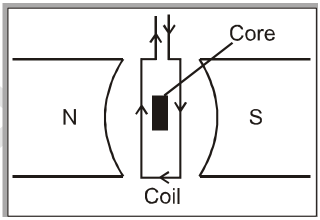
(i) When $\theta = 0$, U = – MB (minimum or stable equilibrium position)

(ii) When $\theta = \pi$, U = + MB (maximum or unstable equilibrium position)

(iii) When $\theta = \frac{\pi}{2}$, potential energy is zero.

20. MOVING COIL GALVANOMETER

A moving coil galvanometer consists of a rectangular coil plaed in a uniform magnetic field produced by cylindrical pole pieces to make the magnetic field radial. Torque on coil τ = N I AB where N is number of turns. A is area of coil. If k is torsional rigidity of material of suspension wire, then for deflection θ, torque $\tau = k\theta$



$\therefore$ For equilibrium

NIAB = k θ

$\Rightarrow$ $$\theta = \frac{NAB}{k} I$$

Clearly deflection in galvanometer is directly proportional to current, so the scale of galanometer is linear.

21. SENSITIVITY OF GALVANOMETER :

21.1 Current sensitivity :

It is defined as the deflection of coil per unit current flowing in it.

Sensitivity $$S_I = \left(\frac{\theta}{I}\right) = \frac{NAB}{k}$$

21.2 Voltage sensitivity :

It is defined on the deflection of coil per unit potenial difference across its ends

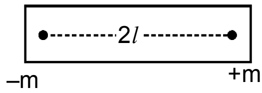
i.e., $$S_v = \frac{\theta}{V} = \frac{NAB}{kR}$$

where R is resistance of galvanometer.

Clearly for greater sensitivity number of turns N, area A and magnetic field strength B should be large and torsional rigidity k of suspension should be small.

22. MAGNETIC DIPOLE MOMENT OF A CURRENT LOOP AND REVOLVING ELECTRON

Magnetic dipole moment of a magnet, M = m 2l amp-m^2 where m is pole strength, 2l is separation between poles. Magnetic dipole moment of a current loop.



M = N I A amp-m^2

Magnetic dipole moment of revolving electron = I A = fe πr^2

where f is frequency, r = radius of orbit

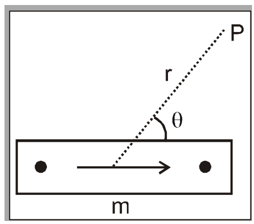
$$M = \frac{e}{2m_e} L \, amp - m^2$$

23. Magnetic field intensity due to a magnetic dipole

Magnetic field intensity at a general point having polar coordinates (r, θ) due to a short magnet is given by

$$B = \frac{\mu_0}{4\pi}\frac{M}{r^3}\sqrt{1+3\cos^2\theta}$$

Special Cases



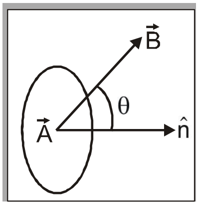
(i) At axial point θ = 0, $B_{axis} = \frac{\mu_0}{4\pi}\frac{2M}{r^3}$

(ii) At equation point θ = 90° $B_{eqn.} = \frac{\mu_0}{4\pi}\frac{M}{r^3}$

E.M.I.

1. MAGNETIC FLUX

Consider a closed curve enclosing an area A (as shown in the figure). Let there be a uniform magnetic field $\vec{B}$ in that region. The magnetic flux through the area $\vec{A}$ is given by



$$\phi = \int \vec{B}.d\vec{A} \Rightarrow \phi = \vec{B}.\vec{A} = BA\cos\theta$$

Unit $\rightarrow$ weber (Wb).

2. FARADAY'S LAW OF ELECTROMAGNETIC INDUCTION

Whenever the flux of magnetic field through the area bounded by a closed conducting loop changes, an emf is produced in the loop. The emf is given by

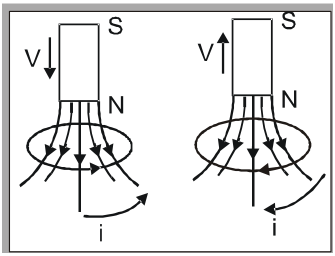
$$\varepsilon = -\frac{d\phi}{dt} \qquad \text{where} \quad \phi = \int \vec{B}.d\vec{A}$$

the flux of magnetic field through the area. If resistance of loop is R then

$$i = \frac{\varepsilon}{R} = -\frac{1}{R}\frac{d\phi}{dt}$$

3. LENZ'S LAW

According to lenz's law of the flux associated with any loop changes than the induced current flows in such a fashion that it tries to oppoose the cause which has produced it.



4. Induced emf generates :

(i) By changing the magnetic field

(ii) By changing the Area

(iii) By changing the angle between Area vector & magnetic field.

5. Motional Emf

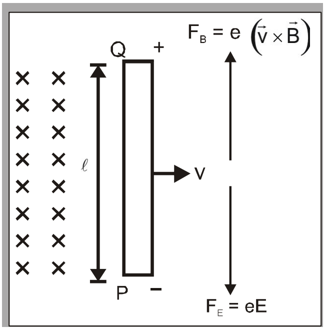
at equilibrium : -

$$|q(\vec{v}\times\vec{B})| = |q\vec{E}|$$

$$\vec{E} = \vec{v}\times\vec{B}$$

$$\int dE = -\vec{E}.d\vec{l}$$

$$E = \int(\vec{v}\times\vec{B}).d\vec{l}$$



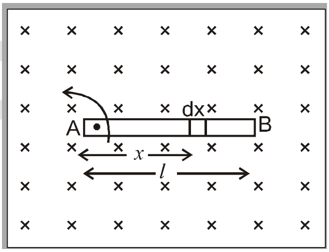
So emf developed across the ends of the rod moving $\perp$ to magnetic field with velocity $\perp$ to the rod is

$$\varepsilon = vB\ell$$

6. Induced emf in a rotating rod :

$$\int dE = \int_0^{\ell} B\omega x dx$$

$$V_A - V_B = \frac{B\omega l^2}{2}$$



7. INDUCED ELECTRIC FIELD

This electric field is produced by the changing magnetic field and not by charged particles. The electric field produced by the changing magnetic field is nonelectrostatic and nonconservative in nature. We cannot define a potential corresponding to this field. We call it induced electric field. The lines of induced electric field are closed curves. There are no starting and terminating points of the field lines.

If $\vec{E}$ be the induced electric field, the force on the charge q placed in the field of $q\vec{E}$. The work done per unit charge as the charge moves through $d\vec{l}$ is E.

$d\vec{l}$. The emf developed in the loop is, therefore,

$$\varepsilon = \oint \vec{E}.d\vec{l}$$

Using Faraday's law of induction,

$$\varepsilon = -\frac{d\phi}{dt}$$

or, $$\oint \vec{E}.d\vec{l} = -\frac{d\phi}{dt} \qquad ...(6)$$

The presence of a conducting loop is not necessary to have an induced electric field. As long as $\vec{B}$ keeps changing, the induced electric field is present. If a loop is there, the free electrons start drifting and consequently an induced current results.

8. SELF INDUCTION :

If current in the coil changes by ΔI in a time interval Δt, the average emf induced in the coil is given as

$$\varepsilon = -\frac{\Delta(N\phi)}{\Delta t} = -\frac{\Delta(LI)}{\Delta t} = -\frac{L\Delta I}{\Delta t}$$

The instantaneous emf is given as

$$\varepsilon = -\frac{d(N\phi)}{dt} = -\frac{d(LI)}{dt} = -\frac{LdI}{dt}$$

S.I unit of inductance is wb/amp or Henry (H)

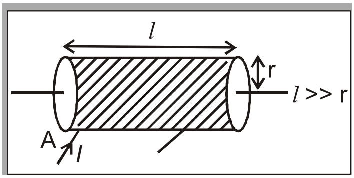
L - self inductance is +ve quantity.

L depends on :

(1) Geometry of loop

(2) Medium in which it is kept. L does not depend upon current. L is a scalar quantity.

8.1 Self Inductance of solenoid



$L = \mu_0 n^2 \pi r^2 \ell$

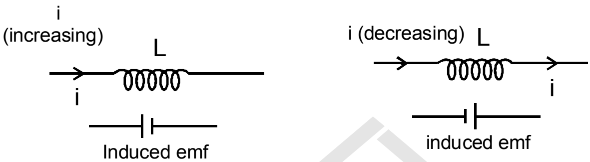
n = no. of turns/length

r = radius

l = length

Inductance/volume = $\mu_0 n^2$

9. INDUCTOR

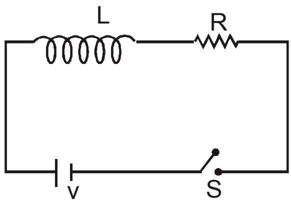


Over all result

$V_A - L\dfrac{di}{dt} = V_B$ $\qquad L\dfrac{di}{dt}$

Energy stored in inductor $\quad U = \dfrac{1}{2}Li^2$

10. L – R CIRCUIT



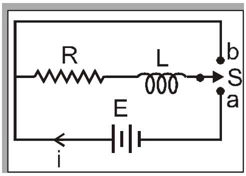
at t = 0 inductor behaves as open switch.

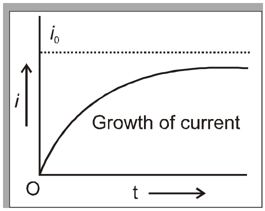
at t = ∞ inductor behaves as plane wire.

10.1 Growth of Current

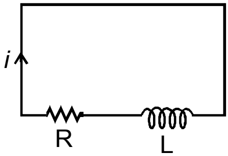
The maximum current in the circuit i_0 = E/R.

So $\qquad i = i_0 \left\{1 - \exp\left(-\dfrac{R}{L}t\right)\right\}$

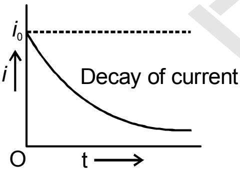




10.2 Decay of Current



or $i = i_0 \exp\left(-\frac{R}{L}t\right) = i_0 \quad \exp\left(-\frac{t}{\tau}\right)$



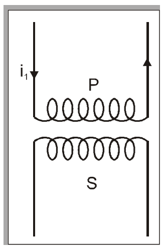
11. MUTUAL INDUCTANCE

Let the current through the primary coil at any instant be i_1. Then the magnetic flux ϕ_2 in the secondary at any time will

be proportional to i_1 i.e., $\phi_2 \propto i_1$

Therefore the induced e.m.f. in secondary

when i_1 changes is given by



$$\varepsilon = -\frac{d\phi_2}{dt} \text{ i.e., } \varepsilon \propto -\frac{di_1}{dt}$$

$$\therefore \quad \varepsilon = -M\frac{di_1}{dt} = -\frac{dMi_1}{dt} \Rightarrow \phi_2 = M\, i_1$$

M = Mutual inductance

Unit of Mutual inductance is Henry (H)

12. COMBINATION OF INDUCTORS

(i) Series combination

$\therefore\ V = V_1 + V_2$

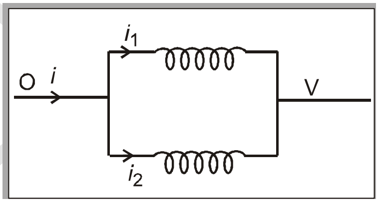
$$L_{eq}\frac{di}{dt} = L_1\frac{di}{dt} + L_2\frac{di}{dt}$$

$L_{eq} = L_1 + L_2 + ...$

Parallel Combination of inductor

$i = i_1 + i_2$

$$\Rightarrow \qquad \frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$$

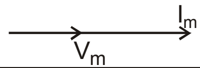
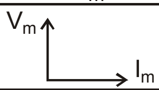
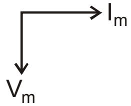


$$\frac{v}{L_{eq}} = \frac{v}{L_1} + \frac{v}{L_2}$$

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + ...$$

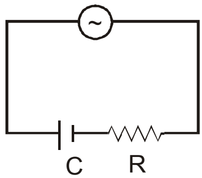
ALTERNATING CURRENT

1. Summary

AC source connected with	ϕ		Z	Phasor Diagram
Pure Resistor	0	V_R is in same phase with i_R	R	
Pure Inductor	$\pi/2$	V_L leads i_L	X_L	
Pure Capacitor	$\pi/2$	V_C leads i_C	X_C	

2. RC series circuit with an ac source :

$v_s = v_m \sin\omega t$



Let $\quad i = I_m \sin(\omega t + \phi)$

$\Rightarrow \quad V_R = iR = I_m R \sin(\omega t + \phi)$

$$V_C = I_m X_C \sin\left(\omega t + \phi - \frac{\pi}{2}\right) \Rightarrow V_S = V_R + V_C$$

or

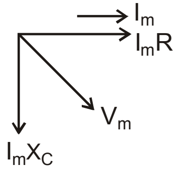
$$V_m \sin(\omega t + \varphi) = I_m R(\omega t + \varphi) + I_m X_C\left(\omega t + \varphi - \frac{\pi}{2}\right)$$

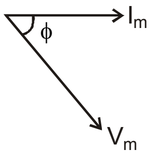
$$v_m = \sqrt{(I_m R)^2 + (I_m X_c)^2 + 2(I_m R)(I_m X_c)\cos\frac{\pi}{2}}$$

or

$$I_m = -\frac{V_m}{\sqrt{R^2 + X_C^2}} \Rightarrow Z = \sqrt{R^2 + X_c^2}$$

Using phasor diagram also we can find the above result.

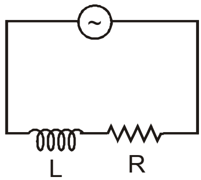




$$\tan\phi = \frac{I_mX_c}{I_mR} = \frac{X_c}{R}$$

3. LR SERIES CIRCUIT WITH AN AC SOURCE

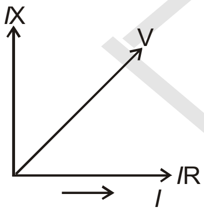
$v_s = v\sin\omega t$

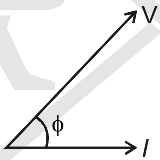


From the phasor diagram

$$V = \sqrt{(IR)^2 + (IX_L)^2} = I\sqrt{(R)^2 + (X_L)^2} = IZ$$

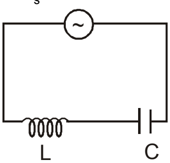
$$\tan\varphi = \frac{IX_L}{IR} = \frac{X_L}{R}$$

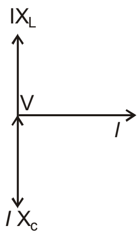


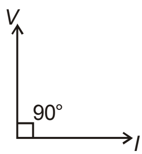


4. LC SERIES CIRCUIT WITH AN AC SOURCE

$v_s = v\sin\omega t$



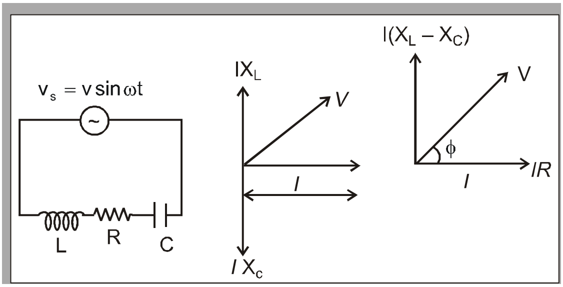




From the phasor diagram

$$V = I|(X_L - X_C)| = IZ \qquad \varphi = 90^\circ$$

5. RLC SERIES CIRCUIT WITH AN AC SOURCE



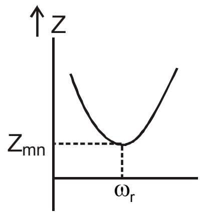
From the phasor diagram

$$V = \sqrt{(IR)^2 + (IX_L - IX_C)^2} = I\sqrt{(R)^2 + (X_L - X_C)^2} = IZ$$

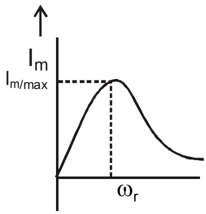
$$Z = \sqrt{(R)^2 + (X_L - X_C)^2}$$

$$\tan\varphi = \frac{I(X_L - X_C)}{IR} = \frac{(X_L - X_C)}{R}$$

6. RESONANCE



Amplitude of current (and therefore I_{rms} also) in an RLC series circuit is maximum for a given value of V_m and R, if the impedance of the circuit is minimum, which will be when $X_L - X_C = 0$. This condition is called resonance.



So at resonance :

$X_L - X_C = 0$

or $\qquad \omega L = \dfrac{1}{\omega C}$

or $\qquad \omega = \dfrac{1}{\sqrt{LC}}$

PART-VI
MODERN PHYSICS

MODERN PHYSICS

De Broglie Waves

$$\lambda = \frac{h}{P} = \frac{h}{m\nu}$$

Photoelectric Effect

(a) Photons

Electromagnetic wave energy is emitted and absorbed in package of definite size called photons. The energy E of photon is proportional to wave frequency f.

i.e. $E \propto f$ or $E = hf = \frac{hc}{\lambda}$

Where h is called the Plank's constant

($h = 6.63 \times 10^{-34}$ Js)

c is speed of light in vacuum, and

λ is the wavelength of light in vacuum.

The expression can be simplifed in the following form for practical applications.

$$E(eV) = \frac{12400}{\lambda(\text{Å})}$$

The momentum p of a photon of energy E is given by $p = \frac{E}{c}$ where c is the speed of light. The direction of momentum is the direction of propagation of light.

(b) Einstein's photoelectric equation

The maximum kinetic energy of the photoelectron is given by

$$\frac{1}{2}mv_{max}^2 = hf - W$$

Where f is the frequency of the incident photon, and

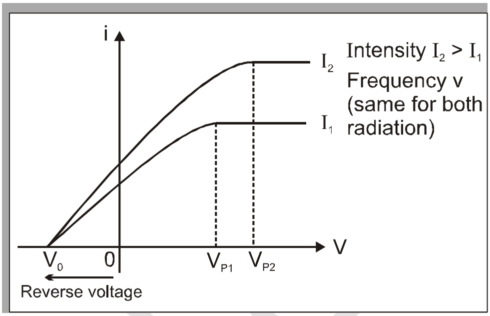
W is the work function of metal surface.

$$W = hf_0$$

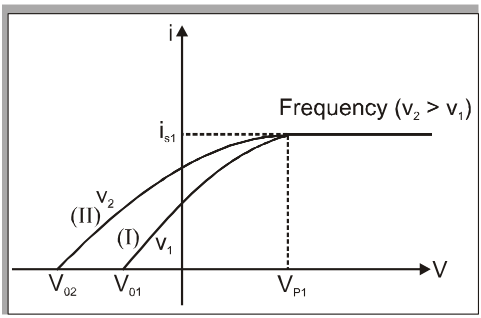
where f_0 is the threshold frequency.

(i) Photocurrent i is directly proportional to intensity of incident radiation and there is no threashold intensity provided the frequency is greater than the threshold frequency.

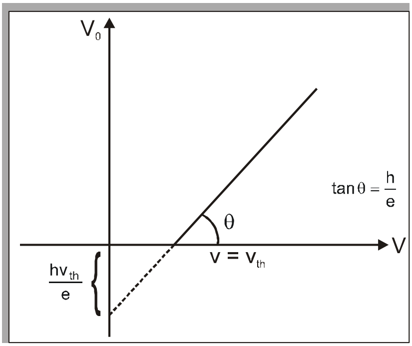
(ii) The figure shows variaton of potential V of the anode with respect to the cathode for a constant light frequency. The stopping potential V_s is independent of light intensity I.



(iii) The figure shows the variaton of photocurrent i as a function of the potential V of the anode with respect to cathode for two different light frequencies f_1 and f_2 (> f_1) with the same intensity. The stopping potential increases linearly with frequency.



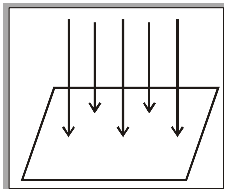
(iv) The figure shows that the maximum kinetic energy $\frac{1}{2}mv_{max}^2$ of ejected electron is directly proportional to the frequency f of the incident light if it is greater than the threshold frequency.



$$\frac{1}{2}mv_{max}^2 = eV_0 = h(v - v_{th})$$

- **FORCE DUE TO RADIATION (PHOTON)**

(A) Case (I) :



$a = 1, \quad r = 0$

pressure = $\frac{F}{A} = \frac{IA}{cA} = \frac{I}{c}$

Case : (II)

when r = 1, a = 0

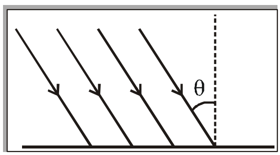
pressure $P = \frac{F}{A} = \frac{2IA}{cA} = \frac{2I}{c}$

Case : (III)

When 0 < r < 1 $\quad$ a + r = 1

Now pressure P $= \frac{IA}{c}(1+r) \times \frac{1}{A} = \frac{I}{c}(1+r)$

(B) Case - I a = 1, r = 0



Pressure = $\frac{F\cos\theta}{A}$ $P = \frac{IA\cos^2\theta}{cA} = \frac{I}{c}\cos^2\theta$

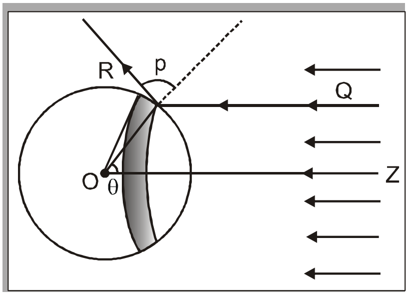
Case II When r = 1, a = 0

Pressure $= \frac{2IA\cos^2\theta}{cA}$, $P = \frac{2I\cos^2\theta}{c}$

Case III $0 < r < 1, \quad a + r = 1$

Pressure

$$= \frac{I\cos^2\theta}{c}(1-r) + \frac{I\cos^2\theta}{c}2r = \frac{I\cos^2\theta}{c}(1+r)$$



(C) Pressure = $\frac{\pi r^2 I}{c}$

Note that integration is done only for the hemisphere that faces the incident beam

ATOMIC LINE SPECTRA

In 1885, Balmer discovered a formula for the wavelengths of certain spectral lines observed coming from excited hydrogen.

$$\frac{I}{\lambda} = R_\infty\left[\frac{1}{2^2} - \frac{1}{n^2}\right] \text{ for } n = 3, 4, 5 \,.......$$

where, $R_\infty = 1.097 \times 10^7\ m^{-1}$ is called the **Rydberg constant.** Later on, other similar formulae were also discovered for hydrogen atoms. These are

Lyman Series,

$$\frac{I}{\lambda} = R_\infty\left[\frac{1}{1^2} - \frac{1}{n^2}\right] \text{ for } n = 2, 3, 4 \,........$$

Paschen Series

$$\frac{I}{\lambda} = R_\infty\left[\frac{1}{3^2} - \frac{1}{n^2}\right] \text{ for } n = 4, 5, 6 \,........$$

Brackett Series

$$\frac{I}{\lambda} = R_\infty\left[\frac{1}{4^2} - \frac{1}{n^2}\right] \text{ for } n = 5, 6, 7 \,.......$$

The Bohr Model

(a) The electrons move in circular orbits and the centripetal force is provided by the Coulomb force of attraction between then nucleus and electron.

(b) Only certain orbits are allowed in which the angular momentum of the electron is an integral multiple of $\frac{h}{2\pi}$

that is, $mvr = \frac{nh}{2\pi}$ where n = 1, 2, 3, ...

(c) Energy is emitted or absorbed when the electron jumps from one level to another. For a transition from a higher orbit to a lower orbit, the photon is emitted and the energy difference is hf, where f is the frequency of the photon.

(d) Photon absorption causes a transition of the electron to a higher orbit but only if the enegy and frequency are just appropriate.

Bohr's Formulae

(a) The radius of the nth orbit is defined as

$$r_n = 0.53\frac{n^2}{Z}\text{Å}$$

(b) The speed of electron in n th orbit is

$$v_n = \frac{Z}{n}\left(\frac{c}{137}\right)$$

Where c = 3 × 10^8 ms^{-1} is the speed of light.

or $$v_n = \frac{2.19\times10^6 Z}{n} ms^{-1}$$

The speed of electron is independent of its mass.

(c) The total energy of an atom is the sum total of kinetic and potential. If the electron is in the n th shell, then

$$E = -13.6\frac{Z^2}{n^2}(eV) \quad \text{or} \quad E = K + U = -K = \frac{U}{2}$$

and the total energy is directly proportional to the mass of the electron.

If the potential energy is assumed to be zero at n = 1, then the total energy of the atom will be

$$E = -\frac{13.6Z^2}{n^2} + 27.2Z^2 \qquad \text{or} \qquad E = \left(2 - \frac{1}{n^2}\right)13.6Z^2$$

(d) The wavelenth of photon emitted when electron jumps from n = n_2 to n = n_1 is given by

$$\frac{1}{\lambda} = \frac{\Delta E}{hc} = \frac{E_2 - E_1}{hc} = \frac{13.6Z^2}{hc}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

or $$\frac{1}{\lambda} = R_\infty Z^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

If the nucleus of atom is not considered to be stationary, then equations for radius energy and wavelength may be modified as

$$r_n = 0.53\frac{n^2}{Z}\left(1 + \frac{m}{M}\right)\text{Å}$$

$$E_n = -13.6\frac{Z^2}{n^2}\left(\frac{M}{m+M}\right)\text{eV}$$

$$\frac{1}{\lambda} = \frac{R_\infty}{1 + \frac{m}{M}}Z^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]m^{-1}$$

where m is the mass of electron, and M is the mass of nucleus.

Note : (i) Total number of emission lines from some excited state n_1 to another energy state

n_2 (< n_1) is given by $\frac{(n_1 - n_2)(n_1 - n_2 + 1)}{2}$

For example total number of lines from n_1 = n to n_2 = 1 are $\frac{n(n-1)}{2}$

(ii) As the principal quantum number n is increased in hydrogen and hydrogen like atoms, some quantities are decreased and some are increased. The table given below show which quantities are increased and which are decreased.

Increased	Decreased
Radius	Speed
Potential energy	Kinetic energy
Total energy	Angualr speed
Time period	
Angular momentum	

X-Rays

X-rays are highly penetrating electromagnetic radiations of wavelength of the order of 1 Å

That is, $\lambda_{min} = \frac{hc}{1/2mv^2} = \frac{hc}{eV}$

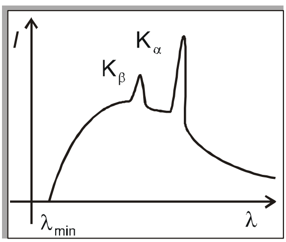
The cut-off wavelength is independent of the target material. It depends only on the kinetic energy of the electron beam.

The characteristic spectrum depends only on the target material, and it is independent of the kinetic energy of the electron beam.

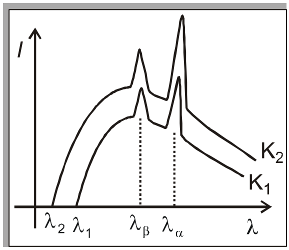
The figure shows the variation of intensity I of X-rays as a function of their wavelength λ. The graphs shows a minimum wavelength λ_{min} (called out-off wavelength) but there is no maximum wavelength. The graph shows two basic features.

(a) a continuous curve with λ_{min} shows continuous spectrum of X-rays

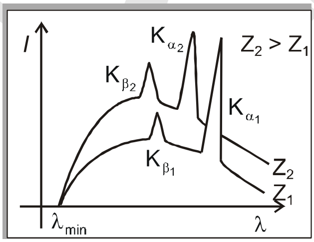
(b) two sharp peaks of high intensity show characteristic spectrum.



(c) Variation of *I* versus λ the kinetic energy of the electron beam is increased from K_1 to K_2 keeping the same target. It is observed that cutoff wavelength of continuous spectrum decreases ($\lambda_2 < \lambda_1$) but the characteristic spectrum remains unchanged.



(d) Variation of *I* versus λ The target material is changed ($Z_2 > Z_1$) but the kinetic energy of the electron beam remains unchanged. It is observed that the cut-off wavelength remains same but the characteristic spectrum is the unique property of tha target material.



Moseley's Law

$$\sqrt{f} = a(Z - b)$$

$$\frac{1}{\lambda} = R_\infty (Z-1)^2 \left[1 - \frac{1}{n^2}\right]$$

n = 2, 3, 4 ....

Radioactivity

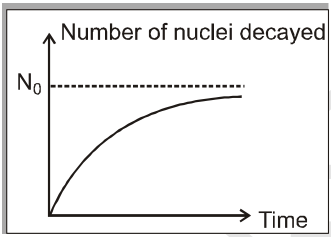
(a) After n half lives,

(i) number of nuclei left = $N_0\left(\frac{1}{2}\right)^n$

(ii) fraction of nuclei left = $\left(\frac{1}{2}\right)^n$ and

(iii) percentage of nuclei left = $100\left(\frac{1}{2}\right)^n$

(b)



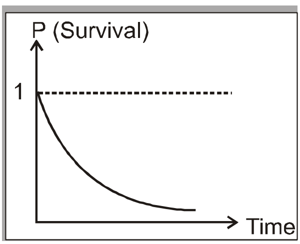
Number of nuclei decayed after time t,

$= N_0 - N$

$= N_0 - N_0\, e^{-\lambda t} = N_0\,(1 - e^{-\lambda t})$

The corresponding graphs is as shwn in figure.

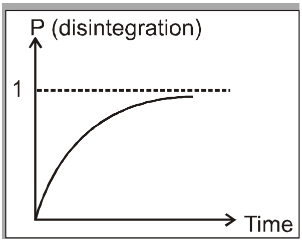
(c)



Probability of a nucleus for survival after time t,

$$P\,(\text{survival}) = \frac{N}{N_0} = \frac{N_0 e^{-\lambda t}}{N_0} = e^{-\lambda t}$$

The corresponding graph is shown in figure.

(d) 

Probability of a nucleus to disintegrate in time t is,

$$P\,(\text{disintegration}) = 1 - P\,(\text{survival}) = 1 - e^{-\lambda t}$$

The corresponding graph is as shown .

(e) Half life and mean life are related to each other by the relation.

$$t_{1/2} = 0.693\ t_{av} \quad \text{or} \quad t_{av} = 1.44\ t_{t/2}$$

(f) As we said in point number (2), Number of nuclei decayed in time t are $N_0\,(1 - e^{-\lambda t})$ So, to avoid it we can use.

$$\Delta N = \frac{\lambda N}{\Delta t}$$

where, ΔN are the number of nuclei decayed in time Δt, at the instant when total number of nuclei are N. But this can be applied only when $\Delta t << t_{1/2}$.

(g) In same interval of time, equal percentage (or fraction) of nuclei are decayed (or left undecayed).